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Solving Linear Fractional Programming Problems With Triangular L-R Fuzzy Numbers Coefficients

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ABSTRACT

In this paper, in real-life situations, the parameters of the linear programming problem model may not be defined precisely, because of the globalization of the market, uncontrollable factors, …etc. For this reason, it was presented an algorithm for solving fuzzy linear fractional programming (FLFP) problems, where coefficients of the objective function and constraints are triangular L-R fuzzy numbers. The FLFP problem can be reduced to a linear fractional programming problem using the ranking function for all triangular L-R fuzzy numbers coefficients, and then using the variable transformation method to obtain an optimal solution with optimum fuzzy objective function. This enables us to obtain many proposed solutions instead of a unique solution, which enables the decision-maker to make the best decisions. A numerical example is given for the sake of illustration.

حل مشاكل البرمجة الخطية الكسرية بمعاملات ذات أعداد ضبابية ثلاثية L-R

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الكلمات المفتاحية:
البرمجة الخطية الكسرية
الحل الامثل
العدد الضبابي الثلاثي *L-R*
دالة الترتيب
طريقة تحويل المتغير

الملخص

في مواقف الحياة الواقيعية ، نواجه غالبا حالة من عدم اليقين نتيجة لعدم القدرة على تحديد معاملات النموذج وكذلك متغيرات القرار بدقة وذلك بسبب عولمة السوق، والعوامل التي لا يمكن السيطرة عليها، ... الخ. ولهذا السبب ولعدم دقة البيانات تم تقديم خوارزمية لحل مسائل البرمجة الخطية الكسرية الضبابية (*FLFP*) ، حيث تكون معاملات دالة الهدف والقيود عبارة عن أرقام ضبابية ثلاثية *L-R* . يمكن اختزال مشكلة *FLFP* إلى مشكلة برمجة كسرية خطية وذلك باستخدام دالة الترتيب لجميع معاملات الأعداد الثلاثية *L-R*، ومن ثم استخدام طريقة تحويل المتغير للحصول على الحل الأمثل مع دالة الهدف الضبابة المثلى. وهذا يمكننا من الحصول على العديد من الحلول المقترحة بدلا من حل وحيد. مما يمكن متخذ القرار من أختيار أفضل الحلول و اتخاذ القرارات المناسبة. وكذلك قدمت امثلة عددية لتوضيح هذه الخوارزمية.

1. Introduction

We need linear fractional programming in many real-world problems such as production planning, work distribution in hospitals and in industry, where decision-makers face problems in making decisions that optimize various parameters such as the profit-to-cost ratio, the number of patients to the number of hospitals, as well as the quantity Inventory to sales quantity, where the objective function is a ratio between two linear functions.

Jayalakshmi and Pandian [1] proposed a new method namely, the denominator objective restriction method for finding an optimal solution to linear fractional programming problems.. Jian Abdul Alim, et al [2] proposed and described the addition, subtraction, multiplication, and division of two L-R fuzzy numbers Zhou, et al [3] proposed a specific type of LR fuzzy number involving the triangular fuzzy number. Ammar and Emsimir [4], A mathematical model for

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solving fuzzy integer linear programming problems with fully rough intervals. Iden and Rasha [5] employed two ranking functions for fuzzy variables to solve multi-objective programming problems. Muamer and Eljerbi, [6] proposed a new representation of Triangular Fuzzy Rough Numbers and defined its rank function. Chauhan et al [8] provides an α -cut-based method that solves linear fractional programming problems with fuzzy variables and unrestricted parameters. Sheikhi and Ebadi [9] present a novel method for solving Fractional Transportation Problems (FTPs) with fuzzy numbers using a ranking function. The proposed method introduces a transformation technique that converts an FTP with fuzzy numbers into an FTP with crisp numbers by employing the robust ranking technique. Bas and Ozkok [10] simplified the intricate structure of the FFLFrLPP into a crisp Linear Programming Problem (LPP) while accommodating the inherent fuzziness. Notably, unlike literature, the proposed technique avoided variable transformation, which is highly competitive in addressing fuzzy-based problems.

In this paper, we propose an algorithm to solve linear fractional programming problems with triangular L-R fuzzy numbers coefficients. we use the ranking function for all coefficients to obtain linear fractional programming problem, for solving this problem based on the variable transformation method.

2. Preliminaries

In this section, the definition of fuzzy numbers and basic operations for triangular $L-R$ fuzzy numbers are given. For more details see [3,5,6].

Definition 1. A fuzzy number is described as any fuzzy set with membership function $\mu_A(x): \mathbb{R} \longrightarrow [0,1]$ which satisfies

1. $\mu_A(x)$ is upper semi-continuous,
2. $\mu_A(x) = 0$ outside the interval $[a, d]$,
3. There exist are real numbers b, c such that $a \leq b \leq c \leq d$, and
   i. $\mu_A(x)$ is increasing on $[a, b]$,
   ii. $\mu_A(x)$ is decreasing on $[c, d]$,
   iii. $\mu_A(x) = 1$, $b \leq x \leq c$

The set of all these fuzzy numbers is denoted by $F(\mathbb{R})$. we now present some equivalent parametric forms.

Definition 2. For a fuzzy number $\tilde{A}$, we say that its type $L-R$ if there exist two functions L (for left) and R (for right), and two scalers $p > 0,\ q > 0$ with membership function

$$\mu_A(x) = \begin{cases} L\left(\dfrac{m-x}{p}\right) & x \leq m \\ R\left(\dfrac{x-m}{q}\right) & x \geq m \\ 0 & otherwis \end{cases}$$

Where the real number m is called core (peak) of $\tilde{A}$ and p , q are called left and right spreads respectively. Symbolically, $\tilde{A}$ is denoted by $\tilde{A} = (m, p, q)_{LR}$.

Example 1. Suppose $L(x) = max\{0,\ 1-x\}$ and $R(x) = e^{-2x}$, $p = 3$, $q = 4$, $m = 5$ Then $\tilde{A} = (5, 3, 4)_{LR}$ denotes an L-R fuzzy number with membership function

$$\mu_A(x) = \begin{cases} \dfrac{x-2}{3} & x \leq 5 \\ e^{(5-x)/2} & x \geq 5 \\ 0 & otherwis \end{cases}$$

Remark: If $L(x)$ and $R(x)$ are both linear functions on the domains $\{x: 0 \leq L(x) \leq 1\}$ and $\{x: 0 \leq R(x) \leq 1\}$, the corresponding L-R fuzzy number is a triangular fuzzy number.

Definition 3. A triangular fuzzy number $\tilde{A}$ determined by the (a, m, b) of real numbers with $a < m < b$ has the membership function

$$\mu_A(x) = \begin{cases} \dfrac{x-a}{m-a} & a \leq x \leq m \\ \dfrac{x-b}{m-b} & m \leq x \leq b \\ 0 & otherwis \end{cases}$$

the class of all triangular fuzzy number is called Triangular fuzzy number space, which is denoted by TF(N).

Definition 4. A triangular fuzzy number $\tilde{A} = (a, m, b)$ is said to be triangular $L-R$ fuzzy number if its membership function given by

$$\mu_A(x) = \begin{cases} L\left(\dfrac{m-x}{m-a}\right) & a \leq x \leq m \\ R\left(\dfrac{x-m}{b-m}\right) & m \leq x \leq b \\ 0 & otherwis \end{cases}$$

which can be denoted as $(m, p, q)_{LR}$ such that $p = m - a, q = b - m$ where the linear functions $L(x)$ and $R(x)$ defined as:

$$L(x) = R(x) = max\{0, 1-x\}$$

Also, the membership function of triangular $L-R$ fuzzy number is shown in Figure 1.

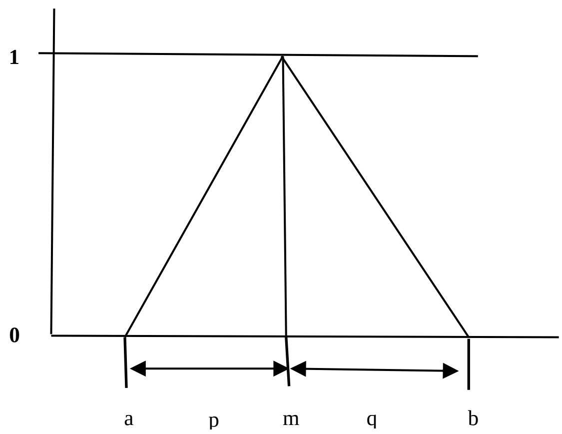


Fig-1: membership function **of** triangular $L-R$ fuzzy number

Definition 5. For any triangular $L-R$ fuzzy number $\tilde{A} = (m, p, q)_{LR}$, for all $\alpha \in [0,1]$ we get a crisp interval by $\alpha - leval$ operation defined as:

$$\tilde{A}_\alpha = [A_\alpha^L, A_\alpha^U] = [m + (\alpha - 1)p, m + (1-\alpha)q]$$

Definition 6. A triangular $L-R$ fuzzy number $\tilde{A}=(m\,,p,q)_{LR}$ is said to be positive if and only if $m-\ p\geq 0$, Noted that $\tilde{0}$ is equivalent to (0,0,0).

Example 2. A triangular fuzzy number $\tilde{A}=(3,5,8)$ can be written as $(5\ ,\ 2\ ,\ 3)_{LR}$ has the membership function

$$\mu_A(x)=\begin{cases}1-\dfrac{5-x}{2} & 3\leq x\leq 5\\ 1-\dfrac{x-5}{3} & 5\leq x\leq 8\\ 0 & otherwis\end{cases},$$

$$\tilde{A}_\alpha=[5+2(\alpha-1)\,,5+3(1-\alpha)]$$

2.1. Operations on triangular $L-R$ fuzzy number:

Suppose two nonnegative triangular $L-R$ fuzzy numbers $\tilde{A}_1\ and\ \tilde{A}_2$ defined as $\tilde{A}_1=(m_1\,,p_1,q_1)_{LR}\ and\ \tilde{A}_2=(m_2\,,p_2,q_2)_{LR}$, then

i. $\tilde{A}_1\oplus\tilde{A}_2=(m_1+m_2\,,p_1+p_2\,,q_1+q_2\,)_{LR}$

ii. $\tilde{A}_1\ominus\tilde{A}_2=(m_1-m_2\,,p_1-q_2\,,q_1-p_2\,)_{LR}$

iii. $\tilde{A}_1\otimes\tilde{A}_2=(m_1.m_2\,,m_1p_2+m_2p_1-p_1p_2\,,m_1q_2+m_2q_1+q_1q_2\,)_{LR}$

iv. $\frac{\tilde{A}_1}{\tilde{A}_2}=\ \tilde{A}_1\otimes(\tilde{A}_2)^{-1}=(m_1\,,p_1,q_1)_{LR}\otimes\ (m_2\,,p_2,q_2)_{LR}^{-1}$ Such that

$$(m_2\,,p_2,q_2)_{LR}^{-1}=(\frac{1}{m_2}\,,\frac{q_2}{m_2(m_2+q_2)}\,,\frac{p_2}{m_2(m_2-p_2)})$$

2.2. Ranking function

The ranking function is used to comparing of triangular $L-R$ fuzzy numbers, A ranking function is a map from TF(N) into real line defined as

$R:TF(N)\longrightarrow\mathbb{R}$. There are many ranking functions, we will review some of them.

Definition 7. let $\tilde{A}_1=(m_1\,,p_1,q_1)_{LR}$ be triangular $L-R$ fuzzy number, then their mid-points are $M(\tilde{A}_{1\alpha})=\frac{1}{2}[2m_1+(\alpha-1)\,p_1+(1-\alpha)q_1]$

Definition 8. let $\tilde{A}_1=(m_1\,,p_1,q_1)_{LR},\ \tilde{A}_2=(m_2\,,p_2,q_2)_{LR}$ are two triangular $L-R$ fuzzy numbers, then the distance of $\tilde{A}_1\ and\ \tilde{A}_2$ defined as

$$\begin{cases}d(\tilde{A}_1,\tilde{A}_2)=\displaystyle\int_0^1[\ M(\tilde{A}_{1\alpha})-M(\tilde{A}_{2\alpha})]d\alpha\\ \qquad=(m_1-m_2)+\dfrac{(q_1-q_2)-(p_1-p_2)}{4}\end{cases}$$

Definition 9. The signed distance of triangular $L-R$ fuzzy number $\tilde{A}=(m\,,p,q)_{LR}$ defined as $d(\tilde{A},\tilde{0})=m+\frac{q-p}{4}$.

Definition 10. A ranking function of triangular $L-R$ fuzzy number $\tilde{A}=(m,p,q)_{LR}$ defined as $\mathcal{R}(\tilde{A})=\frac{1}{2}\int_0^\alpha\ (A_\alpha^L+A_\alpha^L)\,d\alpha$

$$for\ \alpha=1\,,\ \ \mathcal{R}(\tilde{A})=m+\frac{q-p}{4}$$

Let $\tilde{A}$ $and\ \tilde{B}$ be two triangular $L-R$ fuzzy numbers, then

i. $\tilde{A}\lesseqgtr\tilde{B}\quad iff\quad \mathcal{R}(\tilde{A})<\mathcal{R}(\tilde{B})$

ii. $\tilde{A}\gtreqless\tilde{B}\quad iff\ \ \mathcal{R}(\tilde{A})>\mathcal{R}(\tilde{B})$

iii. $\tilde{A}\ \cong\ \tilde{B}\quad iff\ \ \mathcal{R}(\tilde{A})=\mathcal{R}(\tilde{B})$

2.3 Linear Fractional Programming Problem

The general linear fractional programming (LFP) problem is defined as follows:[7]

$$\begin{cases}Max\ f=\dfrac{p(x)}{q(x)}=\dfrac{c^T\mathrm{x}\ +\ \alpha}{d^T\mathrm{x}\ +\ \beta}\\ s.t\ \ x\in S=\{\,\mathrm{x}\in R^n:Ax\leq B\,,\ \ x\geq 0\,\}\end{cases},\qquad(1)$$

Where $c^T,d^T\in R^n\,,\alpha\,,\beta\ \in R\,,A=(a_{ij})_{m\times n},B=(b_i)_{m\times 1}$

$$and\ d^Tx\ +\ \beta>0$$

2.4. Variable Transformation Method

For solving linear fractional programming, usual assume that the denominator is positive everywhere in S and using variable transformation by assumption

$$w=\frac{1}{d^T\mathrm{x}\ +\ \beta}$$

Thus, we obtain the objective function as follows

$\sum_{i=1}^n c^Tx_iw\ +\ \alpha w$

Now using another variable y_i where $y_i=x_iw\ \ ,i=1,2,\dots n$, the linear fractional programming problem is transformed into a linear programming problem in the form

$$\begin{cases}Max\quad f=\ c^T\mathrm{y}\ +\ \alpha w\\ s.t\qquad \mathrm{Ay}-Bw\leq 0\\ \qquad(d^T\mathrm{x}\ +\ \beta)w=1\\ \qquad 0\leq\mathrm{y}\in R^n\ ,\qquad 0\leq w\in R\end{cases},\qquad(2)$$

Definition 11. A point $x^*\in R^n$ is said to be optimal solution of the linear fractional programming problem (1) if there does not exist $x\in R^n$ such that

$$\frac{\boldsymbol{p}(\boldsymbol{x}^*)}{\boldsymbol{q}(\boldsymbol{x}^*)}\leq\frac{\boldsymbol{p}(\boldsymbol{x})}{\boldsymbol{q}(\boldsymbol{x})}$$

3. Problem Formulation:

The fuzzy linear fractional programming (FLFP) problem is defined as follows:

$$\begin{cases}Max\ \ \tilde{f}\ =\dfrac{\tilde{p}\,(x)}{\tilde{q}\,(x)}=\dfrac{\tilde{c}^{\,T}x\ +\tilde{\alpha}}{\tilde{d}^{\,T}x\ +\ \tilde{\beta}}\\ S.\mathrm{t}\\ \displaystyle\sum_{j=1}^n\widetilde{a_{ij}}\,x_j\ \leq\ \widetilde{b_i}\quad,i=1,2,\dots m\\ \tilde{c},\ \ \tilde{d}\,,\tilde{\alpha}\,,\tilde{\beta}\,,\ \ \widetilde{a_{ij}}\ and\ \ \widetilde{b_i}\ \in\tilde{F}(R)\,,\ \ \tilde{d}^{\,T}x\ +\ \tilde{\beta}>0\end{cases},\qquad(3)$$

Where all coefficients are triangular $\boldsymbol{L-R}$ fuzzy numbers.

Now the fuzzy linear fractional programming problem (3) can be reduce used as the ranking function for all coefficients to obtain the following problem:

$$\begin{cases} Max \ \ \tilde{f} \ = \dfrac{\Re(\ \tilde{c}^T)\ \mathrm{x}\ + \Re\ (\tilde{\alpha})}{\Re(\ \tilde{d}^T)\ \mathrm{x}\ + \Re(\tilde{\beta})}, \\ S.t \\ \sum_{j=1}^{n} \Re(\widetilde{a_{ij}})\ x_j \ \le \Re(\ \widetilde{b_i}\ ) \quad , i = 1,2, \dots m, \\ ,x_j \ge 0 \ \ , \quad \Re(\ \tilde{d}^T)\ x\ + \Re(\tilde{\beta}) > 0 \end{cases} \quad (4)$$

To solve this problem can use variable transformation by assumption

$$w = \frac{1}{\Re(\ \tilde{d}^T)\ x\ + \Re(\tilde{\beta})}$$

If we make the additional variable changes $y_i = x_i w$, For all $i = 1,2, \dots$ The linear fractional programming problem (4) with variable changes as formulation

$$\begin{cases} Max \ \ \tilde{f} \ = \Re(\ \tilde{c}^T)\ y + \Re\ (\tilde{\alpha})\ w \\ S.t \\ \sum_{j=1}^{n} \Re(\widetilde{a_{ij}}) y_j - \Re(\ \widetilde{b_i}) w \le 0 \quad , i = 1,2, \dots m, \\ \Re(\ \tilde{d}^T) y_i\ + \Re(\tilde{\beta}) w = 1 \\ y_i\ ,\ w \ \ge 0 \end{cases} \quad (5)$$

Theorem. If (y^*, w) is an optimal solution of problem (5) then $x^* = \frac{y^*}{w}$ is the optimal solution for problem (3).

The proof of this theorem is very similar to the proof theorem (4-1) given by Pandian and Jayalakshmi in [1].

3.1 Algorithm solution for FLFP problem

The proposed algorithm for solving the FLFP problem (3) as follows:

Step1. Find the ranking function for all coefficients in the FLFP problem.

Step2. Reducing the fuzzy linear fractional programming problem to the linear fractional programming problem

Step3. Using variable transformation, a linear fractional programming problem is transformed into a linear programming problem.

Step4. Find the optimal solution using the solver.

3.2 Numerical examples:

Example 1. A furniture factory manufactures three types of products, $A, B, and\ C$ with an uncertain profit of $(\widetilde{30}\ ,\widetilde{40}\ ,\widetilde{35}\ )$ dinars per unit respectively, However, the uncertain costs for each unit of the above products are given by ($\widetilde{10}\ ,\widetilde{12}\ ,\tilde{9}$) dinars per unit respectively. The environmental coefficients such as the profit and cost (due to market situations). The following table shows the number of hours required to produce one unit of furniture and the time available for production:

Types	Department		
	Production	Manufacturing	Encapsulation
A	3	2	1
B	4	1	3
C	2	2	2
Available hours	60	70	50

We need to determine how much of product, $A, B, and\ C$ can be manufactured in order to maximize the total profit. Hence, the problem given above can be formulated as the following fuzzy linear fractional linear programming problem:

$$\begin{cases} Max \ \ \tilde{f}\ (x_1, x_2\ ,\ \ x_3) = \dfrac{\tilde{p}(x)}{\tilde{q}(x)} = \dfrac{\widetilde{30}\ x_1 + \ \widetilde{40}\ x_2 + \ \widetilde{35} x_3}{\widetilde{10}\ x_1\ + \ \widetilde{12}\ x_2 + \ \tilde{9}\ x_3}, \\ S.t \\ 3x_1 + \ 4x_2 + 2x_3 \le \ 60 \\ 2x_1 + x_2 + 2x_3 \ \le \ 70 \\ x_1 + 3x_2 + 2x_3 \le 50 \\ x_1 \quad , \ x_2 \ \ , \ \ x_3 \ \ge \ \ 0 \end{cases}$$

If we assume the profit and cost are triangular $L - R$ fuzzy number

$$\begin{cases} \widetilde{30}\ =\ (30\ ,\ \ 5\ ,7)_{LR} \ \ , \ \ \widetilde{40}\ =\ (40, 8, 10)_{LR} \ \ , \ \ \widetilde{35}\ =\ (35, 7, 8\ )_{LR}, \\ \widetilde{10}\ =\ (10,\ \ 3\ ,\ \ 2\ )_{LR} \ \ , \ \ \widetilde{12}\ =\ (12,\ \ 2\ ,3\ )_{LR} \ \ , \ \ \tilde{9}\ =\ (9, 4,\ \ 6\ )_{LR} \end{cases}$$

For $\boldsymbol{\alpha} \in [\mathbf{0}, \mathbf{1}]$, we take an $\boldsymbol{\alpha - cut}$ for the objective function, then the above problem can be written as follows:

$$\begin{cases} Max\tilde{f}\ (x_1, x_2, x_3) = \dfrac{\begin{matrix}[30 + 5(\alpha - 1)\ ,30 + 7(\alpha - 1)]x_1 + \\ [40 + 8(\alpha - 1)\ ,40 + 10(\alpha - 1)\ ]x_2 + \\ [35 + 7(\alpha - 1)\ ,35 + 8(\alpha - 1)]x_3\end{matrix}}{\begin{matrix}[10 + 3(\alpha - 1)\ ,10 + 2(\alpha - 1)]x_1\ + \\ [12 + 2(\alpha - 1)\ ,12 + 3(\alpha - 1)]x_2 + \\ [9 + 4(\alpha - 1)\ ,9 + 6(\alpha - 1)]x_3\end{matrix}}, \\ S.t \qquad 3x_1 + \ 4x_2 + 2x_3 \le \ 60 \\ 2x_1 + x_2 + 2x_3 \ \le \ 70 \\ x_1 + 3x_2 + 2x_3 \le 50 \\ x_1 \quad , \ x_2 \ \ , \ \ x_3 \ \ge \ \ 0 \end{cases}$$

Now using rank function of the coefficients for the objective function for $\boldsymbol{\alpha} = 1$, then the fuzzy linear fractional linear programming problem reduced to linear fractional linear programming problem can be written as follows:

$$\begin{cases} Max \ \ \tilde{f}\ (x_1, x_2\ ,\ \ x_3) = \dfrac{30.5\ x_1 + \ \ 40.5\ x_2 + \ 35.25 x_3}{9.75\ x_1\ + \ 12.25\ x_2 + \ 9.5\ x_3}, \\ S.t \\ 3x_1 + \ 4x_2 + 2x_3 \le \ 60 \\ 2x_1 + x_2 + 2x_3 \ \le \ 70 \\ x_1 + 3x_2 + 2x_3 \le 50 \\ x_1 \quad , \ x_2 \ \ , \ \ x_3 \ \ge \ \ 0 \end{cases}$$

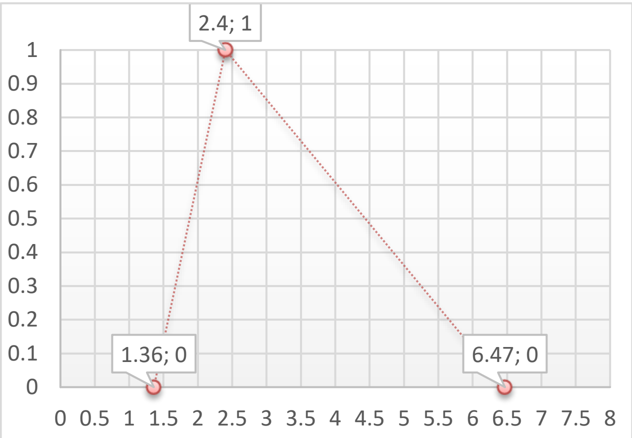
Now we will use the variable transformation where $y_i = x_i w$ For $i = 1,2,3$ to obtain the following linear programming problem:

$$\begin{cases} Max\ f'(y_1, y_2,\ y_3, w) = 30.5\, y_1 + 40.5\, y_2 + 35.25 y_3 \\ \qquad , \\ S.t \\ 9.75\, y_1 + 12.25\, y_2 + 9.5\, y_3 = 1 \\ 3y_1 + 4y_2 + 2y_3 - 60\, w \le 0 \\ 2y_1 + y_2 + 2y_3 - 70\, w \le 0 \\ y_1 + 3y_2 + 2y_3 - 50\, w \le 0 \\ y_1\ ,\ y_2\ ,\ y_3, w \ge 0 \end{cases}$$

the optimal solution using excel solver is

$y_1 = 0.009091\ ,\ y_2 = 0\ ,\ y_3 = 0.040909\ ,\ w = 0.001818$

there for it is $x_1 = 5\ ,\ x_2 = 0\ , x_3 = 22.5$, where



$$\tilde{f} = \frac{(937.5,\ 182.5,\ 215)_{LR}}{(252.5,\ 105,\ 145)_{LR}} = (3.7, 1.8, 4.1)_{LR}$$

It can also be displayed as follows:

Figure 2. The membership function of triangular $L-R$ fuzzy number for example 1.

Example 2.

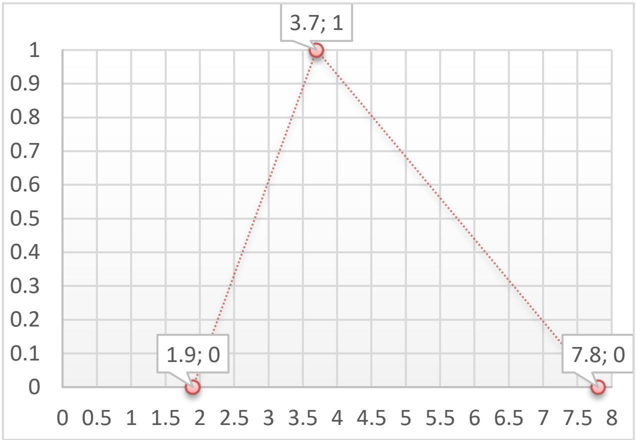
Consider the following fuzzy linear fractional programming problem:

$$\begin{cases} Max\ \ \tilde{f}(x_1, x_2) = \dfrac{\tilde{4}\ x_1 + \tilde{3}\ x_2 + \tilde{9}}{\widetilde{1.5}\ x_1 + \widetilde{1.5}\, x_2 + \tilde{5}} \\ \qquad , \\ S.t \\ \tilde{1} x_1 + \tilde{3}\, x_2 \le \widetilde{30} \\ \tilde{1}\, x_1 - \tilde{3}\, x_2 \le \widetilde{15} \\ x_1\ ,\ x_2 \ge 0 \end{cases}$$

Where

$$\begin{cases} \tilde{4} = (4,\ 1, 1)_{LR}\ ,\ \tilde{3} = (3,\ 2, 1)_{LR}\ ,\ \tilde{9} = (9, 2, 2)_{LR} \\ \widetilde{1.5} = (1.5,\ 1, 0.5)_{LR}\ ,\ \widetilde{1.5} = (1.5,\ 0.5,\ 0.5)_{LR}\ , \\ \tilde{5} = (5,\ 1, 1)_{LR}\ \tilde{1} = (1,\ 1, 2)_{LR}\ ,\ \ \tilde{3} = (3,\ 2, 1)_{LR} \\ \widetilde{30} = (30,\ 10,\ 15)_{LR}\ ,\widetilde{15} = (15, 5, 5)_{LR} \end{cases}$$

Solution: we calculate the rank function of the coefficients for the objective function and the constraints



$$\begin{cases} \Re(\tilde{4}) = \Re(4,\ 1, 1)_{LR} = 4 + \frac{1-1}{4} = 4\ ,\quad \Re(\tilde{3}) = 2.75 \\ \Re(\tilde{9}) = 9\ ,\ \Re(\widetilde{1.5}) = 1.375\ ,\ \Re(\widetilde{1.5}) = 1.5\ ,\ \Re(\tilde{5}) = 5 \\ \Re(\tilde{1}) = 1.25\ ,\ \Re(\tilde{3}) = 2.75\ ,\ \Re(\widetilde{30}) = 31.25,\ \Re(\widetilde{15}) = 15 \end{cases}$$

Now the fuzzy linear fractional programming problem can be reduced to the following linear fractional programming problem:

$$\begin{cases} Max\ f = \dfrac{4\ x_1 + 2.75\ x_2 + 9}{1.375\ x_1 + 1.5\, x_2 + 5} \\ \qquad , \\ S.t \\ 1.25\, x_1 + 2.75\, x_2 \le 31.25 \\ 1.25\, x_1 - 2.75\, x_2 \le 15 \\ x_1\ ,\ x_2 \ge 0 \end{cases}$$

Now we will use the variable transformation where $y_i = x_i w$ For $i = 1,2$ to obtain the following linear programming problem

$$\begin{cases} Max\ f(y_1, y_2, w) = 4\ y_1 + 2.75\ y_2 + 9w \\ \qquad , \\ S.t \\ 1.375\ y_1 + 1.5\, y_2 + 5w = 1 \\ 1.25\, y_1 + 2.75\, y_2 - 31.25\, w = 0 \\ 1.25\, y_1 - 2.75\, y_2 - 15w = 0 \\ y_1\ ,\ y_2\ ,\ w \ge 0 \end{cases}$$

the optimal solution using excel solver is

$y_1 = 0.55814\ ,\ y_2 = 0,\ w = 0.046512$

there for it is $x_1 = 12\ ,\ x_2 = 0$, where

$$\tilde{f} = \frac{(57,\ 14,\ 14)_{LR}}{(23,\ 13,\ 7)_{LR}} = (2.4,\ 1.04,\ 4.07)_{LR}$$

It can also be displayed as follows:

Figure 3. The membership function of triangular $L-R$ fuzzy number for example 2.

By comparing the results of the proposed method with existing methods [1, 8, 9, 10], we can conclude that all objective values and optimal solutions of the existing methods fall into the objective values and optimal solutions of our proposed method, indicating the reliability of FLFP With Triangular L-R Fuzzy Numbers Coefficients in searching for all efficient solutions. In other words, the objective values and optimal solutions always cover all objective values and optimal solutions for the same problem whatever the solution method is used. Therefore, less information would be lost within the computation process of the proposed method than existing methods.

Conclusions and Future Scope

In this paper, we used a new representation for fuzzy approximate numbers, as these numbers give more opportunity to decision-makers at different levels of significance. The rank function was used to convert the fuzzy approximate number into a confirmed number at a certain level of significance. Proposing an algorithm to solve the problem of linear fractional programming with fuzzy extrapolated coefficients and explaining the algorithm with a numerical example.

In future work, the same model can be extended for trapezoidal L-R fuzzy number, we considered parameters and variables to be trapezoidal L-R fuzzy number.

For further studies, nonlinear fractional programming problems in nature and may have multiple objective functions having parameters as L-R fuzzy number can be investigated.

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