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Integration of MPC and SOLADRC to Optimize PWR Performance

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ABSTRACT

Controlling pressurized water reactors (PWRs) is a challenging task due to their inherent nonlinearity, modeling uncertainties, time-varying parameters, and multi-input multi-output (MIMO) nature. These factors require advanced control strategies to maintain safe and efficient operations. This paper focuses on the nonlinear controller design for PWRs, where external disturbances and system uncertainties are the primary challenges. To enhance the control performance of PWRs, the authors propose an integrated approach using Model Predictive Control (MPC) and Second Order Linear Active Disturbance Rejection Control (SOLADRC). MPC is utilized to optimize the reactor core by predicting the dynamic behavior of the system and proactively adjusting the control inputs. SOLADRC is employed to provide robust regulation of key operating parameters, such as reactor power, coolant temperature, and pressure, in the presence of both known and unknown disturbances. Comprehensive simulations and analysis demonstrate the effectiveness of the integrated MPC-SOLADRC approach in improving the PWR's operational efficiency, flexibility, and safety margins. The results show that the combined MPC and SOLADRC strategy outperforms traditional control methods in terms of set point tracking, disturbance rejection, and constraint handling. This work contributes to the advancement of control optimization techniques for MIMO systems, paving the way for enhanced safety, reliability, and economic performance of PWR.

تكامل التحكم التنبؤي النموذجي (MPC) مع التحكم النشط لرفض الاضطرابات من الدرجة الثانية (SOLADRC) لتحسين أداء مفاعل الماء المضغوط (PWR)

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الكلمات المفتاحية:
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إنتاج الطاقة الكهربائية

الملخص

التحكم في مفاعلات الماء المضغوط (PWRs) يعد مهمة صعبة بسبب عدم خطيتها الجوهرية، وعدم اليقين في النمذجة، والمعاملات المتغيرة مع الزمن، وطبيعتها متعددة المدخلات والمخرجات (MIMO). هذه العوامل تتطلب استراتيجيات تحكم متقدمة للحفاظ على عمليات آمنة وفعالة. تركز هذه الورقة على تصميم المتحكمات غير الخطية لمفاعلات الماء المضغوط، حيث تمثل الاضطرابات الخارجية وعدم اليقين في النظام التحديات الأساسية. لتعزيز أداء التحكم في مفاعلات الماء المضغوط، يقترح المؤلفون نهجًا متكاملًا باستخدام التحكم التنبؤي النموذجي (MPC) والتحكم الخطي المكافئ النشط في الاضطرابات من الدرجة الثانية (SOLADRC). يتم استخدام التحكم التنبؤي النموذجي لتحسين قلب المفاعل عن طريق التنبؤ بالسلوك الديناميكي للنظام وتعديل مدخلات التحكم بشكل استباقي. يتم استخدام التحكم الخطي المكافئ النشط لتوفير تنظيم قوي للمعاملات التشغيلية الرئيسية، مثل قوة المفاعل، ودرجة حرارة المبرد، والضغط، في وجود الاضطرابات المعروفة وغير المعروفة. تبرهن المحاكاة والتحليلات الشاملة على فعالية النهج المتكامل للتحكم التنبؤي النموذجي والتحكم الخطي المكافئ النشط في تحسين كفاءة التشغيل ومرونة مفاعل الماء المضغوط ومرونة الأمان. تظهر النتائج أن استراتيجية الدمج بين التحكم التنبؤي النموذجي والتحكم الخطي المكافئ النشط تتفوق على أساليب التحكم التقليدية من حيث تتبع النقطة المحددة، رفض الاضطرابات، والتعامل مع القيود.

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يساهم هذا العمل في تعزيز تقنيات تحسين التحكم لأنظمة MIMO، مما يمهد الطريق لتحقيق أداء أعلى في الأمان، الموثوقية، والأداء الاقتصادي لمفاعلات الماء المضغوط.

1. Introduction

Pressurized water reactors (PWRs) are a widely used type of nuclear power plant, accounting for the majority of operational nuclear reactors worldwide. Effective control of PWRs is critical to ensure safe and efficient operation. These small power generating reactors (PWRs) use water to cool and regulate their cores, and they are carefully engineered to produce electricity while meeting strict safety and environmental regulations. Small PWRs function as Multiple Input Multiple Output (MIMO) systems, controlling multiple outputs like reactor temperature and pressure in addition to multiple inputs like coolant flow rate and reactor power level simultaneously [1]. Complex systems like these require sophisticated control techniques to maintain responsive and safe operations in the face of changing circumstances and disruptions [2,3]. Traditional control methods for PWRs often rely on proportional-integral-derivative (PID) controllers, which can struggle to handle the complex dynamics and nonlinearities inherent in PWR systems. This paper investigates the application of two advanced control methods - model predictive control (MPC) and Second Order Linear Active Disturbance Rejection Control (SOLADRC)for PWR control [4,5].

MPC has been extensively studied for PWR control due to its ability to handle constraints and multivariable control problems. Early work by Muske and Badgwell [6] demonstrated the use of MPC for controlling the power and coolant temperature in a PWR, showing its advantages over classical PID control. Khosa et al. [7] developed an MPC controller for a PWR core power and coolant temperature control, incorporating state and input constraints. They showed the MPC controller's ability to handle set-point changes and load disturbances. Kang et al. [8] proposed an MPC-based control scheme for integrated control of the reactor core, primary coolant system, and secondary steam system in a PWR, improving overall plant efficiency and safety margins. Several studies have also explored the use of economic MPC formulations for PWR control, aiming to optimize operational costs while maintaining safety constraints [9,10]. Active Disturbance Rejection Control (ADRC) has been investigated as a robust control approach for nuclear power plant applications, including PWRs, due to its ability to handle uncertainties and disturbances. Gao et al. [11] proposed a SOLADRC controller for PWR reactor power regulation, showing its ability to handle parameter uncertainties and external disturbances. They also analyzed the closed-loop stability and robustness of the SOLADRC system. Zhao et al. [12] developed a SOLADRC controller for integrated control of the reactor core, primary coolant system, and secondary steam system in a PWR. They demonstrated the SOLADRC controller's effectiveness in setpoint tracking and disturbance rejection compared to classical control methods. The combination of ADRC and sliding-mode control (SMC) has further enhanced the performance of ADRC, leading to the SOLADRC approach. Guang et al. [13] developed an SMC controller for a PWR reactor power control system, showing its superior disturbance rejection capabilities compared to PID control. The literature review suggests that both MPC and SOLADRC have shown promising results in improving the control performance and robustness of PWR systems compared to traditional PID control approaches. The integration of these advanced control methods with the PWR system dynamics and constraints can lead to enhanced safety, efficiency, and operational flexibility.

Through the integration of MPC and SOLADRC into a single controller, it will seek to improve control capabilities, outperforming traditional approaches in handling complex input signals and optimizing system efficiency. By carefully combining these approaches with decentralized control mechanisms, control engineering principles may be advanced, and difficult operational challenges in high-order systems such as PWRs can be addressed [14,15]. This research aims to improve the operational capabilities of PWRs and contribute to efficient and sustainable energy solutions by thoroughly examining these advanced control systems. The MPC - SOLADRC control strategy ensures the safe and efficient operation of PWRs, permitting nuclear power generation within the framework of sustainable energy practices, by optimizing system performance and stability in the face of nonlinear dynamics and operational restrictions [16,17].

2. Mathematical Model of PWR

The mathematical modeling of PWRs is a complex and multidisciplinary task, involving the integration of reactor physics, thermal-hydraulics, control systems, and other relevant disciplines. The resulting models are often used for design, analysis, and control of PWR systems to ensure safe and efficient operation [15].

- The dynamic behavior of the reactor core can be described by the point kinetics equations, which model the time-dependent neutron population and the concentration of delayed neutron precursors. The point kinetics equations are a set of coupled ordinary differential equations that include terms for neutron generation, neutron absorption, and delayed neutron emission. These equations capture the transient response of the reactor power to changes in reactivity, control rod positions, and other parameters.
- The thermal-hydraulic behavior of the primary coolant system is typically modeled using a set of conservation equations, including mass, momentum, and energy balances. These equations describe the time-dependent pressure, temperature, and flow rate of the coolant as it circulates through the reactor core, steam generators, and other components. The thermal-hydraulic model may also include correlations for heat transfer, pressure drop, and other relevant physical phenomena.
- The fuel rods in the reactor core are often modeled using a one-dimensional heat conduction equation, which describes the temperature distribution within the fuel pellet and the cladding. This model captures the heat generation in the fuel due to fission, heat transfer to the coolant, and the thermal resistance of the fuel-cladding gap. The fuel rod model is coupled with the reactor kinetics and thermal-hydraulic models to provide a comprehensive representation of the core thermal behavior.

The following equations are the basic equations used in the mathematical modeling of a PWR, where the complete model would also include additional equations and correlations for various components, such as the steam generators, pressurizer, and control systems, as well as the coupling between the different subsystems. Table 1 present the description of the parameter of the system [15,16].

$$\begin{cases} \dfrac{dn_r}{dt} = \dfrac{1}{\Lambda}(\rho-\beta)n_r + \lambda c_r \\ \dfrac{dc_r}{dt} = \dfrac{\beta n_r}{\Lambda} - \lambda c_r \\ \dfrac{dT_f}{dt} = \dfrac{1}{\mu_f}\left(fP_0 n_r - \Omega T_f + \Omega T_{c1}\right) \\ \dfrac{dT_{c1}}{dt} = \dfrac{1}{\mu_c}((1-f)P_0 n_r + \Omega T_f - \Omega T_{c1} + 2C_{pc}W_p T_{lp} - 2C_{pc}W_p T_{c1}) \\ \dfrac{dT_{c2}}{dt} = \dfrac{1}{\mu_c}\left((1-f)P_0 n_r + \Omega T_f - \Omega T_{c2} + 2C_{pc}W_p T_{c1} - 2C_{pc}W_p T_{c2}\right) \\ \rho = \rho_{rod} + \alpha_f\left(T_f - T_{f,0}\right) - 0.5\alpha_c\left[\left(T_{c1,0} + T_{c2,0}\right) - \left(T_{c1} + T_{c2}\right)\right] \end{cases} \quad (1)$$

Table 1: PARAMETERS AND DESCRIPTIONS WITH UNITS OF MEASUREMENT [16].

Parameter	Description	Units of Measurement
n_r	Relative neutron density	-
c_r	Delayed neutron precursor's relative density	-
ρ	Total reactivity	-
ρ_{rod}	Reactivity added by control rods	-
λ	Decay constant of delayed neutron precursor	s^{-1}
β	Delayed neutron fraction	-
α_f	Fuel reactivity temperature coefficient	$℃^{-1}$
α_c	Coolant reactivity temperature coefficient	$℃^{-1}$
Λ	Average neutron generation time	s

T_f	Fuel average temperature	℃
T_{c_1}, T_{c_2}	Coolant nodes 1 and 2's outlet temperatures	℃
T_{lp}	Coolant node 1's inlet temperature	℃
Ω	Transfer coefficient between fuel and coolant	$W/$℃
$T_{f_0}, T_{c_{1,0}}, T_{c_{2,0}}$	Starting temperatures of $T_{f_0}, T_{c_{1,0}}, and\ T_{c_{2,0}}$	℃
C_{pc}	Specific heat at constant pressure	$J.kg^{-1}/$℃
P_0	Reactor full power	W
W_p	Coolant flow	kg/s

$n_{r;0}, c_{i;0}, T_{f;0}, T_{c1;0}, T_{c2;0}$, and ρ_0 are the equilibrium values from which $n_r, c_i, T_f, T_{c1}, T_{c2}$, and ρ are derived [17-19].

By formulating the PWR system in state space form, it becomes possible to analyze and design control strategies for optimal performance and safety of the reactor [18].

State vector: neutron flux distribution, temperature profiles, control rod positions.

Input vector: coolant flow rate, control rod adjustments.

Output vector: reactor power level, temperature readings.

The deviations or changes of the variables $n_r, c_i, T_f, T_{c1}, T_{c2}$ and ρ from their respective equilibrium values ($n_0, c_0, T_0, T_{c1,0}, T_{c2,0}$ and ρ_0) are defined as:

$$\begin{cases} \delta n_r = n_r - n_{r0} \\ \delta c_i = c_i - c_{i0} \\ \delta T_f = T_f - T_{f0} \\ \delta T_{c1} = T_{c1} - T_{c1,0} \\ \delta T_{c2} = T_{c2} - T_{c2,0} \\ \delta \rho = \rho - \rho_0 \end{cases} \tag{2}$$

Where δ represents a variable's departure from an equilibrium value [18,19]. Finally, by performing linearization and Laplace transformation, we may derive the state space equation by substituting Eq. (2) into Eq. (1) and disregarding the second-order components in accordance with the small perturbation linearization approach [16].

The following matrices representations of the aforementioned equations can be obtained by implementing Laplace transforms to them: [16]

$$A(s)X(s) = B(s)U \tag{3}$$

$\mathrm{X(s)} = [\mathrm{n_r(s), c_r(s), T_f(s), T_{c_1}(s), T_{c_2}(s)}]'$ where X is the state vector; $\mathrm{U(s)} = [\mathrm{\rho_{rod}(s), T_{lp}(s)}]'$ is the input vector, matrix A and matrix B can be written as [16]

For Laplace transform equations and final matrix formulation, refer to the following source [16].

$$A = \begin{bmatrix} s+\beta/\Lambda & -\beta/\Lambda & -\frac{n_{r0}\alpha_f}{\Lambda} & -0.5\frac{n_{r0}\alpha_c}{\Lambda} & -0.5\frac{n_{r0}\alpha_c}{\Lambda} \\ \lambda & -s-\lambda & 0 & 0 & 0 \\ \frac{fP_0}{\mu_f} & 0 & -s-\frac{\Omega}{\mu_f} & \frac{\Omega}{\mu_f} & 0 \\ \frac{P_0-fP_0}{\mu_c} & 0 & \frac{\Omega}{\mu_c} & -\frac{\Omega+2M}{\mu_c}-s & 0 \\ \frac{P_0-fP_0}{\mu_c} & 0 & \frac{\Omega}{\mu_c} & -\frac{\Omega+2M}{\mu_c} & -\frac{2M}{\mu_c}-s \end{bmatrix}$$

$$, \mathrm{B} = \begin{bmatrix} n_{r0}/\Lambda & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{2M}{\mu_c} & 0 \end{bmatrix}$$

From the 3rd equation we can obtain,

$$\begin{bmatrix} \delta n_r(s) \\ \delta T_{c2}(s) \end{bmatrix} = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} \begin{bmatrix} \delta \rho_{rod}(s) \\ \delta T_{lp}(s) \end{bmatrix} \tag{4}$$

For the small PWR in the current research, the transfer functions $G_{11}(s), G_{12}(s), G_{21}(s)$, and $G_{22}(s)$ at 100% of full power are determined as [16].

$$G_{11}(s) = \frac{52500\,(s+5.279)\,(s+4.499)\,(s+1.12)\,(s+0.07704)}{(s+365.9)(s+5..703)(s+0.03427)(s^2+5.358s+8.095)}$$

$$G_{12}(s) = \frac{-77.7\,(s+8.314)(s+1.458)(s+0.07701)}{(s+365.9)(s+5..703)(s+0.03427)(s^2+5.358s+8.095)}$$

$$G_{21}(s) = \frac{57060\,(s+26.38)(s+8.978)(s+0.07703)}{(s+365.9)(s+5..703)(s+0.03427)(s^2+5.358s+8.095)}$$

$$G_{22}(s) = \frac{17.52\,(s+361.4)\,(s+1.301)\,(s-0.0149)}{(s+365.9)(s+5..703)(s+0.03427)(s^2+5.358s+8.095)}$$

The fundamental dynamics and behavior of the system can be taken into consideration in order to simulate the operation of a tiny Pressurized Water Reactor (PWR). When assessing a PWR's performance, the following factors are often crucial to consider: Neutron kinetics: analysis of the distribution of power inside the core, criticality, reactivity, and variations in the neutron population over time. Analysis of heat transfer: creation, movement, and extraction of heat from the coolant to the core.

Coolant flow includes three factors: heat removal capacity, pressure drop, and flow rate.

The fuel's temperature, burn up, and interaction with the coolant are all factors in its behavior.

The moderator temperature coefficients, boron concentration, and control rods are the three mechanisms used to modulate reactivity.

The PWR system's power generation, stability, and safety can be assessed across a range of operating settings and transients through investigating the dynamics and interactions of these factors.

3. PWR Control Design

Model Predictive Control (MPC) is an advanced control strategy that predicts the future behavior of a system based on a dynamic model. By optimizing a control sequence over a finite time horizon, MPC aims to drive the system to a desired state while considering constraints. The MPC toolbox provides tools and algorithms for designing, simulating, and implementing MPC controllers for various applications. It offers features such as model formulation, simulation, controller design, and performance evaluation, making it a valuable tool for control system engineers [20]. To improve system performance, Active Disturbance Rejection Control (ADRC) is a robust control paradigm that focuses on estimating and rejecting disturbances in real-time. SOLADRC improves disturbance rejection and control performance in systems with nonlinearities and uncertainties by incorporating second-order dynamics in the disturbance observer [21]. MPC based on ADRC combines the predictive capabilities of MPC with the disturbance rejection properties of Active Disturbance Rejection Control. By integrating the disturbance estimation and rejection mechanisms of ADRC into the MPC framework, this approach offers improved robustness, disturbance rejection, and tracking performance for complex systems. The synergistic combination of MPC and ADRC provides a powerful control strategy for challenging control problems where both predictive control and disturbance rejection are crucial [22].

3.1 Model Predictive Control Methodology

The specific design and tuning of the MPC controller for a small PWR require in-depth knowledge of the reactor's dynamics, control objectives, and operational constraints. The MPC framework provides a flexible and systematic approach to address the complex control challenges in small PWR systems.

MPC Formulation:

The MPC law aims to find the optimal control input sequence $\boldsymbol{u(k)}$, $\boldsymbol{u(k+1)}, \ldots, \boldsymbol{u(k+N-1)}$ that minimizes the cost function $\boldsymbol{J(k)}$ over a prediction horizon $\boldsymbol{N}$. The cost function J(k) typically includes:

Output tracking error: Penalizes the difference between the predicted output $\boldsymbol{y(k+i|k)}$ and the reference $\boldsymbol{r(k+i)}$.

Control effort: Penalizes the control input $\boldsymbol{u(k+i)}$ to limit excessive control actions.

Constraints can be imposed on the control inputs and outputs to ensure safe and reliable operation, such as:

Control input limits: $\boldsymbol{u_{\min} \leq u(k+i) \leq u_{\max}}$

Output limits: $\boldsymbol{y_{\min} \leq y(k+i|k) \leq y_{\max}}$

Optimization and Implementation:

MPC optimization problem is solved at each time step $\boldsymbol{k}$ to determine the optimal control input $\boldsymbol{u(k)}$.

Efficient numerical optimization algorithms, such as quadratic programming or interior-point methods, are typically used to solve the MPC problem. The MPC controller is implemented in a receding horizon fashion, where only the first control input $\boldsymbol{u(k)}$ is applied to the plant, and the optimization problem is solved again at the next time step $\boldsymbol{k+1}$ with updated information.

Figure 1 illustrates the configuration of the feedback control system, aimed at stimulating the performance of the implemented controllers and assessing their efficacy across diverse operating scenarios [21, 22].

3.2 Active Disturbance Rejection Control Methodology

ADRC uses a two-layer system in which the first layer uses real-time disturbance estimation and compensation, while the second layer modifies the control input according to the calculated disturbances. By adding a second-order model to better capture dynamic behaviors, the SOLADRC is a particular implementation that improves the disturbance rejection capabilities of conventional ADRC. Because of this, SOLADRC is especially useful in settings like PWRs, where disruptions may originate from a variety of sources and guarantee stable operation and enhanced system resilience.

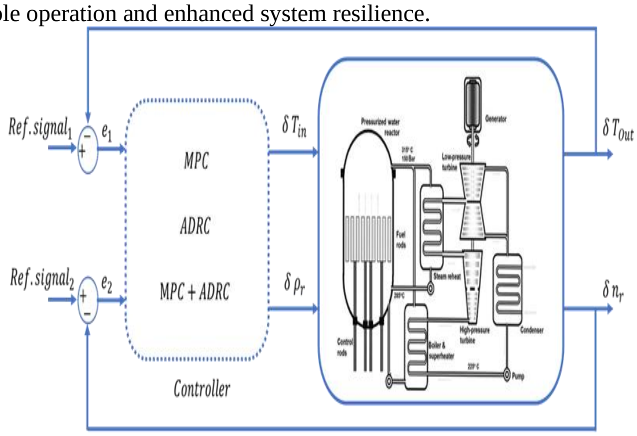


Fig. 1. The diagram illustrates the types of controllers within a MIMO.

The mathematical formulations in traditional second-order control systems can be simplified as follows:

$$\ddot{y} = -a\dot{y} - by + \omega_{drt} + bu \quad (5)$$

With a and b standing for system parameters, y denotes the output, u for the input, and ω_{drt} for the external disturbance variable. Equation (10) can be expressed as follows:

$$\ddot{y} = -a\dot{y} - by + \omega_{drt} + (b - b_0)u + b_0 u = f_r(t, y, y', \omega_{drt}) + b_0 u \quad (6)$$

where the entire disturbances, including internal disturbances, are represented by the formula $f_r\,(t, y, y', \omega_{drt}) = -a\,y' - by + \omega_{drt} + (b - b_0)\,u$, internal disturbances $-a\,y' - by + (b - b_0)\,u$ and the external disturbance ω_{drt}, where. The estimate of b is shown by b_0, The generalized disturbance is denoted by f_r.

$$\begin{cases} \dot{x} = Ax + Bu + Eh \\ \quad y = Cx \end{cases} \quad (7)$$

Where $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ b_0 \\ 0 \end{bmatrix}$, $C = [1 \quad 0 \quad 0]$, $E = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

The linear extended state observer (LESO) of (12) is the state space observer that is built as

$$\dot{z} = Az + Bu + L(y - \hat{y})$$
$$\hat{y} = Cz \quad (8)$$

L stands for the observer gain vector, which can be obtained using well-known techniques such as the pole placement method.

$$L = [\beta_1 \quad \beta_2 \quad \beta_3]^T \quad (9)$$

After the state observer has been properly designed, the controller can be expressed as follows:

$$u = \frac{-z_3 + u_0}{b_0} \quad (10)$$

The plant is reduced to a double integrator with unit gain when the estimation error in z_3 is taken into consideration.

$$\ddot{y} = (f_r - z_3) + u_0 \approx u_0 \quad (11)$$

which a PD controller can easily control

$$u_0 = k_p(r - z_1) - k_p z_1 \quad (12)$$

Here, the set point is denoted by r. It's crucial to remember that $-k_d, z_2$ is utilised rather than $k_d(t - z_2)$.

This method is employed to prevent the differentiation of the set point and to ensure that the closed-loop transfer function becomes a pure second order without a zero. Using the characteristics equation for a second-order system, the values of ω_c and ζ are found. The gains can be chosen in this case as:

$$k_p = \omega_c^2 \quad, and\ k_d = 2\zeta\omega_c \quad (13)$$

The required closed-loop natural frequency and damping ratio are denoted by ω_n and ζ in this case. The selection of ζ is made in order to avoid oscillations. As a result, we can use a simple linear Proportional-Derivative (PD) controller to accomplish our goal. The following is an outline of the linear error control law:

$$\begin{cases} u_0 = k_p(r - z_1) - k_{d_1} z_2 - \cdots k_{d_{n-1}} z_n \\ \qquad u = u_0 - \dfrac{z_{n+1}}{b_0} \end{cases} \quad (14)$$

The following equation shows how the parameters are chosen so that the closed-loop system's n poles are positioned at $-\omega_c$.

$$s^n + k_{d_{n-1}} s^{n+1} + \cdots + k_{d_1} s + k_p = (s + \omega_c)^n$$

Since equation (14) does not require set point differentiation, it is used to convert the closed-loop system into a pure nth-order system devoid of zeros.

***Remark*:**

1. Zero steady-state error is achieved by the disturbance observer-based Proportional-Derivative (PD) controller without the need for an integrator.
2. The model has no bearing on the design. The approximate value of b in (11) is the sole parameter that is necessary. 3. A generalized disturbance is one that combines the impacts of the unknown disturbance with internal dynamics. Active disturbance rejection is made possible by actively estimating and nullifying the additional state that is added to the observer.
4. If it is thought required, a more complex loop-shaping architecture can be used in place of the PD controller in (14).

A linear error control law and a linear extended state observer make up the Linear Active Disturbance Rejection Control (LADRC) system. The enclosed elements inside the drawn line symbolize the overall structure of LADRC, as shown in Figure 2. When combined main idea with the system states, external disturbances and unclear internal dynamics are recognized as the total disturbance, which is viewed as an augmented state. The core idea of Active Disturbance Rejection Control (ADRC) is to obtain the total disturbance and include it into the control algorithm in order to reduce the complexity of the system to a cascade integrator control situation, The figure shown above illustrates centralized control using a unity feedback control structure using ADRC controller [26-29].

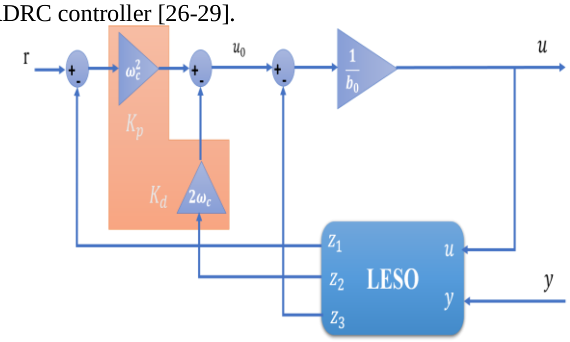


Fig. 2. Fundamental configuration of the SLOADRC.

The following is a continuous linear extended state observer:

$$\begin{cases} \dot{z}_1 = z_2 - \beta_1(z_1 - y(t)) \\ \dot{z}_2 = z_3 - \beta_2(z_1 - y(t)) \\ \qquad \vdots \\ \dot{z}_n = z_{n+1} - \beta_n\big(z_1 - y(t)\big) + b_0 u(t) \\ \dot{z}_{n+1} = z_2 - \beta_{n+1}\big(z_1 - y(t)\big) \end{cases} \quad (15)$$

In order to represent every parameter of the Linear Extended State Observer (LESO) in terms of ω_o, all observer eigenvalues must be set to $-\omega_o$, as shown in the following equation [27,28]:

$$s^n + \beta_1 s^{n+1} + \cdots + \beta_{n+1} s + \beta_n = (s + \omega_o)^n \quad (16)$$

Here, ω_o stands for the LESO's bandwidth. More significantly, only one parameter, ω_o, needs to be adjusted using the LESO tuning.

The plant is reduced to a cascade integrator control system by using the Linear Extended State Observer (LESO) to estimate the overall disturbance All three poles are at the origin in the system (7) for which the LESO is designed. For a given bandwidth of ω_o, it may be deduced that when the observer gains in (9) are relatively small, the observer will display low sensitivity to sounds; a convincing demonstration of this claim would be helpful. On the other hand, the difference between the plant's and the observer's poles determines the gains of the observer. Therefore, it is advised that all three observer poles be positioned at -

ω_o, or comparable, as a smart strategy for efficiency and simplicity, The structure of LESO is shown in the diagram below [27].

$$\lambda(s) = s^3 + \beta_1 s^2 + \beta_2 s + \beta_3 = (s + \omega_o)^3 \tag{17}$$

That is

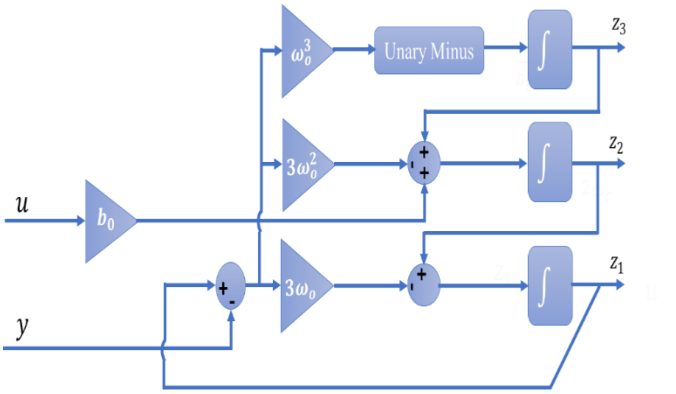
$$\beta_1 = 3\omega_o \ , \qquad \beta_2 = 3\omega_o^2 \ , \qquad \beta_3 = \omega_o^3 \tag{18}$$

$$L = [1042 \quad 3624.8e2 \quad 4.2e7]^T \qquad , \qquad b_0 = 173.80$$

One can easily expand equations (17) and (18) to an nth-order LESO. The following steps can be taken to simply apply this parameterization approach to the Observer for any $A, B, and\ C$ matrix:

a) Transform $\{A, B, C\}$ into $\{A, B, C\}$'s observable canonical form.

b) Calculate observer gain S such that $-\omega_o$ is the alignment point for all



observer poles.

c) To find the observer gain, L, for (A,B,C), use the inverse state transformation.

Since all of the parameters in L are functions of ω_o, making modifications easy and convenient is possible [26-28].

3.3 MPC based on ADRC Methodology

The control input equation and cost function can be merged as follows when combining Model Predictive Control (MPC) with Active Disturbance Rejection Control (ADRC) and Linear Extended State Observer (LESO) into a single controller:

Fig. 3. The diagram depicts the structure of LESO.

Equation of Control Input:

The MPC based ADRC controller with LESO's control input, u, can be written as follows:

$$u = K(\hat{x} - x_{ref}) - \hat{d} \tag{19}$$

Where

u is the control input.

K is the feedback gain from ADRC.

$\hat{x}$ is the estimated system state from LESO.

x_{ref} is the reference state.

$\hat{d}$ is the estimated disturbance from LESO.

The cost function for the MPC+ADRC control considering LESO estimates and disturbance rejection can be formulated as:

$$J = \sum_{k=0}^{N-1}\left(\left\|y_k - y_{ref}\right\|_Q^2 + \|u_k\|_R^2\right) + \left\|\hat{d}_k\right\|_s^2 \tag{20}$$

Where:

J is the cost function to minimize tracking error, control effort, and disturbance estimation error.

$Q, R, and\ S$ are weight matrices for the cost function.

y_k is the predicted output at time step k.

y_{ref} is the reference output.

u_k is the control input at time step k.

$\hat{d}_k$ is the estimated disturbance at time step k.

The MPC based ADRC controller with LESO integration may efficiently optimize control actions while taking disturbance rejection and system stability features into account by combining the control input equation and the cost function.

4. Simulation and results

In order to demonstrate the reference function, this study focuses on analyzing the load behavior with respect to its tracking speed towards the intended reference point as shown in Fig. 4 [20,21].

$$reference\ signal\ (t) = \begin{cases} 1 & , \quad 0 \le t \le 4 \\ -0.875t + 4.5 & , \quad 4 \le t \le 12 \\ -6 & , \quad 12 \le t \le 13.83 \\ 5\sin\left(\dfrac{2\pi}{12.6254}t\right) & , \quad 13.83 \le t \le 20.1106 \\ -6 & , \quad 20.1106 \le t \le 21.95 \\ 1.1429t - 7.1429 & , \quad 21.95 \le t \le 29.95 \\ 1 & , \quad 29.95 \le t \le 35 \end{cases}$$

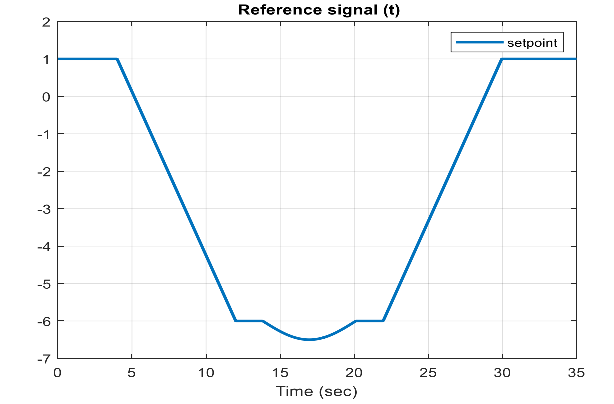


Fig. 4. Set point signal

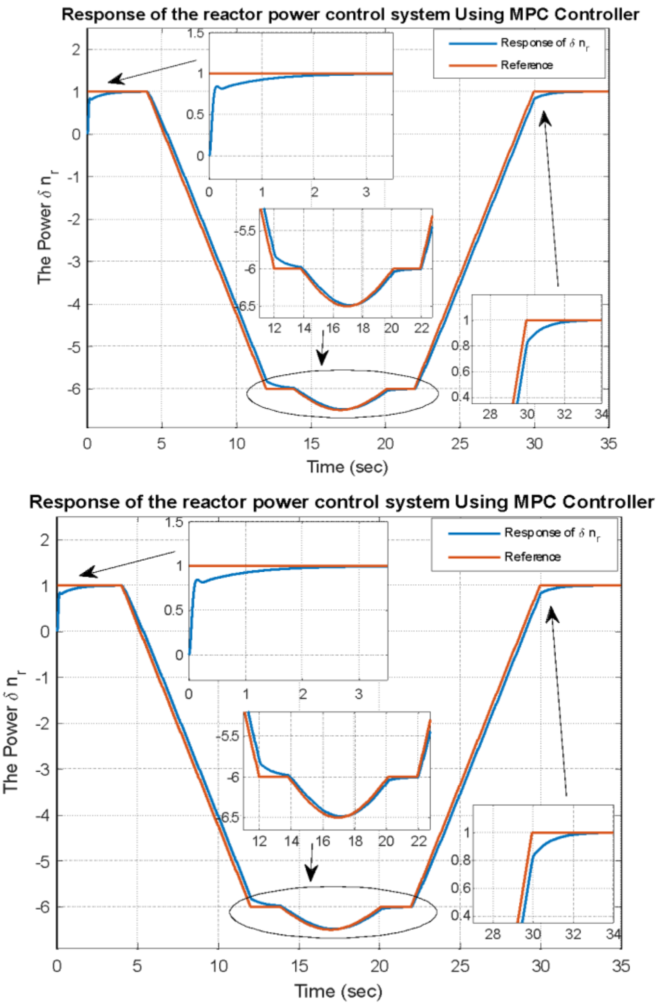


Fig. 5. The power δn_r response using MPC control

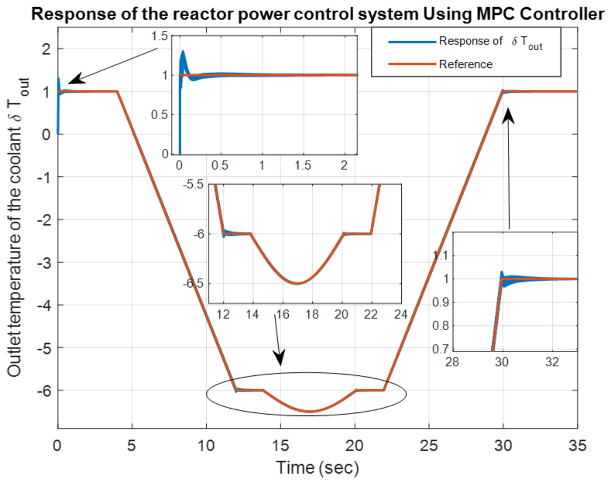


Fig. 6. Coolant Outlet Temperature Response

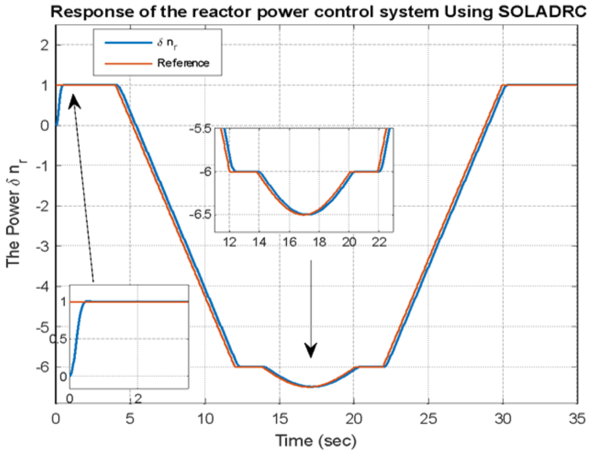


Fig. 7. The power δn_r response using ADRC control

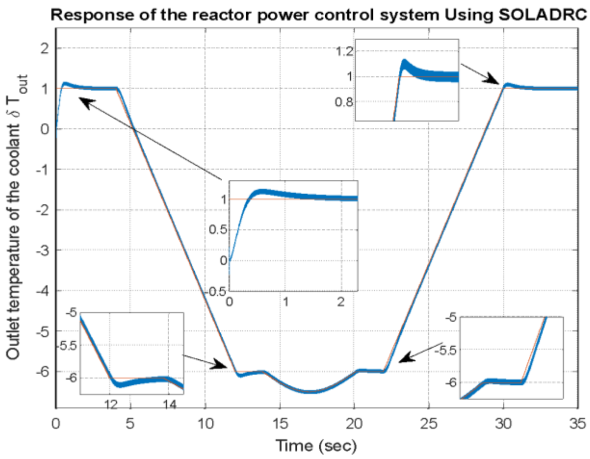


Fig. 8. The Temperature T_{out} response using ADRC control

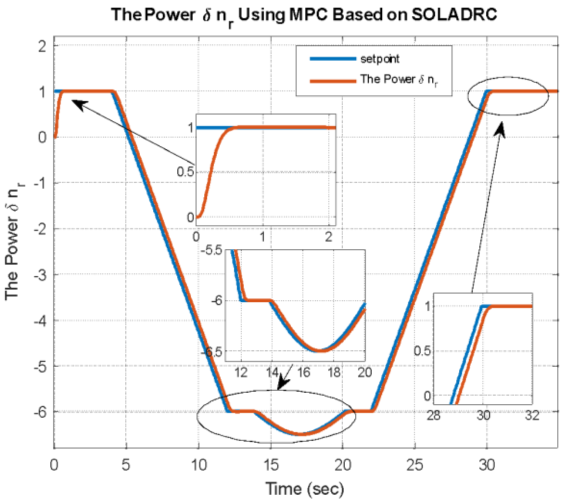


Fig. 9. The Power δn_r response using MPC+ADRC control

The decision to employ a combined Model Predictive Control (MPC) and Second Order Linear Active Disturbance Rejection Control (SOLADRC) strategy for a small Pressurized Water Reactor (PWR) is primarily driven by the potential improvements in transient response, steady-state performance, and overall system stability. Small PWRs are often subject to various operational transients, such as load changes, reactor trips, or response to external disturbances. The effective management of these transients is crucial for ensuring the safety and reliability of the nuclear power plant. From Fig.5 to Fig. 10, the integration of MPC and SOLADRC control offers significant advantages in handling transient scenarios.

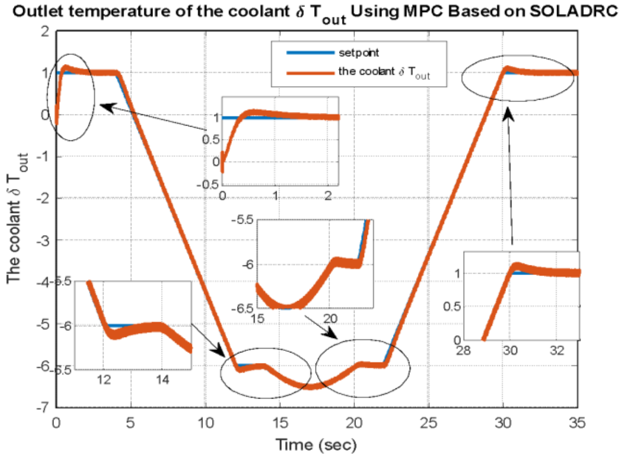


Fig. 10. The Coolant temperature T_{out} response using MPC+ADRC control

The MPC component provides an optimal control framework that can anticipate and respond to upcoming changes in the system. By leveraging the prediction capabilities of MPC, the control system can proactively adjust the manipulated variables (e.g., control rod positions, coolant flow rates) to mitigate the impact of transients on key plant variables like reactor power and coolant temperature. The SOLADRC component further enhances the transient response by effectively estimating and compensating for disturbances and uncertainties that may arise during the transient. The SOLADRC estimator can accurately identify and counteract the effects of parameter variations, fuel burnup, and external disturbances, enabling the control system to maintain tight control over the small PWR's critical variables.

The combined MPC-SOLADRC approach can result in faster response times, reduced overshoots, and better damping of oscillations during transient events. This improved transient performance can contribute to enhanced safety margins, minimized thermal and mechanical stresses on the plant components, and overall better protection of the small PWR's integrity.

In addition to the benefits during transients, the MPC-SOLADRC control strategy can also deliver superior steady-state performance for the small PWR. The MPC component, with its ability to handle multivariable control problems and incorporate operational constraints, can ensure that the small PWR is operated at its optimal setpoints while respecting safety limits and other operational requirements. The online optimization within the MPC framework can continuously adjust the control actions to maintain the desired steady-state conditions, such as reactor power, coolant temperatures, and reactivity levels.

The SOLADRC estimator further enhances the steady-state performance by effectively rejecting the effects of disturbances and uncertainties that can otherwise cause deviations from the desired operating points. This disturbance rejection capability helps the small PWR maintain tighter control over its critical variables, leading to improved efficiency, reduced fuel consumption, and extended equipment lifetime. The combination of MPC and SOLADRC also contributes to the overall stability of the small PWR control system. The MPC component, with its explicit consideration of system constraints and incorporation of feedback from the plant, inherently promotes stable operation. The online optimization and feedback loop within the MPC framework help ensure that the control actions remain within the acceptable operating envelope, preventing the system from reaching unstable regions. Furthermore, the SOLADRC estimator's ability to accurately identify and compensate for disturbances and uncertainties enhances the robustness of the control system. By effectively mitigating the destabilizing effects of parameter variations and external perturbations, the SOLADRC component helps maintain the small PWR's stability, even under challenging operating conditions. The synergistic integration of MPC and SOLADRC results in a control system that is not only optimally tuned for the small PWR's dynamics but also highly resilient to various sources of instability. This enhanced stability, coupled with the improvements in transient response and steady-state performance, makes the MPC-SOLADRC control strategy a compelling choice for small PWR applications.

Overall, the decision to implement MPC and SOLADRC for small PWR control is well-justified by the significant benefits it can provide in terms of transient response, steady-state performance, and system stability. This control approach can contribute to the enhanced safety, reliability, and overall operational excellence of small PWR systems.

5. Conclusion

The combination of SOLADRC and MPC outperforms traditional approaches in the intricate management of tiny Pressurized Water Reactors (PWRs). Because it predicts future system behavior and makes real-time control adjustments, this hybrid approach excels at managing complex signals and dynamic situations. The combination of MPC with SOLADRC, according to the results, improves reactor performance measures including accuracy, energy efficiency, and response time while maintaining resilience and stability. Internal and external disturbances are reduced by SOLADRC's real-time disturbance compensation, and inputs are proactively adjusted by MPC's predictive capabilities. Just SOLADRC improves system performance, demonstrating how effective advanced control is at handling nonlinear systems such as PWRs. These controls satisfy regulatory requirements by improving safety and dependability. This emphasis on state-of-the-art control techniques represents a development in the direction of practical and sustainable energy solutions for the future.

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