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Approximation Of Continuous Set Valued Mapping And Membership Function Rough Topology

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Rough set
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ABSTRACT

In this paper, we introduce the new definition of rough membership function using continues function and we discuss several concepts and properties of rough continuous set value functions as new results on rough the continues function and membership continues function . Moreover, we extend the definition of rough membership function to topology spaces by substituted an equivalence class by continuous functions and prove some theorems on certain types of set value functions and some more general and fundamental properties of the generalized rough sets. Our result generalized the concept of the set valued function by using rough set theory.

التقريب لدالة القيمة المستمرة ودالة العضوية لفضاء طوبولوجي خشن

*فرج أرخيص عبدالنبي أرخيص و أحمد أبراهيم شليتيت

قسم الرياضيات ، جامعة اجدابية ، بنغازي ، ليبيا

الكلمات المفتاحية:
المجموعة الخشنة(الاستقرابية)
التقريب من أعلي
التقريب من أسفل
دالة القمية
دالة العضوية المستمرة

الملخص

في هذه الورقة، نقدم تعريفًا جديدًا لدالة العضوية التقريبية باستخدام دالة مستمرة ونناقش العديد من المفاهيم والخصائص لدالة القيمة الخشنة كنتائج جديدة في التقريب على دالة القيمة المستمرة ودالة العضوية المستمرة. علاوة على ذلك، سنقوم بتوسيع تعريف دالة العضوية التقريبية إلى فضاءات طوبولوجيا خشنة عن طريق استبدال فصول التكافؤ في الطوبولوجي بدوال مستمرة ومن ثم نحاول إثبات بعض النظريات حول أنواع معينة من دوال القيمة وبعض الخصائص الأساسية والأكثر عمومية من الخصائص الأساسية والعامة لتعميم المجموعات التقريبية الخشنة. وقد توصلنا إلي نتائج جيدة ومهمة ومفيدة وهي تعميم او توسيع مفهوم دالة القيمة ا باستخدام نظرية المجموعة الخشنة أو الاستقرابية.

1. Introduction

The rough set theory has been introduced by Z. Pawlak in 1982 [1]. It was coming after a long term in information system and proposed as a good formal tool for modeling and processing incomplete the information in information system. It is introduce as new mathematical method in an incomplete information. Recently, many researchers using the rough set theory in itself and many areas in the

real-life applications. It is coming as an extension of the

Set theory, in which a subset of a universe is described by lower and upper approximations. The upper approximation of a given set is the union of all the equivalence classes, which are subsets of the set, and the upper approximation is the union of all the equivalence classes that are intersection with set non-empty. Many researchers develop this theory and use rough theory in algebra. For example, the notation of rough subring with respect ideal has presented by Davvaz [2]. Algebraic properties of rough sets have been studied by Pomykala [3]. Mordeson [4] used covers of the universal set to defined approximation operators on the power set of the given set. In addition, Davvaz applied the concept of approximation spaces in the theory of algebraic hyper-structures and investigated the similarity between rough membership functions and conditional probability. The rough membership function had defined by equivalence class[5]. In addition, E.F .Luashin el al [6] extend the definition of rough membership function to topology Spaces. The Set valued functions [7] have used in many areas such as Economics [8].Our result is

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generalize the rough set theory in continues functions. Our method is to substitute an equivalence class by continuous functions. Moreover, by our assuming, we will get the new definition of rough membership function using the semi-continues function $F(x)$. That mean, We rewriter the definition of rough membership function by using continues function and we discuss several concepts and properties of continuous set value functions as new results on rough the continues function. In addition, we introduce the new definition of rough membership function in topology spaces by continuous functions and we gives some proves of some theorems and fundamental properties. We hope In future can use our new definition and discussed several concepts and properties of rough continuous set value functions as new results on rough continues function and membership continues function. We can conclude our result can extended the definition of rough membership function to topology spaces by substituted an equivalence class by continuous functions. Our result connects rough sets, topology spaces, fuzzy sets, and semi continues function. We believe our result has many applications in some areas.

The Lower and upper approximations are defined as follows:

Definition 1-1: A set valued function $F: X\rightarrow P(X)$ is function from non-empty X to $P(X)$ the set of all non-empty subsets of X such that $F(x) \neq \phi$ for all $x\in X$. If $B \subset X$, then we define the upper rough approximation by $\overline{F(B)} = \{x \in X|F(x)\subseteq B\}$ and the lower rough approximation by

$\underline{F(B)} = \{x\in X|F(x)\cap B\neq \emptyset\}$. Therefore $(\overline{F(B)}, \underline{F(B)})$

is called F-rough set of X. The boundary is $B(B)= \overline{F(B)} - \underline{F(B)}$, if $B\neq\emptyset$, then $B(B)$ is rough.

Remark 1-1: We define the domain of F by $D_F = \{x\in X: F(x)\neq\emptyset\}$, and the graph of F by $Graph(F) = \{(x, y): y\in F(x)\}$.so, the image of F is a subset of X defined by

$$Im(F)= \cup_{x\in X} F(x) = \cup_{x\in D_F} F(x).$$

Note that, if we define the domain of F by $D_F=\{x\in X: F(x)\neq\emptyset\}$, the set-valued map F is characterized by its graph; $Graph(F)=\{(x, y): y\in F(x)\}$ and the domain of F is the projection of $Graph(F)$ on X. The image of F is a subset of X defined by $Im(F)= \cup_{x\in X} F(x) = \cup_{x\in D_F} F(x)$ It is the projection of $Graph(F)$ on X.

Remark1-2: If $F:X\rightarrow P(X)$, then we called upper semi-continuous mapping on X if the set $\overline{F(A)}$ (resp. $\underline{F(A)}$)

is closed in X where A is closed in X .

Remark1-3: If $F:X\rightarrow P(X)$, then we called upper semi-continuous mapping on X if the set $\overline{F(A)}$ (resp. $\underline{F(A)}$)

is open in X where A is open in X**.**

Example 1-1: Let $X = \{1, 2, 3, 4, 5, 6\}$ and let $F :X\rightarrow P(X)$ where for every $x\in X$, $F(1) = \{1\}$, $F(2) = \{1, 3\}$, $F(3) = \{3,4\}$, $F(4) = \{4\}$, $F(5) =\{1,6\}$, $F(6) = \{1, 5, 6\}$. Let $A = \{1, 3, 5\}$, then $\overline{F(A)} = \{1, 2\}$, and $\underline{F(A)} = \{1, 2, 3, 5, 6\}$,

$B(A)\neq\emptyset$, is rough. $Im(F)= \cup_{x\in X} F(x) = \{1,3,4,5,6\}$.

Let $B=\{2,4,6\}$= then $\overline{F(B)} = \{4\}$, and $\underline{F(B)}$ =

$\{ 3,4, 5, 6\}$, $B(B)\neq\emptyset$, is rough.

Definition1-2: Suppose that F: $X \rightarrow P(X)$ is a set valued function. We define the upper continuous if for all $x\in X$ and any open $V\subset P(X)$ contain $F(x)$, then there is an open $O\subset X$ contain x such that $F(O)\subset V$. And, the lower continuous if for any $x\in X$ and for any open $V\subset P(X)$ such that $F(x)\cap V\neq\phi$ than there is open $O\subset X$ contain x such that $F(O)\cap V\neq\phi$. Therefore, we say F is Continuous if and only F has this property at each point of X.

Example 1-2: Let $X = \{1, 2, 3, 4, 5, 6\}$ and let $F : X\rightarrow P(X)$ where for every $x\in X$, $F(1) = \{1\}$, $F(2) = \{1, 3\}$, $F(3) = \{3,4\}$, $F(4) = \{4\}$, $F(5) =\{1,6\}$, $F(6) = \{1, 5, 6\}$. Let $A = \{1, 3, 5\}$, then $\overline{F(A)} = \{1, 2\}$, and $\underline{F(A)} = \{1, 2, 3, 5, 6\}$.

$B(A)\neq\emptyset$, is rough. $Im(F)=\cup_{x\in X} F(x) = \{1,3,4,5,6\}$. Let $B=\{2,4,6\}$= then $\overline{F(B)} = \{4\}$, and $\underline{F(B)} = \{ 3,4, 5, 6\}$,

$B(B)\neq\emptyset$, is rough.

Definition 1.4. Let F: $X\rightarrow P(X)$ be a set-valued function and A be an event in the function approximation space $S = (X, P)$. Then the lower probability of A is $\mathbf{P}_{\text{alpability}}(A)$= Palpability ($\overline{F(A)}$), and the upper probability is $\mathbf{P}^{\text{alpability}}(A)$ =Palpability ($\underline{F(A)}$).

Note that, respectively. Clearly, $0\leq\mathbf{P}^{\text{alpability}}(A)\leq1$ and $0 \leq \mathbf{P}_{\text{alpability}}(A) \leq 1$.

Example 1-3: We consider example 1-1, for $A = \{1, 3, 5\}$ the upper $\overline{F(A)} = \{1, 2\}$, then Palpability($\overline{F(A)}$) = 2/6 and the lower $\underline{F(A)} = \{1, 2, 3, 5, 6\}$,

then Palpability ($\overline{F(A)}$)=5/6=1

Proposition 1.1[9]: Let $F:X\rightarrow P(X)$ be a set-valued function and A,B be two events in the stochastic approximation space $S=(X,P)$. Then the following holds:

(1)$\mathbf{P}^{\text{alpability}}(\emptyset)=\emptyset=\mathbf{P}_{\text{alpability}}(\emptyset)$;

(2)$\mathbf{P}^{\text{alpability}}(X)=1=\mathbf{P}_{\text{alpability}}(X)$;

(3)$\mathbf{P}_{\text{alpability}}(A\cup B)\leq\mathbf{P}_{\text{alpability}}(A)+\mathbf{P}_{\text{alpability}}(B)-\mathbf{P}_{\text{alpability}}(A\cap B)$;

(4)$\mathbf{P}^{\text{alpability}}(A\cup B)\geq\mathbf{P}^{\text{alpability}}(A)+\mathbf{P}^{\text{alpability}}(B)-\mathbf{P}^{\text{alpability}}(A\cap B)$;

(5)$\mathbf{P}^{\text{alpability}}(A^c)=1-\mathbf{P}^{\text{alpability}}(A)$;

(6)$\mathbf{P}^{\text{alpability}}(A-B)\leq\mathbf{P}^{\text{alpability}}(A)-\mathbf{P}^{\text{alpability}}(A\cap B)$;

(7)$\mathbf{P}^{\text{alpability}}(A)\leq\mathbf{P}^{\text{alpability}}(A)$;

(8)If $A\subseteq B$, then $\mathbf{P}^{\text{alpability}}(A)\leq\mathbf{P}^{\text{alpability}}(B)$ and $\mathbf{P}_{\text{alpability}}(A)\leq \mathbf{P}_{\text{alpability}}(B)$.

Definition 1-5: Suppose that $F:X\rightarrow P(X)$ be a set-valued function. Let A be an event in the stochastic approximation space $S=(X,P)$. The rough probability of A, denoted by $P_*(A)$, is given by:

$$P_*(A) = (\mathbf{P}^{\text{alpability}}(A),\mathbf{P}_{\text{alpability}}(A)).$$

Proposition 1.2: Let $F:X\rightarrow P_*(X)$ be a set-valued function and A be a event in the stochastic approximation space $S =(X,P)$.

1) If F has reflective, then $\mathbf{P}^{\text{alpability}}(A)\leq P(A)\leq \mathbf{P}_{\text{alpability}}(A)$;
2) If F has reflective and transitive properties, then $\mathbf{P}^{\text{alpability}}(\overline{F(A)}) = \mathbf{P}_{\text{alpability}}(A)$ and $\mathbf{P}_{\text{alpability}}(\underline{F(A)}) = \mathbf{P}_{\text{alpability}}(A)$;
3) If A is an exact subset of X, then $\mathbf{P}^{\text{alpability}}(A)= \mathbf{P}_{\text{alpability}}(A)=P(A)$.

Proof. It is unpretentious.

Rough Of Membership Continuous Set Valued Function.

The rough membership function had defined by equivalence class. In addition, E.F .Luashin el al extend the definition of rough membership function to topology Spaces. We will introduce the new definition of rough membership function using the semi-continues function *F*(*x*) as:

$$\mu_A^{F(x)}(x) = \frac{|\{F(x)\}\cap A|}{|F(x)|}, F(x) \in P(X), x \in X \ldots\ldots\ldots(*)$$

Definition2-1: Let $A \subseteq X$, the closure of A is $\bar{A}$ and A° *is interior*, and A^b *is boundary*.

A is exact if $A^b = \emptyset$, otherwise A is rough. And A is exact iff $\bar{A} = A^{\circ}$.

Example 2-1: Let $X = \{1, 2, 3, 4, 5, 6\}$ and let $F : X \to P(X)$ where for every $x \in X$, $F(1) = \{1\}$, $F(2) = \{1, 3\}$, $F(3) = \{3,4\}$, $F(4) = \{4\}$, $F(5) = \{1,6\}$, $F(6) = \{1, 5, 6\}$. Let $A = \{1, 3, 5\}$. Let $A = \{1, 3, 5\}$, then $\overline{F(A)} = \{1, 2\}$, and $\underline{F(A)} = \{1, 2, 3, 5, 6\}$.

So, $B(A) \neq \emptyset$, is rough. $Im(F) = \cup_{x\in X} F(x) = \{1,3,4,5,6\}$. Let $B=\{2,4,6\}$= then $\overline{F(B)} = \{4\}$, and $\underline{F(B)} = \{3,4, 5, 6\}$.

We get, $B(B) \neq \emptyset$, is rough.

We have $A \cap B = \emptyset$. The, $\overline{F(A\cap B)} = \emptyset$, and $\underline{F(A\cap B)} = \emptyset$.

If we suppose that $C=\{1,2,3\}$, then $\overline{F(C)} = \{1, 2\}$, and $\underline{F(C)} = \{1, 2, 3,4, 5, 6\}$.

We can see $A \cap C = \{1,3\}$. We get, $\overline{F(A\cap B)} = \{1,2\}$, and $\underline{F(A\cap B)} = \{1,2,3,5,6\}$.

P-(A/C) =2/2=1, P+(A/C) =5/6,

$\mu_{A\cap C}^{F(x)}(1) = \frac{|\{1\}\cap\{1,3\}|}{|\{1\}|} = \frac{1}{1} = 1$; $\mu_{A\cap C}^{F(x)}(2) = \frac{|\{1,3\}\cap\{1.3\}|}{|\{1,3\}|} = \frac{2}{2} = 1$;

$\mu_{A\cap C}^{F(x)}(3) = \frac{|\{3,4\}\cap\{1.3\}|}{|\{3,4\}|} = \frac{1}{2}$; $\mu_{A\cap C}^{F(x)}(4) = \frac{|\{4\}\cap\{1.3\}|}{|\{4\}|} = \frac{0}{1} = 0$

; $\mu_{A\cap C}^{F(x)}(5) = \frac{|\{1,6\}\cap\{1,3\}|}{|\{1,6\}|}$ $\frac{1}{2}$; $\mu_{A\cap C}^{F(x)}(6) = \frac{|\{1,5,6\}\cap\{1,3\}|}{|\{1,5,6\}|} = \frac{1}{3}$.

$\mu_A^{F(x)}(1) = \frac{|\{1\}\cap\{1,3.5\}|}{|\{1\}|} = \frac{1}{1} = 1$; $\mu_A^{F(x)}(2) = \frac{|\{1,3\}\cap\{1,3.5\}|}{|\{1,3\}|} = \frac{2}{2} = 1$;

$\mu_A^{F(x)}(3) = \frac{|\{3,4\}\cap\{1,3.5\}|}{|\{3,4\}|} = \frac{1}{2}$; $\mu_A^{F(x)}(4) = \frac{|\{4\}\cap\{1,3,5\}|}{|\{4\}|} = \frac{0}{1} = 0$

; $\mu_A^{F(x)}(5) = \frac{|\{1,6\}\cap\{1,3,5\}|}{|\{1,6\}|}$ $\frac{1}{2}$; $\mu_A^{F(x)}(6) = \frac{|\{1,5,6\}\cap\{1,3,5\}|}{|\{1,5,6\}|} = \frac{2}{3}$.

If $B=\{2,4,6\}$

$\mu_B^{F(x)}(1) = \frac{|\{1\}\cap\{2,4.6\}|}{|\{1\}|} = 0$; $\mu_B^{F(x)}(2) = \frac{|\{1,3\}\cap\{2,4.6\}|}{|\{1,3\}|} = 0$;

$\mu_B^{F(x)}(3) = \frac{|\{3,4\}\cap\{2,4.6\}|}{|\{3,4\}|} = \frac{1}{2}$; $\mu_B^{F(x)}(4) = \frac{|\{4\}\cap\{2,4.6\}|}{|\{4\}|} = \frac{1}{1} = 1$

; $\mu_B^{F(x)}(5) = \frac{|\{1,6\}\cap\{2,4.6\}|}{|\{1,6\}|}$ $\frac{1}{2}$; $\mu_B^{F(x)}(6) = \frac{|\{1,5,6\}\cap\{2,4.6\}|}{|\{1,5,6\}|} = \frac{1}{3}$.

Now, for $A \cup B = X$

$\mu_X^{F(x)}(1) = \frac{|\{1\}\cap X|}{|\{1\}|} = \frac{1}{1} = 1$; $\mu_X^{F(x)}(2) = \frac{|\{1,3\}\cap X|}{|\{1,3\}|} = \frac{2}{2} = 1$;

$\mu_X^{F(x)}(3) = \frac{|\{3,4\}\cap X|}{|\{3,4\}|} = \frac{2}{2} = 1$; $\mu_X^{F(x)}(4) = \frac{|\{4\}\cap X|}{|\{4\}|} = \frac{1}{1} = 1$

; $\mu_X^{F(x)}(5) = \frac{|\{1,6\}\cap X|}{|\{1,6\}|} = 1$; $\mu_X^{F(x)}(6) = \frac{|\{1,5,6\}\cap X|}{|\{1,5,6\}|} = \frac{3}{3} = 1$

Then we have $\mu_X^{F(x)} = \sum \mu_A^{F(x)} + \mu_B^{F(x)} = 6$.

We can conclude the next proposition.

Proposition 2.1: If β is a family of pairwise disjoint subsets of X then $\mu_{\cup\beta}^{F(x)}(x) = \sum_{A\in\beta} \mu_A^{F(x)}(A)$ for any $x \in X$.

Proof

We have $\mu_{\cup\beta}^{F(x)}(x) = \frac{|F(x)\cap\cup\beta|}{|F(x)|} = \frac{|\cup\{F(x)\cap A : A\in\beta\}|}{|F(x)|} = \sum_{A\in\beta} \mu_A^{F(x)}(A)$.

Remark 2-1: we can define the fuzzy set by using the equation (*)

$$\check{A} = \{\left(x, \mu_A^{F(x)}(x)\right)\}.$$

Form example1-1, for Let $A = \{1, 3, 5\}$, we can define $\check{A} = \{(1,1), (2,1), (3,1)(4,0)(5,1/2)(6,2/3)\}$. However, the rough membership (*) is very different from rough set theory or Lashin's rough membership function [6].

Proposition 2-2: Let $A \subseteq X$. The rough member function $\mu_A^{F(x)}(x)$ has the following properties:

$\mu_A^{F(x)}(x) = 1$ iff $x \in \overline{F(A)}$; $\mu_A^{F(x)}(x) = 0$ iff $x \in (\underline{F(A)})c$.

Proof

If $x \in \overline{F(A)}$; then $x \in X \mid F(x) \subseteq A$, then $\mu_A^{F(x)}(x) = 1$. If $\mu_A^{F(x)}(x) = 1$, then $x \in \overline{F(A)}$;

We have $x \in (\underline{F(A)})^c \Leftrightarrow F(x) \cap A = \emptyset \Leftrightarrow \mu_A^{F(x)}(x) = 0$.

Example 2-2: From example 2-1, let $X = \{1, 2, 3, 4, 5, 6\}$, Let $A = \{1, 3, 5\}$, then $\overline{F(A)} = \{1, 2\}$, and

$\underline{F(A)} = \{1, 2, 3, 5, 6\}$.

So, $\mu_A^{F(x)}(1) = \frac{|\{1\}\cap\{1,3.5\}|}{|\{1\}|} = \frac{1}{1} = 1$;

$\mu_A^{F(x)}(2) = \frac{|\{1,3\}\cap\{1,3.5\}|}{|\{1,3\}|} = \frac{2}{2} = 1$;

$\mu_A^{F(x)}(3) = \frac{|\{3,4\}\cap\{1,3.5\}|}{|\{3,4\}|} = \frac{1}{2}$; $\mu_A^{F(x)}(4) = \frac{|\{4\}\cap\{1,3,5\}|}{|\{4\}|} = \frac{0}{1} = 0$

; $\mu_A^{F(x)}(5) = \frac{|\{1,6\}\cap\{1,3,5\}|}{|\{1,6\}|}$ $\frac{1}{2}$; $\mu_A^{F(x)}(6) = \frac{|\{1,5,6\}\cap\{1,3,5\}|}{|\{1,5,6\}|} = \frac{2}{3}$.

Let $B=\{2,4,6\}$= then $\overline{F(B)} = \{4\}$, and $\underline{F(B)} = \{3,4, 5, 6\}$.

$\mu_B^{F(x)}(1) = \frac{|\{1\}\cap\{2,4.6\}|}{|\{1\}|} = 0$; $\mu_B^{F(x)}(2) = \frac{|\{1,3\}\cap\{2,4.6\}|}{|\{1,3\}|} = 0$;

$\mu_B^{F(x)}(3) = \frac{|\{3,4\}\cap\{2,4.6\}|}{|\{3,4\}|} = \frac{1}{2}$; $\mu_B^{F(x)}(4) = \frac{|\{4\}\cap\{2,4.6\}|}{|\{4\}|} = \frac{1}{1} = 1$

; $\mu_B^{F(x)}(5) = \frac{|\{1,6\}\cap\{2,4.6\}|}{|\{1,6\}|}$ $\frac{1}{2}$; $\mu_B^{F(x)}(6) = \frac{|\{1,5,6\}\cap\{2,4.6\}|}{|\{1,5,6\}|} = \frac{1}{3}$.

We can extend the concepts of rough set approximations to any subfamily of $P(X)$

Definition2-2: Let $\eta \subseteq P(X)$, we define the η- upper approximation $\overline{F(\eta)}$ = {$A \in P(X)$ | $F(A) \subseteq \eta$ } and η-lower approximation $\underline{F(\eta)}$ ={$A \in P(X)$ |$F(A) \cap \eta \neq \emptyset$}.

Preposition 2-3: $F(F(x))$ ={ $F(x)$ } for all $x \in X$}.

Proof:

We have $\overline{F(F(x))}$=$F(x)$ =$\underline{F(F(x))}$; if $A\in F(F(x))$, then $\underline{F(A)}$= $\underline{F(F(x))}$= $F(x)$ = $\overline{F(F(x))}$= $\overline{F(A)}$= A .

Therefore, $F(F(x))$ ={ $F(x)$ }.

Approximation Of Continuous Set Valued Mapping And Membership Function Rough Topology.

In this section, we will introduce the new definition of rough membership function using the semi-continues function as extend the definition of rough membership function to topology Spaces by E.F .Luashin el al [6].

$$\mu_A^\tau(x) = \frac{|\{F(x)\}\cap A|}{|F(x)|}, F(x) \in P(X), x \in X \text{……….(*)}$$

Example3-1: Let we consider the set X = {1, 2, 3, 4, 5, 6} and let F :$X\to P(X)$ where for every $x\in X$, $F(1)$ = {1,2,3}= $F(2)$, $F(3)$ = {3,4}= $F(4)$, $F(5)$ ={4,5}, and $F(6)$ = {6}. Let A = {1, 2, 3, 4}, Then S:={ {1,2,3},{3,4},{4,5}, {6}},then β={{1,2,3},{3,4},{4,5},{6}, {3},{4} }, We get τ= {X,∅, {1,2,3},{3,4},{4,5},{6},{3},{4},{1,2,3,4},{1,2,3,4,5},{1,2,6},{3,4, 5},{3,4,6},{,6},{4,6},{3,4,5,6}}.

$\mu_A^\tau(1) = \frac{|\{1,2,3)\}\cap\{1,2,3,4\}|}{|\{1,2,3\}|} = \frac{1}{1} = 1$;$\mu_A^\tau(2)$=1;

$\mu_A^\tau(3)$=$\frac{|\{3,4)\}\cap\{1,2,3,4\}|}{|\{3,4\}|} = \frac{2}{2} = 1$;

$\mu_A^\tau(4)$=1 ;$\mu_A^\tau(5)$=$\frac{|\{4,5\}\cap\{1,2,3,4\}|}{|\{4,5\}|}$ =$\frac{1}{2}$;

$\mu_A^\tau(6)$=$\frac{|\{6\}\cap\{1,2,3,4\}|}{|\{6\}|}$ =$\frac{0}{1} = 0$.

Let B = {5, 6 }, we have $\mu_B^\tau(1) = \frac{|\{1,2,3)\}\cap\{5,6\}|}{|\{1,2,3\}|} = 0$;$\mu_B^\tau(2)$=0;

$\mu_B^\tau(3)$=0 ;$\mu_B^\tau(4)$=0 ;$\mu_B^\tau(5)$=$\frac{|\{4,5\}\cap\{5,6\}|}{|\{4,5\}|}$ =$\frac{1}{2}$;

$\mu_B^\tau(6)$=$\frac{|\{6\}\cap\{5,6\}|}{|\{6\}|}$ =$\frac{1}{1} = 1$.

For X. $\mu_X^\tau(1) = \frac{|\{1,2,3)\}\cap X|}{|\{1,2,3\}|}$ =1 =$\mu_X^\tau(2)$;

$\mu_X^\tau(3)$=$\frac{|\{3,4)\}\cap X|}{|\{3,4\}|} = \frac{2}{2} = 1$=$\mu_X^\tau(4)$;$\mu_X^\tau(5)$=$\frac{|\{4,5\}\cap X|}{|\{4,5\}|}$ =1; $\mu_X^\tau(6)$=$\frac{|\{6\}\cap X|}{|\{6\}|}$ =1 .

Proposition 3.1: Suppose that $\beta$is a family of pairwise disjoint subsets of X then $\mu_{\cup\beta}^\tau(x) = \sum_{A\in\beta}\mu_A^\tau(A)$ for any $x \in X$.

Proof

The same way of proof in theory Proposition 2.1.

Note that, we can get the interior and closure of A from the family F of all τ-closed sets:

F={A,∅,{4,5,6},{1,2,5,6},{1,2,3,6},{1,2,3,4,5},{1,2,4,6},{1,2,3,5,6 },{5,6},{6},{4,5},{1,2,6},{1,2,6},{1,2,5}, {1,2,3},{1,2,4,5},{1,2,3,5},{1,2}}

A°={1,2,3}∪{3,4}∪{3}∪{4}={1,2,3,4},

$\bar{A}$= X∩{1,2,3,4,5}={1,2,3,4,5}.

Note that we can get it from rough membership function.$\overline{F(A)}$ = {1, 2,3,4}, and $\underline{F(A)}$ = {1, 2, 3, 4, 5}.

It is clear A is rough from definition 2-1.Also, $B(A)$≠∅, then rough.

Conclusion

The rough sets theory considered as a generalization of the classical sets theory. The main idea of rough set were built by equivalence relation. Occasionally, an equivalence is difficult to be obtained in rearward problems due to vagueness and incompleteness of human knowledge. In the present paper, we generalize rough set theory in continues functions. We substituted an equivalence class by continuous functions. In this paper, we will introduce the new definition of rough membership function using continues function. An addition, we discussed several concepts and properties of rough continuous set value functions as new results on rough the continues function and membership continues function and we extended the definition of rough membership function to topology spaces by substituted an equivalence class by continuous functions. Our result connects rough sets, topology spaces, fuzzy sets, and semi continues function. We believe our result has many applications any some areas.

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