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Structural Evolution of $^{146–158}$ Nd Isotopes Using IBM-2 Hamiltonian

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ABSTRACT

Neodymium isotopes (Z=60) lie in the traditional rotational to transitional-spherical region that occurs at the range of deformed nuclei. The identity of 146−158 Neodymium nuclei have been determined in framework of Interacting Boson Model-2, a simulation program with NPBOS has been used to obtain the theoretical energy levels for Neodymium isotopes. The parameters of the best fit to the measured data are determined. The energy positive parity bands of those isotopes were calculated using (IBM-2) and then it compared with the experimental values. The mentioned isotopes ratios $\frac{E(4_1^+)}{E(2_1^+)}$ have calculated and also E-GOS curves $(\frac{E_\gamma}{I})$ have drawn as a function of the spin I, to classify symmetry of the nuclei. The electromagnetic transition probabilities B(E2) of these nuclei was calculated as well, where a good agreement for low lying energy states were obtained between experimental results and theoretical calculations.The results have been shown that the interested nuclei $^{152\text{-}158}$Nd have rotational characters SU (3), and O (6) symmetry is substantiated for $^{146\text{-}148}$Nd, as well as the critical point X (5) has been determined for ^{150}Nd isotope.

التطور البنيوي لنظائر 146–158 Nd باستخدام هاملتوني IBM-2

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الكلمات المفتاحية:
الحركة الجماعية
النقطة الحرجة
مستويات الطاقة
منحنى E-GOS
نظائر النيوديميوم
نموذج المفاعلة البوزونية-2

الملخص

تمتد نظائر النيوديميوم (Z=60) بين المنطقة الدورانية التقليدية و المنطقة الكروية الإنتقالية الحادثة عن النوى المشوهة. وقد تم تحديد هوية نوى النيوديميوم 146-158 في إطار نموذج المفاعلة البوزونية-2، وقد تم استخدام برنامج محاكاة (NPBOS) للحصول على مستويات الطاقة النظرية للنظائر المذكورة، بحيث يتم تحديد البارامترات الأكثر ملائمة للبيانات المقاسة. تم حساب مستويات الطاقة ذات التماثل الموجب لتلك النظائر باستخدام (IBM-2) ومن ثم مقارنتها مع القيم التجريبية، كما تم حساب نسب النظائر المذكورة $(\frac{E(4_1^+)}{E(2_1^+)})$ ، وكذلك تم رسم منحنيات E-GOS كدالة في الدوران I $(\frac{E_\gamma}{I})$ ، وذلك لتصنيف تناظر النوى. كما تم حساب إحتمالات الإنتقال الكهرومغناطيسي B(E2) لهذه النوى أيضاً. وقد تم الحصول على توافق جيد بين النتائج التجريبية والحسابات النظرية لحالات الطاقة المنخفضة. أظهرت النتائج أن النوى Nd152-158 لها خصائص دورانية (3) SU، والنوى Nd146-148 لها خصائص جاما الناعمة (6) O، كما تم تحديد النقطة الحرجة (5) X للنظير Nd150.

Introduction:

Scientists have made many attempts to explore the factors responsible for the onset of large deformation in nuclei of the mass region A> 100. One of those attempts that has been successful in describing the low-lying nuclear collective motion in medium and heavy mass nuclei, is the Interacting Bosons Model (IBM), presented by Arima and Iachello [1], [2], and Casten [3], which has become

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widely accepted as a tractable theoretical scheme of correlating, describing and predicting low-energy collective properties of complex nuclei, since it reduces the number of states heavily [4], [5]. The building blocks of the IBM is the monopole s and the quadrupole d bosons, reflecting the collective and collective pairs of valence nucleons. The number of bosons is half the number of valence nucleons counted from the nearest closed shells. When departing from the closed shell (Near Magic) with the increase of the valence nucleon number N, the shape changes from spherical vibrator (Sph. Vib.) to deformed rotor (Def. Rot.), passing through the transitional nuclei in between. Each shape results in the characteristic level structure: phonon-like level scheme for a vibrator, and a clear rotational band for a rotor, which are well indicated by the ratio of 4_1^+ to 2_1^+excitation energies, denoted by $R_{4/2}$ [6].

Many studies have been done by using (IBM-1) to identify the energy of different states along the yrast region applying a suitable limit for each isotope. However, this model does not distinguish between proton pairs and neutron pairs [5], [7]. Therefor in this work we have been consider the proton-neutron interacting boson model (IBM-2), which distinguishes between proton and neutron bosons [8]. The IBM-2 is more closely connected with a microscopic picture than the original version of the IBM-1. The IBM-2 is comprised of the so-called proton (neutron) s_π (s_ν) and the proton (neutron) d_π (d_ν) bosons, which reflect collective L = 0^+ and 2^+ proton (neutron) pairs, respectively. The number of proton (neutron) bosons N_ρ is equal to half the numbers of valence protons (neutrons). Since in medium-heavy and heavy nuclei protons and neutrons occupy in the different major shells, one has only to consider proton-proton and neutron-neutron pairs in the IBM-2 framework, neglecting the proton-neutron pair [2], [6], [9].

Spectrum generating algebra for the nucleus is U (6), since all physical observables (Hamiltonian, Transition Operators…) are expressed in terms of the generators of U (6) [10]. The energy ratio of the first $4_1^+$and 2_1^+excited states in even–even nuclei, namely, $R_{4/2}=\frac{E(4_1^+)}{E(2_1^+)}$ is generally regarded as a good indicator of different collective motions and critical point symmetries of a nucleus, especially in low-lying nuclear structure. As is well known, based on some ideal assumptions, the energy ratio $R_{4/2}$ has been predicted to be 10/3 for a well deformed axially symmetric rotor, 2.5 for the γ -unstable limit, and 2.0 for a spherical vibrator, corresponding to SU (3), O (6), and U (5) dynamic symmetries, respectively. Further, the $R_{4/2}$ value will be about 2.9 for the X (5) symmetry (the critical point of the spherical to deformed transition path) [11], [12], [13].

Furthermore, E-GOS curves (E-Gama over spin) are used to understand the evolution of nuclear collectivity, indicating a non-ignorable vibration effect. Moreover, taking the first-order rotation–vibration interaction into account, we further extend the E-GOS curve, which can actually agree with the expected trend [14].

In this paper, the neodymium (Nd) even-even isotopes with atomic number (Z=60) were studied theoretically by using the (IBM-2) to identify the energy levels, the energy ratios and electromagnetic transitions probabilities (B(E2) and B(M1)) for even neutrons (N=86-98) of Nd isotopes. These isotopes calculations are compared with the standard values for the three limits, the vibrations U (5), gamma-soft O (6) and the rotational SU (3).

The (IBM-2) Model Theory;

Nuclear collective excitations are described in terms of N s and d bosons, Spectrum generating algebra for the nucleus is U (6). All physical observables (Hamiltonian, transition operators, …) are expressed in terms of the generators of U (6). Formally, nuclear structure is reduced to solving the problem of N interacting s and d bosons.

The IBM-2 Hamiltonian in this model is given as [15]:

$$H=\varepsilon_d(n_{d\nu}+n_{d\pi})+k(Q_\nu.Q_\pi)+V_{\nu\nu}+V_{\pi\pi}+M_{\nu\pi} \quad (1)$$

Where ε_d is the energy difference between s and d boson, $n_\rho$is the number of d bosons, where ρ corresponds to π(proton) or ν(neutron) bosons, the second term denotes the quadrupole-quadrupole interaction between proton and neutron with strength k, which written as [16], [17]:

$$\hat{Q}_\rho=d_\rho^\dagger s_\rho+s_\rho^\dagger\tilde{d}_\rho+\chi_\rho[d_\rho^\dagger\tilde{d}_\rho]^{(2)} \quad (2)$$

$V_{\nu\nu}, V_{\pi\pi}$ are represented the interaction of identical bosons, and is given by

$$\hat{V}_{\rho\rho}=1/2\sum_{L=0,2,4}C_\rho^{(L)}\left([d_\rho^\dagger d_\rho^\dagger]^{(L)}.\left([\tilde{d}_\rho\tilde{d}_\rho]^{(L)}\right)\right) \quad (3)$$

and $M_{\nu\pi}$ is the Majorana term separates the configurations which are totally symmetric under the interchange of the proton and neutron from those configurations which have mixed symmetry in the protons and neutrons wave function [18], [19], where

$$\hat{M}_{\nu\pi}=\frac{1}{2}\zeta_2\left[(d_\nu^\dagger s_\pi^\dagger-d_\pi^\dagger s_\nu^\dagger).(\tilde{d}_\nu s_\pi-\tilde{d}_\pi s_\nu)\right]+\sum_{k=1,3}\zeta_k\left([d_\nu^\dagger d_\pi^\dagger]^{(k)}[\tilde{d}_\pi\tilde{d}_\nu]^{(k)}\right) \quad (4)$$

E-GOS curves:

Typical E-GOS curves for a perfect harmonic vibrator, γ -soft, and axially symmetric rotors with the first 2^+ excitations of 500, 300, and 100 keV, respectively, are plotted with the isotope E-GOS curve to know the isotope's limit [20]. As mentioned by Regan et al., the gamma-ray decay energies for a perfect harmonic vibrator are given by [21], [22]:

$$\frac{E_\gamma}{I}=\frac{\hbar\omega}{I} \quad (5)$$

While, for an axially deformed structure it written as:

$$\frac{E_\gamma}{I}=\frac{4-2/I}{2J/\hbar^2} \quad (\mathbf{6})$$

However, for γ -soft rotor it can be obtained with this formula:

$$\frac{E_\gamma}{I}=\frac{\mathrm{E}(2_1^+)}{4}\left(\mathbf{1}+\frac{\mathbf{2}}{\boldsymbol{I}}\right) \quad (\mathbf{7})$$

Where, (I) is the spin and, (J) is the static moment of inertia.

Electric Transition Probability B(E2):

For general one body operator, the reduced matrix element which does not depend on the L_Z=M quantum number (electromagnetic transition operator ($T^{(E2)}$)) In IBM-2, is given by [16]:

$$T^{(E2)}=e_\pi Q_\pi+Q_\nu e_\nu \quad (8)$$

Where $e_\pi$and $e_\nu$are boson effective charges measured by eb units, depending on the boson number N_ρ, those parameters are free and can take any an incentive to suit the test information.

The B(E2) ratios, defined as follows [6]:

$$R_1=\frac{B(E2;\ 4_1^+\to 2_1^+)}{B(E2;\ 2_1^+\to 0_1^+)} \quad (9)$$

$$R_2=\frac{B(E2;\ 2_2^+\to 2_1^+)}{B(E2;\ 2_1^+\to 0_1^+)} \quad (10)$$

$$R_3=\frac{B(E2;\ 0_2^+\to 2_1^+)}{B(E2;\ 2_1^+\to 0_1^+)} \quad (11)$$

which, as we shall show, exhibit the limit of the isotopes.

Results and discussion;

The (IBM) provides a unified description of collective nuclear states in terms of a system of interacting bosons [23]. The ${}^{146-158}_{60}Nd$ isotopes have 5 proton bosons number ($N_\pi=5$ proton bosons outside the closed shell 50) with 2-8 neutron bosons ($N_\nu=2,3,4,5,6,7,8$), respectively, measured from the closed shell at N=82.

The energy spectra are obtained by diagonalizing the IBM-2 Hamiltonian using the NPBOS code. An attempt has been made to reduce the number of free parameters and produce the smoothness variation of the parameters [18]; Used parameters are listed in Table-1, While the parameters, $k,\chi_\rho\ and\ \varepsilon_\rho$, as well as the Majorana parameters ζ_k with k =1,2,3 were treated as free parameters and their values were estimated by fitting with the experimental values [12], [24]. Using the parameters in Table 1, the estimated energy levels can be obtained. As can be seen in fig (1. a, b and c), the agreement between experimental results and theoretical calculations for low lying energy states; is quite good and the general features are reproduced well. We observe little discrepancy between theoretical and experimental results for high spin states ($8_1^+, 10_1^+$) [25].

In general, sharp change from a γ-unstable to an axially deformed

rotor structure of the heavier neodymium isotopes can be inferred from Fig.2, Here the excitation energy ratio of the 4_1^+state to the 2_1^+state, which is one of the usual benchmarks used in assessing collectivity, is reported as a function of the neutron number (N), with the values predicted by the U (5), O (6) and SU (3) symmetries. This figure suggests that $^{152\text{-}158}$Nd is inore deformed than $^{146\text{-}148}$Nd, It also shows that for $^{152\text{-}158}$Nd the $(E_{4_1^+}/E_{2_1^+})$ ratios are far from the U(5) limit; for $^{146\text{-}148}$Nd the ratios reach the O(6) limit, 2.5, while $^{152\text{-}158}$Nd the ratios arise to reach the SU(3), rotational limit, 3.3, in contrast the ratios for ^{150}Nd were very closely follow the X(5), limit (critical point symmetry), 2.91.[26], [27]
Figure (3) shows the theoretical limits plotted for three schematic nuclei: vib., 500 keV; γ-unstable, 300 keV; and rotor, 100 keV, with respect to isotope's E-GOS curve. We can note that the $^{146\text{-}148}$Nd isotopes curves follow the γ-unstable (O (6)) limit, but $^{152\text{-}158}$Nd isotopes curves follow the axially deformed rotor (SU (3)) limit, in contrast for ^{150}Nd isotope was not far from SU (3) limit, and it classified as X (5) limit, these results enhance what was inferred from the use of energy ratio classification.
For more precisely, another classification based on the electrical transition probability (R1, R2, R3) Which is obtained by calculating the initial value of the boson effective charge, has been used. Here we have found that the proton effective charge $e_\pi = 0.09\ eb$ and the neutron effective charge $e_\nu = 0.26\ eb$. And since the standard values of the ratios (R1, R3) were equal, for the two limits, SU (3) and O (6), Therefore, these two classifications of the ratios were canceled, and only the (R2) ratio was used as a classification test here. The standard (R2) ratio is 2 in the U (5), zero in the SU (3) and 1.42 in the O (6) limit. Results showed that the ^{150}Nd isotope close to follow the SU (3) limit, because of the calculated R2 was equal to 0.313 and the experimental value equal to 0.086, which closest to the rotational limit. Table-2 shows the limit classification of $^{146\text{-}158}$Nd isotopes. The values for ^{150}Nd are remarkably close to X (5) predictions and show a significant deviation from the rotor values.

Figures:

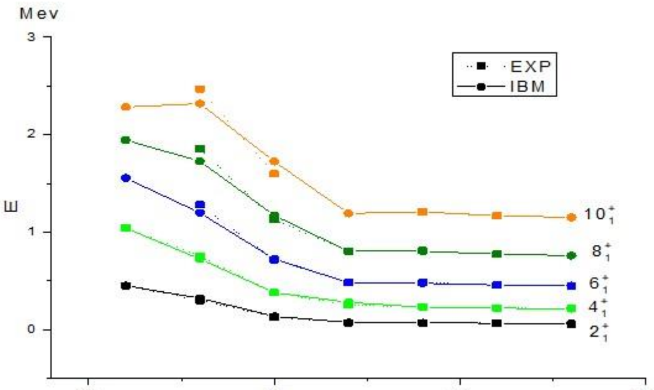


Fig. 1.a. Comparison between the measured and the calculated yrast band energies of 5 levels of $_{60}$Nd isotopes with even numbers of neutrons 86 up to 98

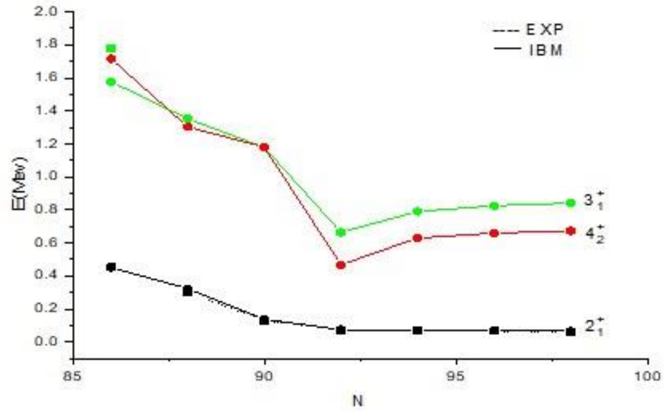


Fig. 1.b measured and the calculated γ band energies

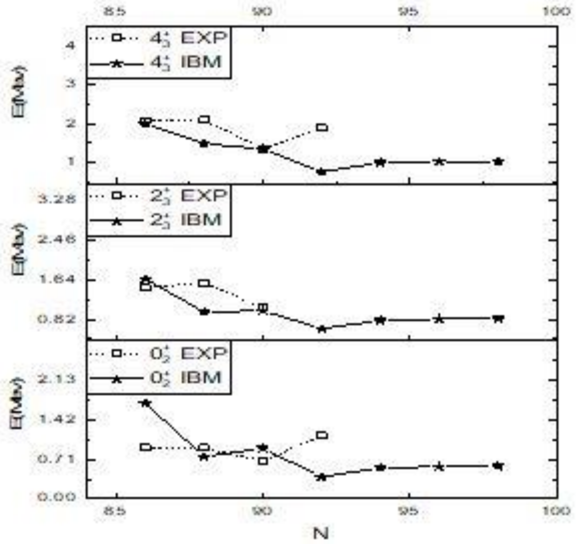


Fig. 1.c measured and the calculated β band energies

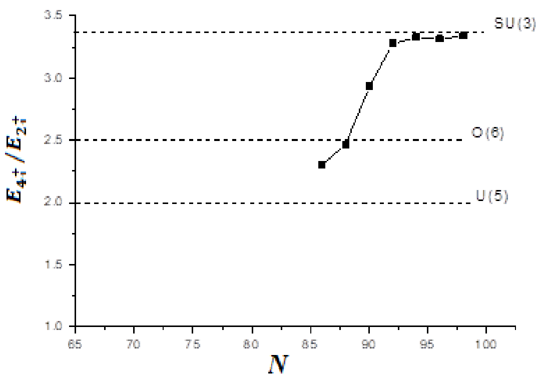
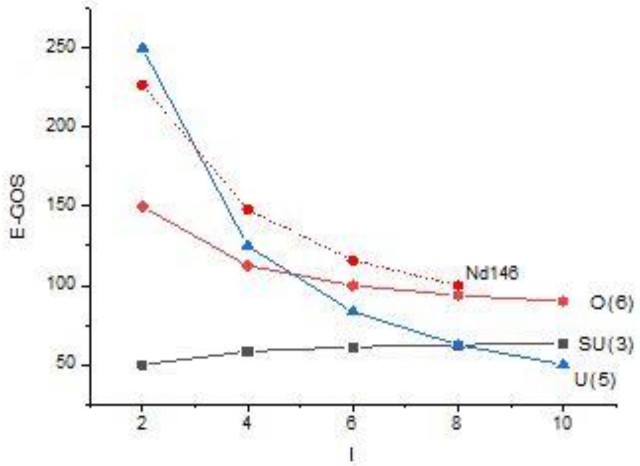
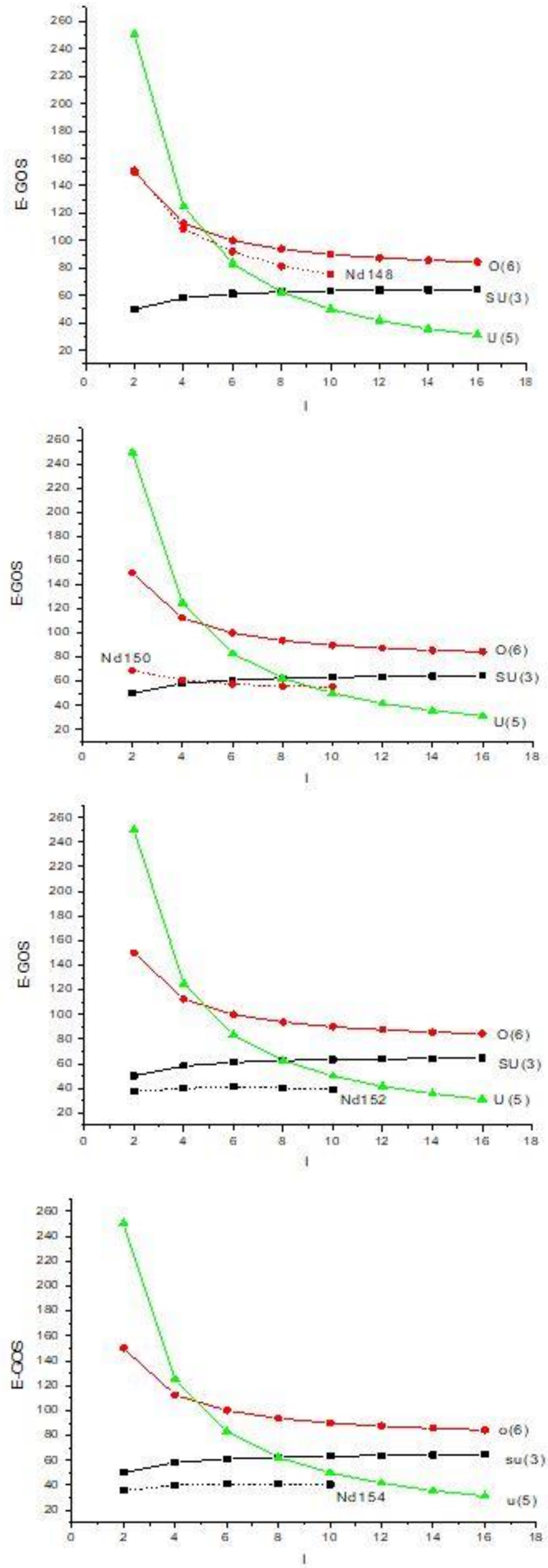


Fig.2. Systematic of $(E_{4_1^+}/E_{2_1^+})$ for Nd isotopes as a function of neutron number





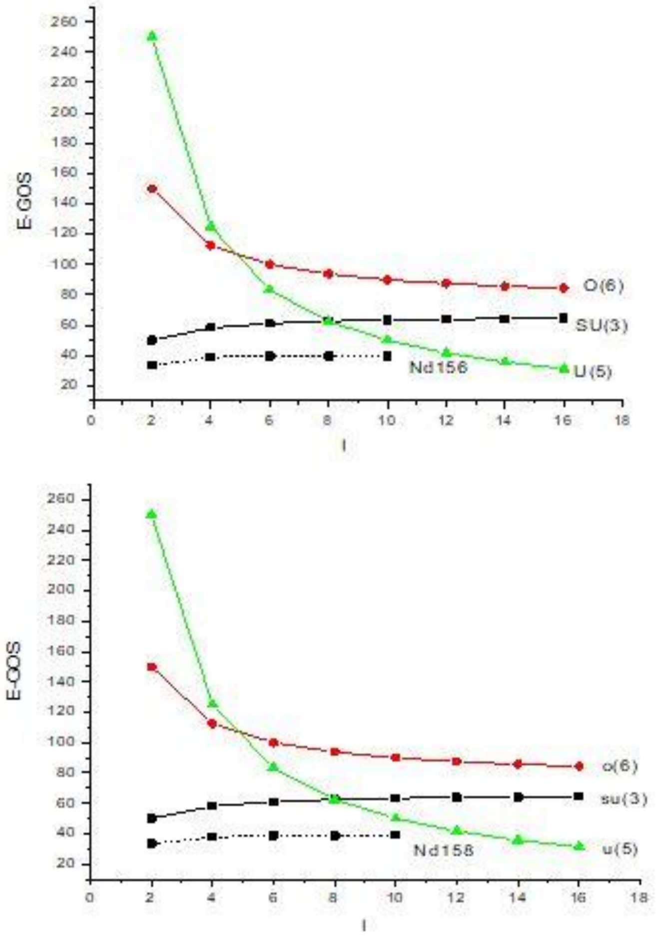


Fig.3. E-GOS curves for $^{146-158}$Nd isotopes with typical E-GOS curves

Tables:

Table 1: Hamilton Parameters for (146-158) Neodymium isotopes

$^{A}_{Z}$X	$^{146}_{60}$Nd	$^{148}_{60}$Nd	$^{150}_{60}$Nd	$^{152}_{60}$Nd	$^{154}_{60}$Nd	$^{156}_{60}$Nd	$^{158}_{60}$Nd
ε_d	0.805	0.650	0.555	0.347	0.375	0.382	0.388
κ	-0.098	-0.070	-0.093	-0.068	-0.067	-0.067	-0.067
χ_π	-1.200	-1.200	-1.200	-1.200	-1.200	-1.200	-1.160
χ_ν	-1.100	-1.100	-1.200	-1.100	-1.100	-1.100	-1.030
ξ_2	-0.00	0.00	0.00	-0.130	-0.100	-0.100	-0.100
	-0.600	0.0	8.0	0.0	0.0	0.0	0.0
CL_π	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	-0.05	0.0	0.0	0.0	0.0	0.0
	0.900	0.0	1.00	0.0	0.0	0.0	0.0
CL_ν	-0.0	0.0	0.00	0.0	0.0	0.0	0.0
	-0.0	0.0	0.00	0.0	0.0	0.0	0.0

Table 2: Isotopes $^{146-158}$Nd classification

SU (3)	O (6)	X (5)
Nd^{152}_{60}	Nd^{146}_{60}	Nd^{150}_{60}
Nd^{154}_{60}	Nd^{148}_{60}	
Nd^{156}_{60}		
Nd^{158}_{60}		

Conclusion

The results of this work show that the IBM-2 provides a good description of even-even Nd nuclei. The calculated excitation energies of the yrast – band, $\boldsymbol{\beta}$ band and $\boldsymbol{\gamma - band}$ clarify very good agreement with the experimental data. The results obtained for $E_{4_1^+}/E_{2_1^+}$values indicate that the shape transition for these isotopes is from O (6) toward SU (3) and the occurrence of E-GOS curve confirm that $^{146-158}$Nd isotopes under this study exist in region O (6)-SU (3). The results of the electromagnetic transition probability also confirmed that.

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