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Exact Calculations for One Photon Micromaser with Coherent Inputs II

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ABSTRACT

We introduce results for the second part of our calculations for the action of a single atom and one photon micromaser when atoms are coherently injected into a microwave cavity, and for the case when the atomic probabilities $|\alpha| \neq 0$ and $|\beta| \neq 0$. During the presence of the atom inside the cavity the field couples to a heat bath at temperature T. The results of this second part and those of the first part of our work give a complete information for the behaviour of the micromaser field for the two cases of injected atoms namely $|\alpha|=1, |\beta|=0$ and $|\alpha| \neq 0, |\beta| \neq 0$. We have found that the evolution of the field after a sufficient number of atoms had passed the cavity shows a steady state when the repetition time T_p is much greater than the interaction time t_int. We also found that the trapping state at n = 3 plays an important role in the early dynamics of the field within the cavity. Finally, we found that the field evolution towards a mixed state and not to a pure state. This prevention of purity because of: the presence of decay of the field inside the cavity, the presence of the trapping state at n = 3 (centered between the two trapping states at photon number n = 0 and at a photon number n = 15) prevents the field from developing towards a pure state, and finally the presence of the black body radiation field initially in the cavity at temperature T.

Acronyms
Maser: an acronym for microwave amplification by stimulated emission of radiation,
JCM: an acronym for Jaynes Cummings Model.

الحسابات المضبوطة لميكروميزر الفوتون الواحد بمدخلات متماسكة2

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الكلمات المفتاحية:
التراكب المتماسك
حالات الاصطياد
حالة الاستقرار
زمن التفاعل
زمن التكرار
عدد الفوتون
ميكروميزر

الملخص

نقدم نتائج الجزء الثاني للحسابات الخاصة بأداء ميكروميزر الذرة المفردة والفوتون الواحد عندما تحقن الذرات الى داخل فجوة ميكروويفية بشكل متماسك وفي الحالة التي تكون فيها الاحتمالات الذرية وحالاتها السفلية والعلوية $|\alpha| \neq 0$ و $|\beta| \neq 0$. عندما تكون الذرة داخل الفجوة يستمر الاقتران بالحمام الحراري عند درجة حرارة T. نتائج هذا الجزء الثاني إضافة الى نتائج الجزء الأول من بحثنا تعطي معلومات كافية ووافية عن تصرفات المجال الميكروميزري في جميع حالات الذرات المقذوفة داخل الفجوة وفي احتمالين إثنين الأول عندما $|\alpha|=1, |\beta|=0$ والثاني عندما $|\alpha| \neq 0, |\beta| \neq 0$. لقد وجدنا أن تطور المجال بالفجوة بعد عبور عدد كاف من الذرات يظهر حالة استقرار بالمجال وذلك عندما يكون الزمن بين الذرات المقذوفة T_p أكبر بكثير من زمن التفاعل t_int بين الذرة والمجال داخل الفجوة. وجدنا أيضا أن الحالة الأولى للاصطياد (الحالة الفخية) عند n = 3 تلعب دورًا مهمًا في الديناميكيات المبكرة للمجال داخل الفجوة. وأخيرا وجدنا أن تطور المجال في الغالب نحو حالة مختلطة وليس نحو حالة نقية. وهذا المنع من الوصول للحالة النقية بسبب الآتي: اضمحلال المجال داخل الفجوة ، ووجود حالة الاصطياد (الحالة الفخية) عند n = 3 (المتمركزة بين الحالتين الفخيتين أي حالتي الاصطياد عند العدد

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الفوتوني n = 0 والعدد الفوتوني n = 15) تمنع المجال من التطور نحو حالة نقية، وأخيرا وجود مجال إشعاع الجسم الأسود ابتدائيا في الفجوة الميكروويفيةعند درجة الحرارة T.

Introduction

The quantum theory of the micromaser was first developed by P.Filipowicz et. al.1986 [1]. This theory is a direct application of the quantum theory of the laser to a problem of two level Rydberg atoms interacting with a single mode radiation field in a microwave cavity [2-8]. In most studies of the one photon micromaser theory the injection of atoms assumes to have Poisson statistics and all atoms always entered the cavity in their excited (upper) states.

This device, the micromaser, exhibits highly non-classical features such as a sub-Poisson statistics for the field, quantum revivals and trapping states. The experimental realization of a micromaser has been made possible because of the enormous progress in both the construction of superconducting cavities and the progress in laser technology which have been offered a high excitation of Rydberg atoms (two-level atoms) that can be injected into a superconducting cavity of high enough quality factor Q to made the observation of these features of the micromaser are experimentally possible and realized [9-12].

In this second part of our work (following our first part A.M.Kremid,2019 [13]) we assume that the atoms are pumped into the cavity in a coherent superposition of their upper and lower atomic states , the cavity is at finite temperature T, and the coupling with a heat bath is on even when the cavity is empty of atoms. Our concern will be about the effects of the black body radiation in the cavity (as an initial thermal field n_{th}) and the effects of the damping rate on the evolution of the micromaser cavity field coupled to a heat bath at temperature T. for the measurement of the purity of the state reached by the cavity field we employ the entropy $S = T_r\, \rho \ln \rho$

The objectives of our present study are getting results when the injected atoms are pumped in their upper and lower states with probabilities $|\alpha|\neq 0$ and $|\beta|\neq 0$ (with a condition $|\alpha|^2 + |\beta|^2 = 1$) instead of the case of our first part where $|\alpha|=1$ and $|\beta|=0$.

This work is organized as following: first we will give the main equations of our model which are given in detail and calculated at our first part of this work A.M.Kremid, 2019[13], (we will repeat some of these equations and there solutions here for the benefit of readers), following that the solutions to those equations of motion will be introduced, and the numerical results and their discussions are given, then the conclusion to this work is given and at the last the references are introduced.

The Model

The original micromaser consists of two level Rydberg atomic beam pumped into a high-Q microwave cavity containing a single mode radiation field such that only one atom at any given time is present inside the cavity and the atom flies through the cavity in a very short time compared to the time between any two successive atoms in the atomic beam. It has been assumed in the original theory, that no coupling between the single mode and the heat bath during the interaction time t_{int} so when the atom is inside the cavity the problem is well described by the Jaynes- Cummings Hamiltonian only and when the atom exits the cavity the coupling between the heat bath and the single mode is switched on.

In this work we introduce a different approach to this problem where at any point in time of the motion we solve for the total density operator ρ for the atom plus the field, namely, the coupling between the cavity field and the heat bath is switched on throughout the whole motion and not only when the cavity is empty of atoms. The damping process is governed by the master equation of the damped harmonic oscillator given by equation, G.S.Agarwal, 1974 [14]

$$\dot{\rho}(t) = -\kappa(n_{th}+1)\left[a^\dagger a\rho + \rho a^\dagger a + a\rho a^\dagger\right] - \kappa n_{th}\left[aa^\dagger\rho + \rho aa^\dagger + a\rho a^\dagger\right], \quad (1)$$

where ρ is a density matrix operator, $\kappa = \frac{1}{2}\omega Q^{-1}$ is the cavity damping constant with Q is the cavity quality factor. The cavity damping time is $T_c = (2\kappa)^{-1}$, $a^\dagger$ and a are the creation and annihilation operators for the cavity field respectively and n_{th} is the average thermal photon number in the cavity

We illustrate the case when atoms enter the cavity in a coherent superposition state, and we use the Jaynes Cummings model (JCM) [15] as a fundamental model where the total Hamiltonian of the system (atom + field) is given by E.T.Jaynes et. al. 1963 [15]

$$H = \hbar\omega a^\dagger a + \hbar\omega_0\sigma_z + \hbar g(a^\dagger\sigma_- + a\sigma_+) \quad (2)$$

For the case of the single cavity mode is coupled to the heat bath at temperature T>0, and the atom is coupled to this bath only via this cavity mode, the total density operator ρ for atom plus field satisfies the master equation for the high Q-cavity which is given by

$$\dot{\rho}(t) = -i[H-\rho] - \kappa(n_{th}+1)\left[a^\dagger a\rho(t) - 2\,a\rho(t)a^\dagger + \rho(t)a^\dagger a\right] - \kappa n_{th}\left[aa^\dagger\,\rho(t) - 2a^\dagger\rho(t)a + \rho(t)aa^\dagger\right]$$

(3) Where H is the J- C Hamiltonian eqn.(2) when and only when there is a single atom in the cavity. On the other hand when there is no atom in the cavity the above differential equation becomes:

$$\dot{\rho}(t) = -i[H_0-\rho] - \kappa(n_{th}+1)\left[a^\dagger a\rho(t) - 2\,a\rho(t)a^\dagger + \rho(t)a^\dagger a\right] - \kappa n_{th}\left[aa^\dagger\,\rho(t) - 2a^\dagger\rho(t)a + \rho(t)aa^\dagger\right]$$

(4)

where $H_0 = \hbar\omega a^\dagger a$, is the single mode cavity field Hamiltonian.

Since the two–level atom is in a coherent stat i.e.

$$|\psi\rangle = \alpha\,|e\rangle + \beta|g\rangle \quad (5)$$

with

$$|\alpha|^2 + |\beta|^2 = 1 \quad (6)$$

Since the coupling between the atom with upper state $|e\rangle$ and lower state $|g\rangle$ and the single mode radiation field is present during the motion, the space is spanned by states $|g\rangle|0\rangle$, $|e\rangle|n\rangle$, $|g\rangle|n+1\rangle$ for n=0,1,2,….. here |0> is a vacuum state. In this case we should solve for the four coupled elements: $\rho_{e,n;e,m}$, $\rho_{g,n+1;g,m+1}$, $\rho_{e,n;g,m+1}$, $\rho_{g,n+1;e,m}$

The differential equations for these coupled elements can be written in matrix form as:

$$\dot{\psi}^{(k)}(n,t) = \bar{A}^{(k)}(n)\,\psi^{(k)}(n,t) + B^{(k)}(n)\,\psi^{(k)}(n+1,t) + C^{(k)}(n)\,\psi^{(k)}(n-1,t) \quad (7)$$

In which the $\psi^{(k)} \equiv \psi(n,m)$ and $k = m - n$ represents the degree of off-diagonality.

The 4- column vectors $\psi^{(k)}(n,t)$ are

$$\psi^{(k)}(n,t) = \begin{pmatrix} \rho_{e,n-1;e,m-1(t)} \\ \rho_{g,n;g,m(t)} \\ \rho_{e,n-1;g,m(t)} \\ \rho_{g,n;g,m-1(t)} \end{pmatrix}, \quad (8)$$

and $\bar{A}^{(k)}(n)$, $B^{(k)}(n)$, $C^{(k)}(n)$ are 4x4 matrices.

For the complete calculations and manipulations of differential equations and adoption of some useful techniques for obtaining solution we refer to our first part, namely A.M, Kremid. 2019 [13].

The Solution:

Now we solve the equations of motion for the density matrix ρ which are expressed in the matrix form eq.(7).

First of all we assume that all atoms are prepared in the coherent superposition of their upper and lower states namely:

$$\rho_{atom} = |\psi\rangle\langle\psi|$$

$$\rho_{atom} = |\alpha|^2\rho_{e,e} + \alpha^*\beta\rho_{g,e} + \alpha\beta^*\rho_{e,g} + |\beta|^2\rho_{g,g} \quad (9)$$

where eq.(5) has been used and the single mode cavity radiation field is initially in a diagonal thermal state

$$\rho_{n,m} = \delta_{n,m}\frac{(n_{th})^n}{(n_{th}+1)^{n+1}} \quad (10)$$

For n=0, m=0, we put

$$\psi^{(0)}(0,t) = \begin{pmatrix} 0 \\ \rho_{g,0;g,0}\ (t) \\ 0 \\ 0 \end{pmatrix} \quad (11)$$

At t=0 and for (n=0, m=0)

$$\psi^{(0)}(0,0) = \begin{pmatrix} 0 \\ |\beta|^2 \frac{1}{(n_{th}+1)} \\ 0 \\ 0 \end{pmatrix}, \quad (12)$$

and for (n>0 , m>0),

$$\psi^{(k)}(n,0) = \begin{pmatrix} |\alpha|^2 \frac{(n_{th})^{n-1}}{(n_{th}+1)^n} \\ |\beta|^2 \frac{(n_{th})^n}{(n_{th}+1)^{n+1}} \\ 0 \\ 0 \end{pmatrix} \quad (13)$$

After some manipulations we reach to an equation for the density matrix ρ for the micromaser cavity field in photon number basis, the density matrix elements of the master equation for damping are,

$$\dot{\rho}^{(k)}(n,t) = 2\kappa(n_{th}+1)\left[\sqrt{(n+1)(n+k+1)}\rho^{(k)}(n+1,t) + \left(n+\frac{k}{2}\right)\rho^{(k)}(n,t)\right] + 2\kappa n_{th}\left[\sqrt{(n)(n+k)}\rho^{(k)}(n-1,t) - \left(n+1+\frac{k}{2}\right)\rho^{(k)}(n,t)\right] \quad (14)$$

The solution to the total differential equation of the field density matrix can be expressed as, P.Bogar, *et. al.*, (1995) [16]:

$$\rho^{(k)}(n,t) = exp(-\kappa kt)\sum_{i=0}^{n}\sum_{j=n-i}^{\infty} C_{n,j,i}^{(k)} \frac{A^j}{B^{j+k+1}}\left(\frac{A'}{\boldsymbol{A}}\right)^{n-i}\left(\frac{B'}{\boldsymbol{B}}\right)^{i}\rho^{(k)}(n,0) \quad (15)$$

where

$$C_{n,j,i}^{(k)} = (-1)^i \binom{i+j+k}{i}\binom{j}{n-i}\left(\frac{\binom{j+k}{n+k}}{\binom{j}{n}}\right)^{\frac{1}{2}},$$

$A = (n_{th}+1)(1-\exp(-2\kappa t))$,

$B = 1 + n_{th}(1-\exp(-2\kappa t))$,

$A' = \exp(-2\kappa t) - n_{th}(1-\exp(-2\kappa t))$ and

$B' = -n_{th}(1-\exp(-2\kappa t))$.

After obtaining all $\psi^{(k)}(n,t)$'s (n=0,1,2,….) we calculate the reduced density matrix elements of the cavity field by tracing over the atomic variables by using the relation

$$\rho_f^{(k)}(n) = Tr_{atom}\rho^{(k)}(n) \quad (16)$$

The diagonal elements of the density matrix $\rho^{(0)}(n)$ can be used for the calculation of the following physical observables

1- The average photon number of the micromaser field

$$< n >= Tr(n\rho^{(0)}(n)) \quad (17)$$

2- The normalized variance in the photon number

$$v = \sqrt{\frac{<n^2>-<n>^2}{<n>}} \quad (18)$$

3-The entropy of the micromaser field

$$S = -Tr[\rho\ln(\rho)]$$

$$S = -\sum_{n=0}^{\infty}\rho_{nn}\ln\rho_{nn} \quad (19)$$

This equation is for the case when ρ is diagonal that is when $|\alpha|^2 = 1$ only but when $|\alpha|^2 \neq 1$ the density matrix becomes ρ_{nm} instead of ρ_{nn} namely

$$S = -\sum_{n,m}\rho_{nm}\ln\rho_{nm} \quad (20)$$

Numerical Results and Discussion

We will now give numerical results for the case when $|\alpha| \neq 0$ and $|\beta| \neq 0$, that is when the atoms are pumped into the cavity in a coherent superposition of their upper and lower states. The calculations we are working with will be in a two dimensional space, namely the computational work needs a dealing with every $\rho_{n,m}$ where *n* and *m* are running from 0 to 40.

For the quality factor of the cavity $Q = 5\times10^{10}$, $gt_{int} = 1.54$, $gT_p = 308$, and the temperature of the cavity is T=0.5 K in which the initial average photon number inside the cavity is $n_{th} = 0.15$ and with regular inputs, the cavity field evolves towards a steady state as shown in **Fig.(1)**. Initially the trapping state at *n=m=3* for $gt_{int} = 1.57$ appears to play a significant role in the dynamics. The average photon number in the cavity field rapidly increases from black body at *N=0* to reach *<n> =2.31* at *N=11* then rises steadily to higher values as *N* increases.

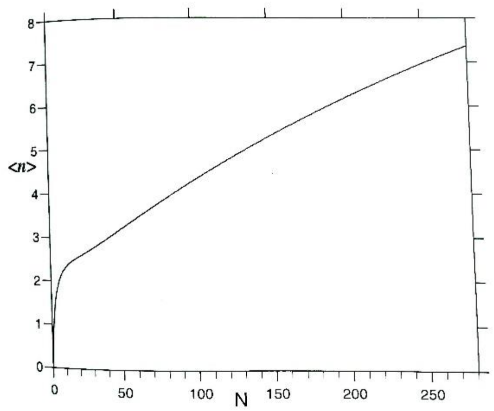

Fig.(1) The average photon number <n> in the cavity field as a function of the number of atoms N for the case of regular inputs $|\alpha|^2$=0.8 , $Q = 5\times10^{10}, gt_{int} = 1.54$, $gT_p = 308$ and the initial average thermal photon number in the cavity is $n_{th} = 0.15$

The corresponding variance *v* **Fig**.(**2**), drops from black body to v = 0.558 at *N=11.* Past this point *v* increases steadily to *v=1.95* at *N=169* then it decreases slowly afterwards.

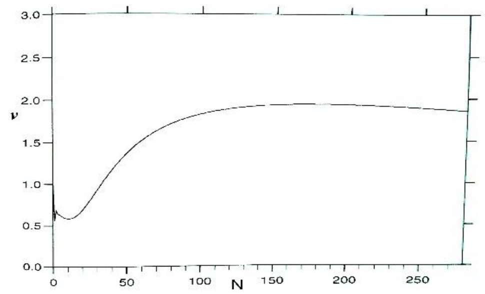

Fig.(2) The variance in the photon number as a function of the number of atoms N for the case of regular inputs , the other parameters are those of Fig.(1)

For this set of parameters the field evolves towards a mixed state rather than a pure state as shown by the entropy (increasing *S*) in **Fig.(3**).

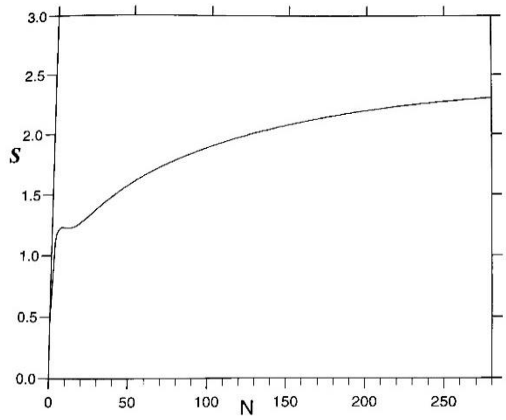

Fig.(3) The entropy S of the cavity field as a function of the number of atoms N for the case of regular inputs , the other parameters are those of Fig.(1)

Moreover, the probability that the atom exits the cavity in its upper state $S_z \equiv P_{|e>}$ is very small for $1 \leq N \leq 4$ as shown in **Fig.(4**), but very soon it becomes large and reaches its maximum at *N=20* and beyond this point it decreases slowly as *N* increases

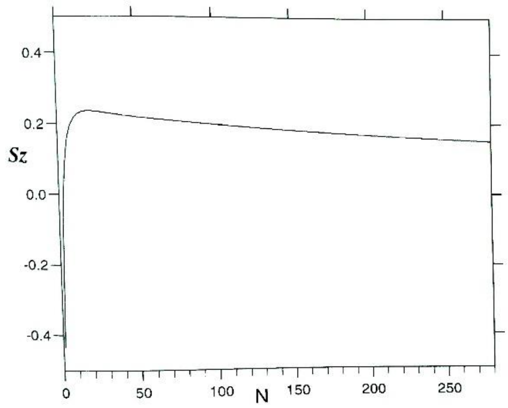

Fig.(4) The atomic probability of upper state $S_z \equiv P_{|e>}$ as a function of the number of atoms N for the case of regular inputs, the other parameters are those of Fig.(1)

In **Fig.(5)** we show the moduli in the density matrix $\rho_{n,m}$at the end of the 50th atom. It is evident from these figures that, the cavity field evolves towards a steady state regime rather than to a pure state or trapping state regime. In this steady state regime the down-going trapping state at *n=m=0* is still effective even at this large number of atoms (but its probability is expected to be finished at very large *N*).

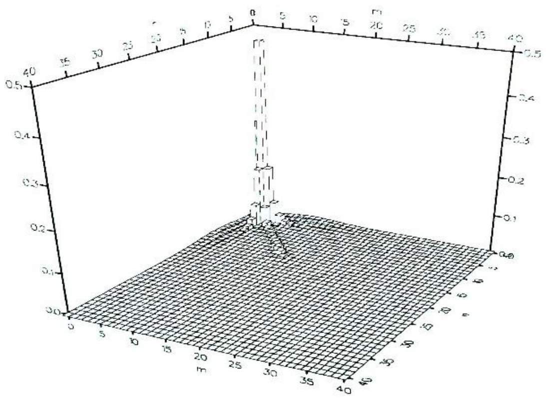

Fig.(5) The Moduli of the density matrix elements after 50 atoms have passed the cavity the parameters are those of Fig.(1**)**

However, the trapping state at *n=m=3* (the first up-going trapping state) apparently plays an important role for the evolution of the micromaser and since this trapping state lies in between a down trapping state at n=m=0 and an upper trapping state at n=m=15, then the situation becomes complicated and consequently no evolution towards a pure state J.J. Slosser, *et. al*., (1990) [17]. Later (after a large number of atoms have passed through the cavity) the trapping state at *n=m=15* becomes important for the micromaser evolution as well (its probability increases). Therefore, up to this, (still limited), number of atoms, the evolution of the micromaser is apparently towards a steady state equilibrium where on one hand the trapping states at *n=m=0* and at *n=m=3 are slowly reduced*. On the other hand, the trapping state at *n=m=15* is still growing by increasing *N*. No other trapping states are expected to emerge and so finally a steady state regime will be reached.

The presence of black body radiation initially in the cavity prevents the evolution towards a pure state because this radiation destroys the coherence induced in the cavity field. Furthermore there is another factor prevents the evolution of the cavity field towards a pure state which is the cavity field damping, L. Ladron *et. al*. (1997)[18], and since the coupling between the cavity field and the heat bath is switched on for all times then there is no chance at all for the evolution of the field to be towards a pure state.

In future works we will try to eliminate those factors which prevent the evolution to a pure state to reach a trapping states regime rather than a steady state regime.

Conclusion

We conclude our work for the action of the one atom micromaser when the atoms inject coherently into a microwave cavity that coupled with a heat bath even when the cavity does not empty of atoms. The evolution of the cavity field shows a steady state equilibrium when the condition that the repetition time T_p is much grater than the interaction time t_{int}. In this steady state regime the cavity field reaches a steady state after a sufficient number of atoms have passed through the cavity. Evidently the only responsible parameter for the qualitative changes is the repetition time T_p namely when $T_p \gg t_{int}$ then the micromaser field evolution is towards a steady state equilibrium. We will investigate in another future work the case when the repetition time T_p is reduced to the order of interaction time t_{int} where we expect the evolution of the cavity field is controlled by the trapping state dynamics and not by steady state one.

Moreover the first going-up trapping state at n=3 plays a significant role in the early dynamics of the micromaser field for this type of injection of atoms i.e the coherent superposition case but the evolution to a pure state is failed or prevented by some factors like the presence of a cavity field damping , the presence of the trapping state at n=3 in between the n=0 trapping state and trapping state at n=15 prevents the field from evolving towards a pure state and finally the presence of black-body radiation field initially in the cavity destroys coherence induced in the field and no pure state reached by the field.

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