SEBHA UNIVERSITY JOURNAL OF PURE & APPLIED SCIENCES VOL.20 NO. 4 2021
DOI: 10.51984/JOPAS.V20I4.1763

جامعة سبها للعلوم البحتة والتطبيقية مجلة
Sebha University Journal of Pure & Applied Sciences

Journal homepage: www.sebhau.edu.ly/journal/index.php/jopas

A study on some relations on multi-fuzzy (left-right)ideals of rings

*Kholoud. W. Saleem [a], Suzan. A. A. Abu-harbid [a], and Abrahim.A. A.Tentush [b]

[a]Mathematics Department. Science Faculty. Omar Al-Mukhtar University. Al-guba-Libya
[b]Mathematics Department. Science Faculty. Al-Zawia University. Ganzour

Keywords:
Multi-fuzzy set
Multi fuzzy ring
Multi fuzzy left(right) ideals
homomorphism of multi-fuzzy ideal of a ring
Cartesian product of multi-fuzzy sets.

ABSTRACT

multi fuzzy set theory is an extension of fuzzy set theory, which deals with the multi dimensional fuzziness. In this paper, we introduce the concepts of multi-fuzzy rings, multi fuzzy ideal, image of multi- fuzzy ideal of a ring under homomorphism, Cartesian product of multi-fuzzy set, and some related properties and theorems are investigated. The purpose of this study is to implement the fuzzy set theory and rings theory in multi-fuzzy rings.

دراسة بعض العلاقات حول المثاليات(اليساريه- اليمينية) للحلقات متعددة الضبابية

*خلود وليد سليم [1] و سوزان عبدالله ابو هربيد [1] و ابراهيم عبدالله تنتوش [2]

[1]قسم الرياضيات، كلية الاداب والعلوم، جامعة عمر المختار، القبة، ليبيا
[2]قسم الرياضيات،كلية العلوم، جامعة الزاوية، ليبيا

الكلمات المفتاحية:
مجموعة متعددة الضبابية
الحلقة متعددة الضبابية
المثاليات (اليمينية) اليسارية متعددة الضبابية
التشاكل المثالي للحلقة متعددة الضبابية
الضرب الديكارتي للمجموعات متعددة الضبابية.

الملخص

تعد نظرية المجموعات الضبابية المتعددة امتداداًلنظرية المجموعات الضبابية، التي تتعامل مع الضبابية متعددة الابعاد. في هذا البحث نقدم مفاهيم الحلقات متعددة الضبابية، والمثاليات متعددة الضبابية، والصورة المثالية متعددة الضبابية للحلقة تحت تأثير التشاكل، والضرب الديكارتي للمجموعات متعددة الضبابية. وبعض خصائص النظريات ذات الصلة التي تمت دراستها، حيث الغرض من هذه الدراسة هو تطبيق نظرية المجموعة الضبابية ونظرية الحلقات في الحلقات متعددة الضبابية.

1. Introduction

After the introduction of the fuzzy set by Zadeh[1] and Rosenfeld [2] several researchers explored the generaliz- ation of notion of fuzzy set. Since then, the study of fuzzy algebraic structures has been pursued in many directions such as groups, modules, vector space and so on. The idea of anti fuzzy subgroup was introduced by Biswas [3] which was extended by many researchers. The concept of a fuzzy ideal of a ring was introduced by Liu [4] in 1982. Subsequently, Yue [5], Mukharjee and Sen [6] Swamy and Raju [7], Dixit et al [8], Raj Kumar [9], and Zie [10] developed the theory of fuzzy rings. Since then many researchers explored on the generalization of the notions of fuzzy set and its application to many mathematical branches. F.A.Azam, A.A. Mamum, and F.Nasrin [11] apply the idea of Biswas to the theory of ring. They introduced a notion of anti fuzzy ideal of a ring. The notion of fuzzy sub near-ring and fuzzy ideals was introduced by Abou-Zaid [12].

Sabu Sebastian and T.V. Ramakrishnan [13,14] introduced the theory of multi-fuzzy sets in terms of multi-dimensional membership functions and investi- gated some properties of multi level fuzziness. After introducing multi-fuzzy subsets of a crisp set, they have also introduced and studied some elementary properties of multi-fuzzy subgroups. R. Muthuraj and S. Balamurugan [15] introduced the concept of multi-anti fuzzy subgroup and discussed some of its properties. In [16], we extended the concept of multi-anti fuzzy subgroup to multi-anti fuzzy ideal of a ring. and introduced a notion of multi-anti fuzzy ideal of a ring and some of its properties are discussed.

In this paper, we introduce the concepts of multi-fuzzy rings, multi

*Corresponding author:
E-mail addresses: Kholoudwaleed83@gmail.com, (S. A. A. Abu-harbid) sanasuzansana@gmail.com, (A. A. A.Tentush) none
Article History : Received 24 July 2021 - Received in revised form 24 August 2021 - Accepted 15 September 2021

fuzzy(left-right)ideal, image of multi-fuzzy ideal of a ring under homomorphism, Cartesian product of multi-fuzzy set, and some related properties and theorems are investigated and study some of their related properties.

2. Preliminaries

In this section, we site the fundamental definitions that will be used in the sequel.

Definition 2.1[15-16] Let X be a non-empty set. A multi-fuzzy set A of X is defined as $A = \{< x,\ u_A(x) >:\ x \in X\}$. Where $u_A = (u_1, u_2, \dots, u_k)$, that is, $u_A(x) = (u_1(x), u_2(x), \dots, u_k(x))$ and $u_i: X \to [0,1]$, ∀ i = 1,2, …, k. Here k is the finite dimension of A. Also note that, for all i, $u_i(x)$ is a decreasingly ordered sequence of elements. That is, $u_1(x) \geq u_2(x) \geq \cdots \geq u_k(x),\ \forall\ x \in X$.

Definition 2.2[17-18] Let $\boldsymbol{k}$ be a positive integer and **A** and **B** be a multi-fuzzy set of dimension $\boldsymbol{k}$ on $\boldsymbol{X}$. That is, $\boldsymbol{A} = \{(\boldsymbol{x}, \boldsymbol{u_1(x)}, \boldsymbol{u_2(x)}, \dots, \boldsymbol{u_k(x)}), \boldsymbol{x} \in \boldsymbol{X}\}$ and

$\boldsymbol{B} = \{(\boldsymbol{x}, \boldsymbol{v_1(x)}, \boldsymbol{v_2(x)}, \dots, \boldsymbol{v_k(x)}), \boldsymbol{x} \in \boldsymbol{X}\}$ where

$\boldsymbol{u_j(x)}, \boldsymbol{v_j(x)} \in \boldsymbol{I}$, $\forall\, \boldsymbol{j} = \boldsymbol{1}, \boldsymbol{2}, \dots, \boldsymbol{k}$.

Then we have the following relations and operations for all $x \in X$

i. $A = B$ iff $u_i(x) = v_j(x)\ \forall\ j = 1,2, \dots, k$.
ii. $A \subseteq B$ iff $u_i(x) \leq v_j(x)\ \forall\ j = 1,2, \dots, k$.
iii. $A \cup B = \{(x, max\{u_1(x), v_1(x)\}, \dots, max\{u_k(x), v_k(x)\}); x \in X\}$
iv. $A \cap B = \{(x, min\{u_1(x), v_1(x)\}, \dots, min\{u_k(x), v_k(x)\}); x \in X\}$
v. The multi-fuzzy complement of multi-fuzzy set A is $C(A) = \{(x, c(u_1(x)), c(u_2(x), \dots, c(u_k(x))) : x \in X\} = \{(x, 1-u_1(x), 1-u_2(x), \dots, 1-u_k(x)), x \in X\}$ where $C(u_i(x))$ is the complement of $u_i(x)$ for all $i = 1, 2, 3, \dots, k$.

Exampel 2.3. let $X = \{a, b, c, d\}$. Then $A = \{(0.5, 0.5, 0.3, 0.1, 0.1)/b, (0.65, 0.2, 0.2, 0.2)/d\}$ is a fuzzy multiset of X. or equivalently, we can write it as:

$$A\ (a) = A\ (c) = 0, \qquad A\ (b) = (0.5, 0.5, 0.3, 0.1, 0.1),$$
$$A\ (d) = (0.65, 0.2, 0.2, 0.2).$$

Example 2.4. Let $X = \{a, b, c, d\}$ and define the fuzzy multisets A, B, C of X as follows:

$$A = \{(0.5, 0.5, 0.3, 0.1, 0.1)/b, (0.65, 0.2, 0.2, 0.2)/d\}$$

$$B = \{(0.5, 0.5, 0.3, 0.1, 0.1)/b, (0.5, 0.1)/c, (0.65, 0.2, 0.2, 0.2)/d\}$$

$$C = \{(0.5, 0.3)/a, (0.5, 0.1)/c, (0.35, 0.3, 0.1)/d\}.$$

Then it is clear that:

1) $A \subseteq B \ and\ A \neq B$,

2) $A \cap C = \{(0.35, 0.2, 0.1)/d\}$,

3) $A \cup C = \{(0.5, 0.3)/a, (0.5, 0.5, 0.3, 0.1, 0.1)/b, (0.5, 0.1)/c, (0.65, 0.3, 0.2, 0.2)/d\}$, and

4) $C(A) = \{(1,1,1,1,1)/a, (0.9, 0.9, 0.7, 0.5, 0.5)/b, (1,1,1,1,1)/c, (1, 0.8, 0.8, 0.8, 0.35)/d\}$

Definition 2.5[19] Let k be a positive integer and A and B be multi-fuzzy set of dimension k on $X \times Y$. That is,

$A = \{(x, u_1(x), u_2(x), \dots, u_k(x)), x \in X\}$ and $B = \{(y, v_1(y), v_2(y), \dots, v_k(y)), y \in Y\}$. Then the Cartesian product $A \times B$ of two mutli-fuzzy sets A and B is given by

$A \times B = \{((x, y), min\{u_1(x), v_1(y)\}, min\{u_2(x), v_2(y)\}, \dots, min\{u_k(x), v_k(y)\})\ ; (x, y) \in X \times Y\}$.

Definition 2.6[20] Let $A = \{(x, u_1(x), u_2(x), \dots, u_k(x)), x \in X\}$ where $A_j(x) \in [0,1]$, $\forall\ j = 1,2, \dots, k$ be a multi-fuzzy set on X and $\alpha = (\alpha_1, \alpha_2, \dots, \alpha_k), \alpha_i \in I$ for $i = 1,2, \dots, k$. Then the set $A_\alpha = \{x \in X\ /\ A(x) \geq \alpha\}$ is called level subset of a multi- fuzzy subset A.

Definition 2.7[21] Let f be a mapping from a set X to a set Y, and let A and B be multi-fuzzy subsets in X and Y respectively.

i. $f(A)$, the image of A under f, is a multi-fuzzy subset in Y. For all $y \in Y$, we define,

$$f(A)(y) = \begin{cases} sup_{x \in f^{-1}(y)} A(x) & if\ f^{-1}(y) \neq \emptyset \\ 0, & otherwise \end{cases}$$

ii. $f^{-1}(B)$, is the pre-image of B under f, is a multi-fuzzy set in X. That is,

$f^{-1}(B)(x) = B(f(x))$ for all $x \in R$.

Definition 2.8[22] A multi-fuzzy set A in X has the sup property if, for any subset T of X, there exists $t_0 \in T$ such that

$$A(t_0) = sup_{t \in T}\ A(T)$$

Definition 2.9[23] A multi-fuzzy set A on a ring R is said to be a multi-fuzzy ring on R, if for all $x, y \in R$

(i) $A(x - y) \geq min\ \{A(x), A(y)\}$ and

(ii) $A\ (xy) \geq min\ \{A(x), A(y)\}$.

Definition 2.10[24] A multi-fuzzy ring A on R is said to be

(i) a multi-fuzzy left ideal if

$A\ (xy)\ \geq\ A(y)$, for all $x, y \in R$ and

(ii) a multi-fuzzy right ideal if

$A\ (xy)\ \geq\ A(x)$, for all $x, y \in R$.

Definition 2.11[24] A multi-fuzzy ring $A = \{(x, u_1(x), u_2(x), \dots, u_k(x), \dots) : x \in X\}$ on a ring R is called a multi-fuzzy ideal if it is both a multi-fuzzy left ideal and a multi-fuzzy right ideal.

In other words, a multi-fuzzy set A on R is a multi-fuzzy ideal, if for all $x, y \in R$

(i) $A(x - y) \geq min\ \{A(x), A(y)\}$ and

(ii) $A\ (xy) \geq max\ \{A(x), A(y)\}$

Example 2.12 Let $(R, +, .)$ be a ring defined by the following tables:

+	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

.	0	1	2
0	0	1	2
1	0	1	2
2	0	1	2

$A = \{(1, 1, 0.8, 0.8, 0.1)/0, (0.7, 0.5, 0.3, 0.1)/1, (0.7, 0.5, 0.3, 0.1)/2\}$ is a multi fuzzy left-ideal of R.

Example 2.13 Let $(Z, +, .)$ be a ring of integers under standard addition "+" and with "·" defined by x · y = y for all $x, y \in Z$. Let

$$A(x) = \begin{cases} (1, 0.3, 0.3, 0.2) & x \in \{0, \mp 2, \pm 4, \dots\} \\ 0 & x \in \{\pm 1, \mp 3, \pm 5, \dots\} \end{cases}$$

Then A a multi-fuzzy ring on Z.

3. Main results

In this section we introduce the notion of multi fuzzy (left, right) ideal of a ring R. Further we show that any intersection of multi fuzzy (left, right) ideals is a multi fuzzy (left, right) ideal, Cartesian product of multi-fuzzy set and multi fuzzy (left, right) ideals and (inverse) image of a multi fuzzy (left, right) ideal is a multi fuzzy (left, right) ideal.

Theorem 3.1[24] If $\{A_i / i \in I\}$ is a family of multi fuzzy sub rings of a ring R, then $\cap_{i\in I} A_i$ is a multi fuzzy sub ring of R.

Proof: Let $A = \cap A_i$ and let $x, y \in R$.

(a) $A(x-y) = (\wedge A_i)(x-y)$

$= \wedge_{i\in I} A_i(x-y)$

$\geq \wedge_{i\in I} (A_i x \wedge A_i y)$

$= (\wedge_{i\in I} A_i)x \wedge (\wedge_{i\in I} A_i)y$

$= (\cap_{i\in I} A_i)(x) \wedge (\cap_{i\in I} A_i)(y) = Ax \wedge Ay.$

(b) $A(x\,y) = (\wedge A_i)(xy)$

$= \wedge_{i\in I} A_i(x\,y) \geq \wedge_{i\in I} (A_i x \wedge A_i y)$

$= (\wedge_{i\in I} A_i)x \wedge ((\wedge_{i\in I} A_i)\, y$

$= (\cap_{i\in I} A_i)(x) \wedge (\cap_{i\in I} A_i)(y) = Ax \wedge Ay.$

Theorem 3.2[16] If $\{A_i / i \in I\}$ is a family of multi fuzzy left ideals of a ring R, then $\cap_{i\in I} A_i$ is a multi fuzzy left ideals of R.

Proof: Let $A = \cap_{i\in I} A_i$. Since meet of any family of multi fuzzy sub rings is a multi fuzzy sub ring. So, it is enough to show that $A(xy) \geq A(y)\ \forall x, y \in R$, to show A is a multi fuzzy left ideal.

$A(xy) = (\wedge_{i\in I} A_i)(xy)$

$= \wedge_{i\in I} \left(A_i(x\,y)\right)$

$\geq \wedge_{i\in I} \left(A_i(y)\right)$

$= (\wedge_{i\in I} A_i)(y) = A(x)$

Theorem 3.3[16] If $\{A_i / i \in I\}$ is a family of multi fuzzy right ideals of a ring R, then $\cap_{i\in I} A_i$ is a multi fuzzy right ideals of R.

Proof: Let $A = \cap_{i\in I} A_i$. Since meet of any family of multi fuzzy sub rings is a multi fuzzy sub ring. So, it is enough to show that $A(xy) \geq A(x)\ \forall x, y \in R$, to show A is a multi fuzzy right ideal.

$A(xy) = (\wedge_{i\in I} A_i)(xy)$

$= \wedge_{i\in I} \left(A_i(x\,y)\right)$

$\geq \wedge_{i\in I} \left(A_i(x)\right)$

$= (\wedge_{i\in I} A_i)(x) = A(x).$

Theorem 3.4[16] If $\{A_i / i \in I\}$ is a family of multi fuzzy ideals of a ring R, then $\cap_{i\in I} A_i$ is a multi fuzzy ideal of R.

Proof: It follows from above two theorems.

.

Theorem3.5[25] If A and B are two multi fuzzy left ideal of a ring R, Then $(A \times B)$ is a multi-fuzzy left ideal of $R \times R$.

Proof:

Let A and B be two multi fuzzy left ideal of a ring R.

Let (x_1, x_2) and $(y_1, y_2) \in R \times R$

i. $(A \times B)(x_1 - y_1, x_2 - y_2) =$
$\{min\{A_1(x_1 - y_1), B_1(x_2 - y_2)\}, \ldots, min\{A_k(x_1 - y_1), B_k(x_2 - y_2)\}\}$
$\geq$
$\{min\{min(A_1(x_1), A_1(y_1)), min(B_1(x_2), B_1(y_2))\}, \ldots,$

$min\{min(A_k(x_1), A_k(y_1)), min(B_k(x_2), B_k(y_2))\}\}$
$=$
$\{min\{min(A_1(x_1), B_1(x_2)), min(A_1(y_1), B_1(y_2))\}, \ldots,$

$min\{min(A_k(x_1), B_k(x_2)), min(A_k(y_1), B_k(y_2)))\}\}$
$=$
$\{min\{min(A_1(x_1), B_1(x_2)), \ldots, min(A_k(x_1), B_k(x_2)),$

$min(A_1(y_1), B_1(y_2)), \ldots, min(A_k(y_1), B_k(y_2))\}\}$

$= min\{(A \times B)(x_1, x_2), (A \times B)(y_1, y_2)\}$

$(A \times B)(x_1 - y_1, x_2 - y_2) \geq min\{(A \times B)(x_1, x_2), (A \times B)(y_1, y_2)\}$

ii. $(A \times B)(x_1\, y_1, x_2\, y_2) = \{min\{A_1(x_1\, y_1), B_1(x_2\, y_2)\}, \ldots$

$min\{A_k(x_1\, y_1), B_k(x_2\, y_2)\}\}$
$min\{A_k(x_1\, y_1), B_k(x_2\, y_2)\}\}$

$\geq \{min\{A_1(y_1), B_1(y_2)\}, \ldots, min\{A_K(y_1), B_k(y_2)\}\}$

$= (A \times B)\,(y_1,\ y_2)$

$(A \times B)\,(x_1\, y_1, x_2\, y_2) \geq (A \times B)\,(y_1, y_2)$

Hence (A × B) is a multi-fuzzy left ideal of R.

Theorem 3.6[25] If A and B are two multi fuzzy right ideal of a ring R, Then $(A \times B)$ is a multi-fuzzy right ideal of $R \times R$.

Proof:

Let A and B be two multi fuzzy right ideal of a ring R.

Let (x_1, x_2) and $(y_1, y_2) \in R \times R$

i. $(A \times B)(x_1 - y_1, x_2 - y_2) =$
$\{min\{A_1(x_1 - y_1), B_1(x_2 - y_2)\}, \ldots, min\{A_k(x_1 - y_1), B_k(x_2 - y_2)\}$
$\geq$
$\{min\{min(A_1(x_1), A_1(y_1)), min(B_1(x_2), B_1(y_2))\}, \ldots,$

$min\{min(A_k(x_1), A_k(y_1)), min(B_k(x_2), B_k(y_2))\}\}$
$=$
$\{min\{min(A_1(x_1), B_1(x_2)), min(A_1(y_1), B_1(y_2))\}, \ldots,$

$min\{min(A_k(x_1), B_k(x_2)), min(A_k(y_1), B_k(y_2)))\}\}$
$=$
$\{min\{min(A_1(x_1), B_1(x_2)), \ldots, min(A_k(x_1), B_k(x_2)),$

$min(A_1(y_1), B_1(y_2)), \ldots, min(A_k(y_1), B_k(y_2))\}\}$

$= min\{(A \times B)(x_1, x_2), (A \times B)(y_1, y_2)\}$

$(A \times B)(x_1 - y_1, x_2 - y_2) \geq min\{(A \times B)(x_1, x_2), (A \times B)(y_1, y_2)\}$

ii. $(A \times B)(x_1\, y_1, x_2\, y_2) =$

$\{min\{A_1(x_1\, y_1), B_1(x_2\, y_2)\}, \ldots$

$min\{A_k(x_1\, y_1), B_k(x_2\, y_2)\}\}$

$\geq \{min\{A_1(x_1), B_1(x)\}, \ldots, min\{A_K(x_1), B_k(x_2)\}\}$

$= (A \times B)\,(x_1,\ x_2)$

$(A \times B)\,(x_1\, y_1, x_2\, y_2) \geq (A \times B)\,(x_1, x_2)$

Hence $(A \times B)$ is a multi-fuzzy right ideal of R.

Theorem3.7[24-26] Let f be a homomorphism from a ring R into a ring T and let B be a multi-fuzzy left ideal of T. Then the pre-image, $f^{-1}(B)$ is a multi-fuzzy left ideal left of R.

Proof. Consider a ring homomorphism $f: R \to T$ .Let B be a multi-fuzzy left ideal of T.

i. $f^{-1}(B)(x - y) = Bf(x - y)$

$= B(f(x) - f(y))$

$\geq min\{Bf(x), Bf(y)\}$

$= min\{f^{-1}(B)(x), f^{-1}(B)(y)\}$

$f^{-1}(B)(x-y) \geq min\{f^{-1}(B)(x), f^{-1}(B)(y)\}$

ii. $f^{-1}(B)(xy) = B(f(xy))$

$= B(f(x)f(y))$

$\geq min\{Bf(x), Bf(y)\}$

$= min\{f^{-1}(B)(x), f^{-1}(B)(y)\}$

$f^{-1}(B)(xy) \geq min\{f^{-1}(B)(x), f^{-1}(B)(y)\}$

iii. $f^{-1}(B)(xy) = B(f(xy))$

$= B(f(x)f(y))$

$\geq B(f(y))$

$= f^{-1}(B)(y)$

$f^{-1}(B)(xy) \geq f^{-1}(B)(y)$

Therefore, $f^{-1}(B)$ is a multi-fuzzy left ideal of R.

Theorem3.8[24-26] Let f be a homomorphism from a ring R into a ring T and let B be a multi-fuzzy right ideal of T. Then the pre-image, $f^{-1}(B)$ is a multi-fuzzy right ideal of R.
Proof. Consider a ring homomorphism $f: R \to T$. Let B be a multi-fuzzy right ideal of T.
For all $x, y \in R$.

i. $f^{-1}(B)(x-y) = Bf(x-y)$

$= B(f(x) - f(y))$

$\geq min\{Bf(x), Bf(y)\}$

$= min\{f^{-1}(B)(x), f^{-1}(B)(y)\}$

$f^{-1}(B)(x-y) \geq min\{f^{-1}(B)(x), f^{-1}(B)(y)\}$

ii. $f^{-1}(B)(xy) = B(f(xy))$

$= B(f(x)f(y))$

$\geq min\{Bf(x), Bf(y)\}$

$= min\{f^{-1}(B)(x), f^{-1}(B)(y)\}$

$f^{-1}(B)(xy) \geq min\{f^{-1}(B)(x), f^{-1}(B)(y)\}$

iii. $f^{-1}(B)(xy) = B(f(xy))$

$= B(f(x)f(y))$

$\geq B(f(x))$

$= f^{-1}(B)(x)$

$f^{-1}(B)(xy) \geq f^{-1}(B)(x)$

Therefore, $f^{-1}(B)$ is a multi-fuzzy right ideal of R.

Theorem3.9[24-26] Let f be a homomorphism from a ring R into a ring T, and let B be a multi-fuzzy ideal of T. Then the pre-image, $f^{-1}(B)$ is a multi-fuzzy ideal of R.
Proof
It is clear

Theorem3.10[22] Let f be a homomorphism from a ring R into a ring T and let A be a multi-fuzzy left ideal of a ring R with sup property .Then the image, $f(A)$ is a multi-fuzzy left ideal of a ring T.
Proof. Consider a ring homomorphism $f: R \to T$ .Let A be a multi-fuzzy left ideal of T.
For all $x, y \in R$.

i. $f(A)\big(f(x) - f(y)\big) = f(A)f(x-y)$

$= A(x-y)$

$\geq min\{A(x), A(y)\}$

$= min\{f(A)(x), f(A)(y)\}$

$f(A)(f(x) - f(y)) \geq min\{f(A)(x), f(A)(y)\}$

ii. $f(A)(f(x)f(y)) = fA(f(xy))$

$= A(xy)$

$\geq min\{A(x), A(y)\}$

$= min\{f(A)(x), f(A)(y)\}$

$f(A)(f(x)f(y)) \geq min\{f(A)(x), f(A)(y)\}$

iii. $f(A)(f(x)f(y)) = f(A)f(xy)$

$= A(xy)$

$\geq A(y)$

$= f(A)(f(y))$

$f(A)(f(x)f(y)) = f(A)(f(y))$

Therefore, $f(A)$ is a multi-fuzzy left ideal of T.

Theorem3.11[22] Let f be a homomorphism from a ring R into a ring T and let A be a multi-fuzzy right ideal of a ring R with sup property .Then the image, $f(A)$ is a multi-fuzzy right ideal of a ring T.
For all $x, y \in R$.
Proof. Consider a ring homomorphism $f: R \to T$ .Let A be a multi-fuzzy right ideal of T.

i. $f(A)\big(f(x) - f(y)\big) = f(A)f(x-y)$

$= A(x-y)$

$\geq min\{A(x), A(y)\}$

$= min\{f(A)(x), f(A)(y)\}$

$f(A)\big(f(x) - f(y)\big) \geq min\{f(A)(x), f(A)(y)\}$

ii. $f(A)(f(x)f(y)) = f(A)f(xy)$

$= A(xy)$

$\geq min\{A(x), A(y)\}$

$= min\{f(A)(x), f(A)(y)\}$

$f(A)(f(x)f(y)) \geq min\{f(A)(x), f(A)(y)\}$

iii. $f(A)(f(x)f(y)) = f(A)f(x\,y)$

$= A(xy)$

$$\geq A(x)$$

$$= f(A)(f(x))$$

$$f(A)(f(x)f(y)) \geq f(A)(f(x))$$

Therefore, $f(A)$ is a multi-fuzzy right ideal of T.

Theorem3.12[22] Let f be a homomorphism from a ring R into a ring T, and let A be a multi-fuzzy ideal of a ring R with sup property. Then the image, $f(A)$ is a multi-fuzzy ideal of a ring T.

Proof It is clear.

4. CONCLUSION

In the present paper, by using the idea of multi-fuzzy ideals we have pioneered the notion of multi-fuzzy ideals of a rings and we introduce the concepts of multi-fuzzy rings, image of multi-fuzzy ideal of a ring under homomorphism, Cartesian product of multi-fuzzy set, and some related properties and theorems are investi- gated. It is well known that the concept of fuzzy multiset

is well established in dealing with many real life problems. As a result, we can deal with real life problems involving the concept of fuzzy multiset with a different perspective.

References

[1]- L.A. Zadeh., (1965) Fuzzy Sets, Information and Control, 8,338-353.

[2]- Rosenfeld, A, (1971)Fuzzy groups, Journal of Mathematical Analysis and applications, 35 , 512 – 517.

[3]- Biswas.R., (1990) fuzzy subgroups and Anti-fuzzy subgoups, Fuzzy sets and Systems, 5, 121 – 124.

[4]- W. J. Liu, (1982) “Fuzzy invariant subgroups and fuzzy ideals, Fuzzy Sets and Systems, vol. 8, no. 2, pp. 133–139.

[5]- Z . Yue. (1988) Prime L-fuzzy ideal and primary, FSS, 27 345-350.

[6]- T.K.Mukharjee and S.K.Sen: (1989)Prime fuzzy ideals in ring. FSS, 32337-341.

[7]- U.U. Swami, D. V. Raju : (1991) Irreducibility in algebraic fuzzy systems : Fuzzy sets and systems, 41,233-241.

[8]- V.N. Dixit, R. Kumar, N. Ajma: (1992) On fuzzy rings, FSS, 49 205-213.

[9]- R. Kumar: (1992) certain fuzzy ideals of rings redephined, FSS, 46 (251-260.

[10]- X. Y. Zie.(2008) On prime fuzzy ideals of a semigroup : J.fuzzy Math, (4),231-241.

[11]- Azam.F.A, A.A.Mamun, and F. Nasrin, (2013)Anti fuzzy ideals of a ring, Annals of Fuzzy Mathematics and Informatics, Volume 5, No.2, pp 349 – 360.

[12]- S. Abou-Zaid, (1991) “On fuzzy subnear-rings and ideals” Fuzzy Sets and Systems, vol. 44, no. 1, pp. 139–146.

[13]- Sabu Sebastian and T.V. Ramakrishnan., (2010)Multi-Fuzzy Sets, International Mathematical Forum, 5, no. 50, 2471-2476, Al-Tahan, M.; Hoskova.

[14]- T.V. Ramakrishnan and Sabu Sebastian., (2010) A Study on Multi-fuzzy Sets, International Journal of Applied Mathematics, Vol. 23, No.4, 713-721.

[15]- Muthuraj. R and Balamurugan. S,(2013)Multi-fuzzy group and its level subgroups, Gen.Math.Notes, Vol. 17, No. 1, pp 74 – 81.

[16]- Muthuraj. R and Malarselvi. C, (2014) Multi-anti Fuzzy Ideal of a Ring, Proceedings of the National Converence on Mathematical Modellig, 114 – 120.

[17]- Shinoj, T.K.; Baby, A.; John, S.J.,(2015) On some algebraic structures of fuzzy multisets. Ann. Fuzzy Math. Inform. 9, 77–90.

[18]- Al-Tahan, M.; Hoskova-Mayerova, S.; Davvaz, B.(2020) Fuzzy multi-polygroups. J. Intell. Fuzzy Syst, 38, 2337–2345.

[19]- Miyamoto, S. (2000) Fuzzy Multisets and Their Generalizations, Multiset Processing; Lecture Notes in Computer Science 2235, Germany, pp. 225–236.

[20]- Hoskova-Mayerova., S.; Al Tahan, M. (2021) Anti-Fuzzy Multi-Ideals of Near Ring. Mathematics, 9, 494.

[21]- Mayerova, S.; Davvaz, B., (2019) Some results on (generalized) fuzzy multi-Hv-ideals of Hv-rings. Symmetry, 11, 1376.

[22]- R.Muthuraj, C. Malarselvi., (2015) Homomorphism and Anti-homomorphism of Multi-Fuzzy Ideal and Multi-Anti Fuzzy Ideal of a Ring. IOSR Journal of Mathematics, PP 83-94e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 11.

[23]- Muthuraj. R and Malarselvi. C., (2014) Multi-Fuzzy Prime Ideal of a Ring, Discovery, Volume 21, No. 65.

[24]- Lokavarapu Sujatha., (2014)Multi fuzzy subrings and ideals. Annals of Fuzzy Mathematics and Informatics Volume x, No. x, (mm 201y)m.

[25]- R.Muthuraj, C. Malarselvi., (2017) Cartesian product of multi-fuzzy ideal of a ring, International Journal of Computational and Applied Mathematics. ISSN 1819-4966 Volume 12, Number 1.

[26]- Gezahagne Addis, Nasreen Kausar, Mohammad Munir. (2020) Fuzzy homomorphism theorems on rings. Journal of Discrete Mathematics Sciences and Cryptography.ISSN: 0972-0529(print) ISSN 2169-0065(Online).