SEBHA UNIVERSITY JOURNAL OF PURE & APPLIED SCIENCES VOL.21 NO. 2 2022
DOI: 10.51984/JOPAS.V21I2.1857

مجلة جامعة سبها للعلوم البحتة والتطبيقية
Sebha University Journal of Pure & Applied Sciences

Journal homepage: www.sebhau.edu.ly/journal/index.php/jopas

Analytical Solution of Time-Varying Investment Returns with Random Parameters

*B.O. Osu[a], I.U Amadi[b] & P.A Azor[c]

[a]Department of Mathematics Abia State University, UturuAbia State.
[b]Department of Mathematics/ Statistics Captain ElechiAmadi polytechnic, Rumuola PortHarcourt
[c]Department of Mathematics/ Statistics,Federal University Otuoke,BayelsaState.Nigeria

Keywords:
Risky asset
Risky-less asset
Reinsurer
Insurer
ODE
SDE
return rates

ABSTRACT
In this paper, a combination of deterministic and stochastic systems with its random parameters in the model was considered. The analytical solution to the proposed model is presented in detail which determined the following insurance quantities or variables such as: surplus process of insurance company, risky and risk-less assets. These results were explicitly verified graphically and the effects of the expected rate of return were discussed.

حل تحليلي لعوائد الاستثمار المتغيرة بمرور الوقت مع معلمات عشوائية

*B.O. Osu[a], I.U Amadi[b] & P.A Azor[c]

[a] قسم الرياضيات جامعة ولاية أبيا ولاية أوتورآبيا.
[b] قسم الرياضيات ، الإحصاء الكابتن *Elechi* بوليتكنك مادي ميناء رومولا هاركورت
[c] قسم الرياضيات ، الإحصاء ، الجامعة الفيدرالية أوتوك ، ولاية بايلسا ، نيجيريا

الملخص
في هذا البحث تم الأخذ بعين الاعتبار توليفة من الأنظمة الحتمية والعشوائية مع معاملاتها العشوائية في النموذج. الحل التحليلي للنموذج المقترح مقدم بالتفصيل والذي حدد كميات التأمين أو المتغيرات التالية مثل: عملية فائض شركة التأمين، الأصول الخطرة والأصول الخالية من المخاطر. تم التحقق صراحة من هذه النتائج بيانيا ونوقشت آثار معدل العائد المتوقع.

الكلمات المفتاحية:
أصول محفوفة بالمخاطر
أصول أقل خطورة
معيد تأمين
شركة تأمين
ODE
SDE
معدلات العائد

Introduction

Investments are basically business operations associated with risk which cannot be relegated to the background. Humans factually takes risk surviving; so risk is a vital tool used to effectively manage our portfolio of investment because it determines the variation in returns on the asset and portfolio and gives investor a mathematical basis for investment decisions [1]. A typical example of the risk associated with a security are stocks, bonds, property, etc.

However, because of risk involvement in portfolio of investments, insurance companies came up with the idea of insuring life and properties etc. In fact, insurance companies' share third party in their obligation of financial results. Reinsurance is the practice where by insurer on financial result of its insurance obligation in various ways with reinsurance, risk among several insurance companies inside and outside the country will be divided. So in the event of large losses from financial situation as insurance company will not face a risk, in simple terms reinsurance means division and distribution of risk. Generally, risk is a universal factor as far as human being is concern, because we acquire risky or riskless assets respectively. These factors are better modeled as the trajectory of a diffusion process defined on some underlying probability space, with the geometric Brownian motion the paramount tool used as the established reference model, [2]. Modeling of financial concepts cannot be over emphasized due to its numerous applications in the fast growing field of science and technology.

For instance, [3] considered maximizing the exponential utility and minimizing the ruin probability and result showed the same type of

Corresponding author:
E-mail addresses: Osu.bright@abiastateuniversity.edu.ng ,(I.U Amadi) amadiuchennainno@gmail.com ,(P.A Azor) azor.promise@gmail.com
Article History : Received 16 April 2022 - Received in revised form 03 August 2022 - Accepted 03 August 2022

investment strategy for zero interest rate. [4] studied an optimal reinsurance and investment problem for insurer with jump diffusion risk process. [5] Considered the risk reserve of an insurer and a reinsurer to follow Brownian motion with drift and applied optimal probability of survival problem under proportional reinsurance and power utility preference. In a similar circumstance, [7] took into account the excess loss of reinsurance and investment in a financial market, where optimal strategies were obtained explicitly.

[8] engrossed on an optimal reinsurance investment problem for an insurer with jump diffusion risk model when the stock's price was governed by a CEV model.

[9] worked on strategies of optimal reinsurance and investment for exponential utility maximization under different financial markets. [10] investigated investment problem with multiple risky assets. [1] studied an optimal portfolio selection model for risky assets based on asymptotic power law behavior where security prices follow a Weibull distribution.

The work of [11] analyzed the stability of stochastic model of price change at the floor of a stock market. In their analysis précised steps were derived which helped in determining the equilibrium price and growth rate of stock shares. [12] considered the unstable nature of stock market forces using proposed differential equation model.

[13] considered stochastic analysis of the behavior of stock prices. Results reveal that the proposed model is efficient for the prediction of stock prices. In the same vein, [14] studied the stochastic formulations of some selected stocks in the Nigerian Stock Exchange (NSE), in their findings, the drift and volatility coefficients for the stochastic differential equations were determined and the Euler-Maruyama method for system of SDE'S was used to stimulate the stock prices. [8] built the geometric Brownian Motion and studied the accuracy of the model with detailed analysis of simulated data.

More so [15] worked on stochastic problem of the fluctuation of stock market price. Here conditions for finding the equilibrium price, necessary and sufficient conditions for dynamic stability and convergence to equilibrium of the growth rate of the valued function of shares. On the other hand, [16] considered a stochastic problem of price changes at the floor of stock market. The equilibrium price and the market growth rate of shares were determined from their evaluations.

It is clear that [6] has expressed the reserve of risk of two insurance variables to revolve according to a Brownian motion under which their optimal probability of existence was tackled. Nevertheless, based on the fact that the point of focus is of risky asset, risk free asset and over-plus of company's claims which changes over time between the one who insures and reinsures. Due to these changes affecting stock variables motivated us to proffer precise condition of adding sources of randomness in the formulated model. To this end, an accurate analytical solution was obtained for time varying investment returns. To the best of our knowledge these novel contributions have not been seen elsewhere as this will widen the scope in this dynamic area of financial mathematics.

This paper aims at first, present a time varying investment return which is deterministic and stochastic using drift and random parameters which is aimed at determining the surplus of insurance company for insurer and reinsurer. Secondly an investment of risky asset (stock) and riskless asset (bond) of insurer and reinsurer whose returning rate of time is linear and quadratic.

This paper is prescribed in the following ways: Section 2.1 presents the preliminaries, the mathematical formulation is seen in subsection 2.2, subsection 2.3 is method of solution, results are seen in Section 3, while the discussion is presented in Section 4 and paper is concluded in Section 5.

Preliminaries

Definition 1: A standard Brownian motion is simply a stochastic process $\{B_t\}_{t\in\tau}$ with the following properties:

i) With probability 1, $B_0 = 0$.

ii) For all $0 \le t_1 \le t_2 \dots \le t_n$, the increments

$B_{t_2} - B_{t_1}, B_{t_3} - B_{t_2}, \dots B_{t_n} - B_{t_{n-1}}$ are independent.

iii) For $t \ge s \ge 0, B_t - B_s \sim N(0, t-s)$.

iv) With probability 1, the function $\to B_1$ is continuous.

1. Stock Price Modelling

Theorem 1: (Ito's formula) Let $(\Omega, \beta, \mu, F(\beta))$, be a filtered probability space and $X = \{X, t \ge 0\}$ be an adaptive stochastic process on $(\Omega, \beta, \mu, F(\beta))$ processing a quadratic variation (X) with SDE defined as: $dX(t) = g(t, X(t))dt + f(t, X(t)dW(t)$

$$du(t, X(t)) = \left\{\frac{\partial u}{\partial t} + g\frac{\partial u}{\partial x} + \frac{1}{2}f^2\frac{\partial^2 u}{\partial x^2}\right\}dt + f\frac{\partial u}{\partial x}dW(t),$$

$t \in \Re$ and $u = u(t, X(t) \in C^{1,2}(\mathbb{I} \times \square))$.

Mathematical Formulations

Here, deterministic and stochastic systems are considered following a claim process $C(t)$ of insurance company whose stock price of asset is been traded at a specified time, t .The stochastic process describing the analysis is of the form:

$$dC(t) = \alpha dt + \beta dZ^{(1)}(t), \qquad (1)$$

where α is an expected rate of returns on stock, β is the stock volatility, dt is the relative change in the price during the period of time and $Z^{(1)}$ is a standard Brownian motion defined on a complete probability space $(\Omega, f, (f), P)$.

Also considering the surplus term $R(t)$ of the insurer where the insurance company is obliged to buy proportional reinsurance in order to reduce risk and pay reinsurance premium consistently at a particular rate, [5] based on the above factors the surplus of insurance is governed following differential equation and stochastic differential equation respectively.

$$\frac{dR_K(t)}{dt} = (\theta - \varepsilon\eta^2\alpha)R_k(t) \qquad (2)$$

$$dR'_k(t) = (\theta - \varepsilon\eta^2\alpha)R'_k(t)dt + \beta R'_k(t)dZ^{(1)}(t), \qquad (3)$$

with the following initial condition:

$$R_t(0) = R_{0,}\ t > 0 \text{ and } R_0'(0) = R_0',\ \ t > 0. \qquad (4)$$

Where $R_k(t), R_k'(t)$ are surplus terms of the company claims, θ and η are loadings (ie security risk premium) and ε is random noise due to environmental effects. The process η will be under multiple sources of randomness.

However, suppose the insurer and reinsurer invest their gain in the market is made up of risky asset (stock) and risk free asset (bond) which rate of return follows a linear and quadratic function of time. Let $P_1(t)$, $P_1'(t)$ and $P_2(t), P_2'(t)$ represent prices of risky asset and riskless asset respectively. Deterministic and stochastic systems, Hence, the following mathematical structure is proposed:

$$\frac{dP_1(t)}{dt} = \alpha\varepsilon P_1(t), \qquad (5)$$

$$dP_1'(t) = \beta P_1'(t) dZ^{(1)}(t) + \alpha\varepsilon P_1'(t)dt, \qquad (6)$$

$$P_1(0) = P_0 . t > 0 \text{ and } P_1'(0) = P_0', t > 0\ , \qquad (7)$$

$$\frac{dP_2(t)}{dt} = \alpha^2 \varepsilon P_2(t), \qquad (8)$$

$$dP_2'(t) = \alpha^2 \varepsilon P_2'(t)dt + \beta P_2'(t)dZ^{(1)}(t), \qquad (9)$$

and

$$P_2(0) = P_0 . t > 0 \text{ and } P_2'(0) = P_0', t > 0. \qquad (10)$$

Method of Solution

The model (1) -(9) consist of a system of variable coefficient differential equations and stochastic differential equations whose solutions are not trivial. We adopt the methods of variable separable and Ito's lemma in solving f0r

$R_k(t), R_k'(t), P_1(t), P_1'(t), P_2(t)$ and $P_2'(t)$.

To tackle this problem, we note that $R_k(t), R_k'(t), P_1(t), P_1'(t), P_2(t)$and $P_2'(t) < \infty$ for all $r\epsilon[0,1)$.
From (1), this equation is solved using separable variable, hence

$\frac{dR_k(t)}{R_k(t)} = (\theta - \varepsilon\eta^2\alpha)dt$. Integrating both sides give

$$\int \frac{dR_k}{R_k(t)} dR = \int (\theta - \varepsilon\eta^2\alpha)dt + \emptyset,$$

$$\ln R_k(t) = \theta - \varepsilon\eta^2\alpha + \ln\emptyset \text{ , or}$$

$$\ln\left(\frac{R_k(t)}{\emptyset}\right) = \theta - \varepsilon\eta^2\alpha t$$

Taking log of the both sides yields the following

$$R_k(t) = \emptyset e^{\theta - \varepsilon\eta^2\alpha t}. \qquad (11)$$

Applying the initial condition in (4) yields

$$R_k(t) = R_0 e^{\theta - \varepsilon\eta^2\alpha t} \qquad (12)$$

From (3), the over-plus process of the company claims is completely deterministic and the future can be predicted work of the surplus with sureness. Thus, generalizing and considering a function $f(R_k'(t)$,t) hence it has to do with partial derivatives. Expansion of $f(R_k'(t), \mathrm{d}(R_k'(t)$,t+dt)) in a Taylor series about ($R_k'(t)$,t) gives

$$df = \frac{\partial f}{\partial R_k'(t)} dR_k'(t) + \frac{\partial f}{\partial t}dt + \frac{1}{2}\frac{\partial^2 f}{\partial R_k'(t)} dR_k'(t) + \cdots \qquad (13)$$

Substituting (3) in (13) gives

$$df = \frac{\partial f}{\partial R_k'(t)}\{(\theta - \varepsilon\eta^2\alpha)R_k'(t)dt + \beta R_k'(t)dZ^{(1)}(t)\} + \frac{1}{2}\frac{\partial^2 f}{\partial R_k'(t)} dR_k'^2(t) dR_k'^2(t)$$

$$= \beta R_k'(t)\frac{\partial f}{\partial R_k'(t)} dZ^{(1)}(t) + (\theta - \varepsilon\eta^2\alpha)R_k'(t)\frac{\partial f}{\partial R_k'(t)} + \frac{1}{2}\beta^2 R_k'^2(t)\frac{\partial^2 f}{\partial R_k'(t)} dR_k'^2(t).$$

Now considering the SDE in (3).

Let $f(R_k'(t)) = \ln R_k'(t)$, the partial derivatives becomes

$$\frac{\partial f}{\partial R_k'(t)} = \frac{1}{R_k^{'}(t)},\ \frac{1}{2}\frac{\partial^2 f}{\partial R_k'(t)} = -\frac{1}{R_k'^2(t)}, \frac{\partial f}{\partial t} = 0. \qquad (14)$$

According to theorem 1(Ito's), substituting (14) and evaluating gives

$$df = \left\{(\theta - \varepsilon\eta^2\alpha) - \frac{1}{2}\beta^2\right\} + \beta dZ^{(1)}. \qquad (15)$$

Since the RHS of (15) is independent of $f(R_k(t))$,the stochastic is computed as follows:

$$f(R_k'(t)) = f_0 + \int_0^t \left\{(\theta - \varepsilon\eta^2\alpha) - \frac{1}{2}\beta^2 + \int_0^t \beta dZ^{(1)}(t)\right\}$$

$$= f_0 + ((\theta - \varepsilon\eta^2\alpha) - \frac{1}{2}\beta^2)t + \beta dZ^{(1)}(t).$$

Since $f(R_k'(t)) = \ln R_k'(t)$, a foundation for $R_k'(t)$ becomes

$$\ln R_k'(t) = \ln R_0'(t) + ((\theta - \varepsilon\eta^2\alpha) - \frac{1}{2}\beta^2)t + \beta dZ^{(1)}(t)$$

or

$$\ln\frac{R_k'}{R_0'} = ((\theta - \varepsilon\eta^2\alpha) - \frac{1}{2}\beta^2)t + \beta dZ^{(1)}(t).$$

So that

$$R_k'(t) = R_0'\, exp\left\{((\theta - \varepsilon\eta^2\alpha) - \frac{1}{2}\beta^2)t + \beta dZ^{(1)}(t)\right\}. \qquad (16)$$

This is the complete solution of over-plus of the insurance company for the insurer and the reinsurer. It describes the estimate risk reserve of an insurer and reinsurer which follows Brownian motion with drift.

From (5), this equation is solved using separable variable, hence

$$\frac{dP_1(t)}{P_1(t)} = \alpha\varepsilon dt.$$

Integrating both sides give

$$\int \frac{dP_1(t)}{P_1(t)} = \alpha\varepsilon \int dt + K_1,$$

so that

$. \ln \frac{P_1(t)}{K_1} = \alpha\varepsilon t.$

Taking exponent of both sides gives

$$P_1(t) = K_1 e^{\alpha\varepsilon t}. \quad (17)$$

Applying the initial condition in (7) yields

$$P_1(t) = P_0 e^{\alpha\varepsilon t}, \quad (18)$$

whereP_0 is the initial price of asset at time, t . Now considering a function $f(P_1'(t), t)$,hence it has to do with partial derivatives. Expansion of $f(P_1'(t), dP_1'(t), t+dt)$ in a Taylor series about $(P_1'(t), t)$ gives

$$df = \frac{\partial f}{\partial P_1'(t)} dP_1'(t) + \frac{\partial f}{\partial t} dt + \frac{1}{2} \frac{\partial^2 f}{\partial P_1'^2(t)} dP_1'^2(t) + \cdots \quad (19)$$

Putting (7) in (19) gives

$$df = \frac{\partial f}{\partial P_1'(t)} \{\varepsilon\alpha P_1'(t)dt + \beta P_1'(t) dZ^{(1)}(t)\} + \frac{\partial f}{\partial t} dt + \frac{1}{2} \frac{\partial^2 f}{\partial P_1'(t)} dP_1'^2(t)$$

$$= \beta P_1'(t) \frac{\partial f}{\partial P_1'(t)} dZ^{(1)}(t) + \left(\varepsilon\alpha P_1'(t) \frac{\partial f}{\partial P_1'(t)} + \frac{1}{2}\beta^2 P_1'^2(t) \frac{\partial^2 f}{\partial P_1'(t)} dP_1'^2(t)\right) dt. \quad (20)$$

Let $f(P_1'(t)) = \ln P_1'(t)$, the partial derivatives become

$$\frac{\partial f}{\partial P_1'(t)} = \frac{1}{P_1'(t)}, \ \frac{1}{2} \frac{\partial^2 f}{\partial P_1'(t)} = -\frac{1}{P_1'^2(t)}, \frac{\partial f}{\partial t} = 0. \quad (21)$$

According to theorem 1, substituting (21) gives after simplifying;

$$f(P_1'(t)) = (\alpha\varepsilon - \tfrac{1}{2}\beta^2)dt + \beta dZ^{(1)}. \quad (22)$$

Since the RHS of (22) is independent of $f(P_1(t))$,the stochastic is computed as follows:

$$f(P_1'(t)) = f_0 + \int_0^t \left\{(\alpha\varepsilon - \frac{1}{2}\beta^2 + \int_0^t \beta dZ^{(1)}(t)\right\}$$
$$= f_0 + (\alpha\varepsilon - \frac{1}{2}\beta^2)t + \beta dZ^{(1)}(t).$$

Since $f(P_1'(t)) = \ln P_1'(t)$, a foundation for $P_1'(t)$ becomes

$$\ln P_1'(t) = \ln P_0'(t) + (\alpha\varepsilon - \tfrac{1}{2}\beta^2)t + \beta dZ^{(1)}(t)$$

or

$$\ln \frac{P_1'}{P_0'} = (\alpha\varepsilon - \tfrac{1}{2}\beta^2)t + \beta dZ^{(1)}(t).$$

So that

$$P_1'(t) = P_0' \exp\left\{(\alpha\varepsilon - \tfrac{1}{2}\beta^2)t + \beta dZ^{(1)}(t)\right\}. \quad (23)$$

(18) and (23) are investment of risky asset (stock) of insurer and reinsurer whose rate of return is a linear function of time, t .

From (8) using separable variable, hence

$$\frac{dP_2(t)}{P_2(t)} = \alpha^2 \varepsilon dt.$$

Integrating both sides gives

$\int \frac{dP_2(t)}{P_2(t)} = \alpha^2 \varepsilon \int dt + K_2$, so that

$$\ln \frac{P_2(t)}{K_2} = \alpha^2 \varepsilon t.$$

Taking exponent of the both sides gives

$$P_2(t) = K_2 e^{\alpha^2 \varepsilon t}. \quad (24)$$

To find particular solution, we apply the initial condition in (10) yields

$$P_2(t) = P_0 e^{\alpha^2 \varepsilon t}. \quad (25)$$

To solve the stochastic differential equation (9);

considering a function $f(P_2'(t), t)$,hence it has to do with partial derivatives. Expansion of $f(P_2'(t), dP_2'(t), t+dt)$ in a Taylor series about $(P_2'(t), t)$ gives

$$df = \frac{\partial f}{\partial P_2'(t)} dP_2'(t) + \frac{\partial f}{\partial t} dt + \frac{1}{2} \frac{\partial^2 f}{\partial P_2'^2(t)} dP_2'^2(t) + \cdots. \quad (26)$$

Substituting (9) in (26) gives the following

$$df = \frac{\partial f}{\partial P_2'(t)} \{\varepsilon\alpha^2 P_2'(t)dt + \beta P_2'(t) dZ^{(1)}(t)\} + \frac{\partial f}{\partial t} dt + \frac{1}{2} \frac{\partial^2 f}{\partial P_2'(t)} dP_2'^2(t)$$

$$= \beta P_2'(t) \frac{\partial f}{\partial P_2'(t)} dZ^{(1)}(t) + \left(\varepsilon\alpha^2 P_2'(t) \frac{\partial f}{\partial P_1'(t)} + \frac{1}{2}\beta^2 P_2'^2(t) \frac{\partial^2 f}{\partial P_1'(t)} dP_2'^2(t)\right) dt. \quad (27)$$

Considering the SDE in (9),

let $f(P_2'(t)) = \ln P_2'(t)$, the partial derivatives become

$$\frac{\partial f}{\partial P_2'(t)} = \frac{1}{P_2'(t)}, \ \frac{1}{2} \frac{\partial^2 f}{\partial P_2'(t)} = -\frac{1}{P_2'^2(t)}, \frac{\partial f}{\partial t} = 0. \quad (28)$$

Following theorem 1 and putting (28) in (27) gives

$$f = \left(\alpha^2\varepsilon - \tfrac{1}{2}\beta^2\right)t + \beta dZ^{(1)}(t) \quad (29)$$

Since the RHS of (30) is independent of $f(P_2(t))$,the stochastic is computed as follows

$$f(P_2'(t)) = f_0 + \int_0^t \left\{(\alpha^2\varepsilon - \frac{1}{2}\beta^2 + \int_0^t \beta dZ^{(1)}(t)\right\}$$
$$= f_0 + (\alpha^2\varepsilon - \frac{1}{2}\beta^2)t + \beta dZ^{(1)}(t).$$

Since $f(P_2'(t)) = \ln P_2'(t)$, a foundation solution for $P_2'(t)$ becomes

$$\ln P_2'(t) = \ln P_0'(t) + (\alpha^2\varepsilon - \tfrac{1}{2}\beta^2)t + \beta dZ^{(1)}(t)$$

or

$$\ln \frac{P_2'}{P_0'} = (\alpha^2\varepsilon - \tfrac{1}{2}\beta^2)t + \beta dZ^{(1)}(t). \quad (30)$$

So that

$$P_2'(t) = P_0' \exp\left\{(\alpha^2\varepsilon - \tfrac{1}{2}\beta^2)t + \beta dZ^{(1)}(t)\right\}. \quad (31)$$

(25) and (30) are investments of risk free asset (bond) of the one who insures and reinsures whose return's rate is quadratic function of time.

The solutions of insurance quantities are summarized below:

- Deterministic solution of over-plus for the one who insures and reinsures investments

$$R_k(t) = R_0 e^{\theta - \varepsilon\eta^2}$$

- Stochastic solution of over-plus for the one who insures and reinsures investments

$$R_k'(t) = R_0' \, exp\left\{((\theta - \varepsilon\eta^2\alpha) - \frac{1}{2}\beta^2)t + \beta dZ^{(1)}(t)\right\}$$

- Deterministic solution of investments of risky assets for the one who insures and reinsures whose return's rate is a linear function of time.

$$P_1(t) = P_0 e^{\alpha\varepsilon t}$$

- Stochastic solution of investments of risky assets for the one who insures and reinsures whose return's rate is a linear function of time

$$P_1'(t) = P_0' \exp\left\{(\alpha\varepsilon - \tfrac{1}{2}\beta^2)t + \beta dZ^{(1)}(t)\right\}$$

- Deterministic solution of investments of risky assets for the one who insures and reinsures whose return's rate is a quadratic function of time

$$P_2(t) = P_0 e^{\alpha^2\varepsilon t}$$

- Stochastic solution of investments of risky assets for the one who insures and reinsures whose return's rate is a linear function of time

$$P_2'(t) = P_0' \exp\left\{(\alpha^2\varepsilon - \tfrac{1}{2}\beta^2)t + \beta dZ^{(1)}(t)\right\}$$

Results

This Section presents the graphical results for the problems in (2) - (10) whose solutions is in (12) -(31), the graphical results are obtained using MATHEMATICA. Hence the following parameter values were used in the simulation study:

$R_k(0) = 20.2, \theta = 0.75, \varepsilon = 0.01, \eta = 0.45, t = 4$ and $\beta = 5$;

$P_1(0) = 80.51, \varepsilon = 0.01, t = 6, \beta = 5$ and $dZ = 1$;

$P_2(0) = 50.61, \varepsilon = 0.01, t = 12, \beta = 5$ and $dZ = 1$.

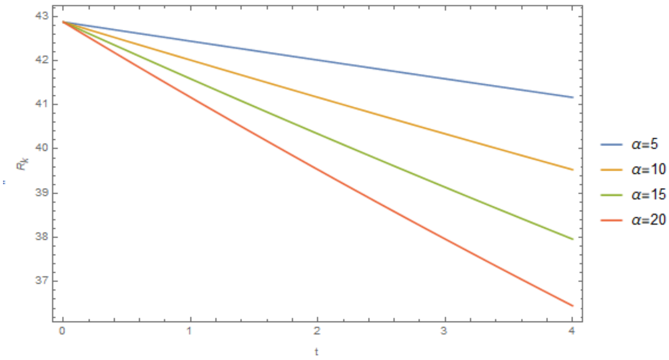

Figure 1: The variation of deterministic over-plus process of company claims of the one who insures and reinsures.

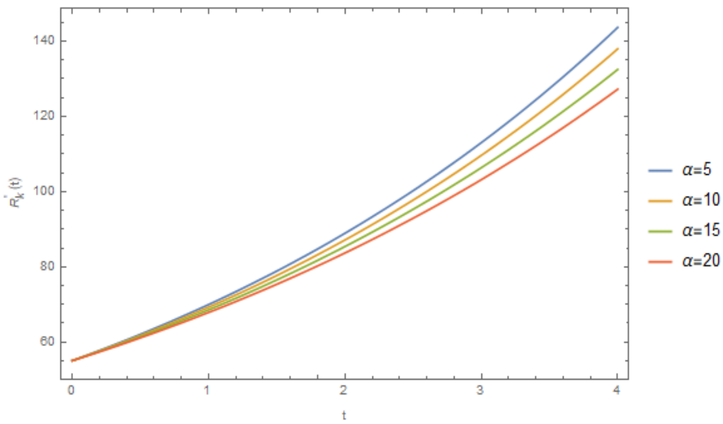

Figure 2: The variation of stochastic over-plus process of company claims of the one who insures and reinsures

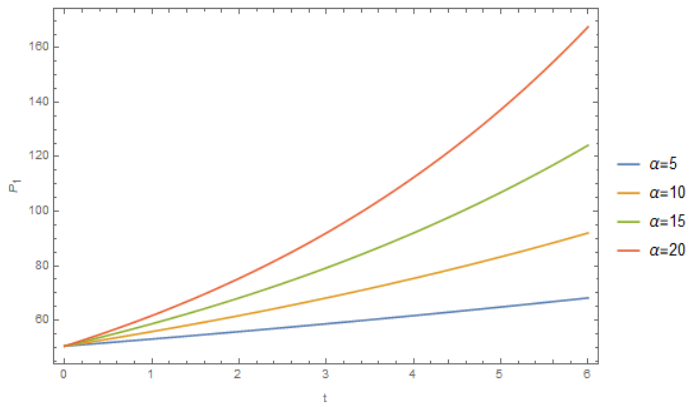

Figure 3: The variation of deterministic risky asset of the one who insures and reinsures of an insurance company

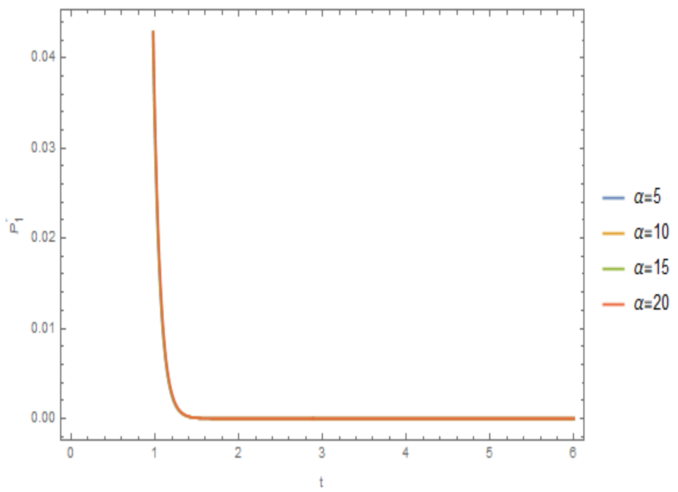

Figure 4: The variation of stochastic risky asset of the one who insures and reinsures an insurance company

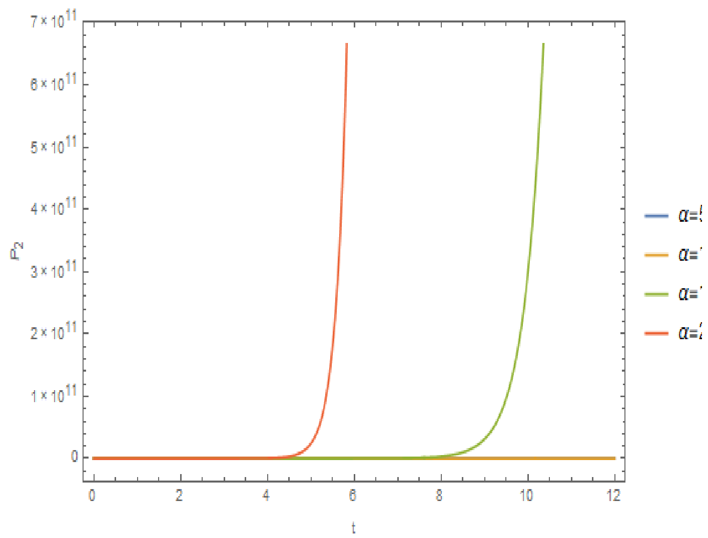

Figure 5: The variation of deterministic risk free asset of the one who insures and reinsures an insurance company

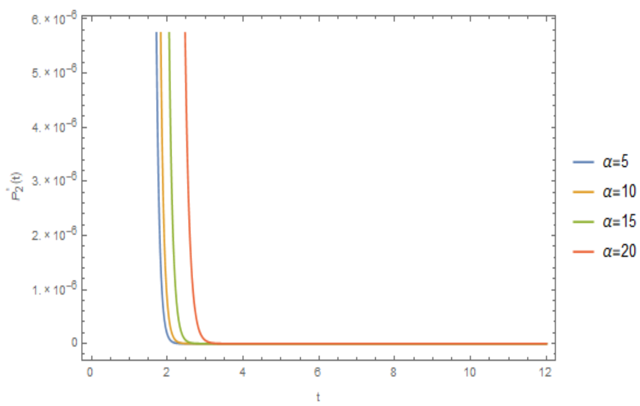

Figure 6: The variation of stochastic risk free asset of the one who insures and reinsures an insurance company

Discussion of results

Figure 1 shows the variation of deterministic over-plus process of company claims of the one who insures and reinsures r. It can be seen that the over-plus process of company claims reduces with increasing return's rate parameter, it is quite obvious in sense that funds are allocated towards public debt; which reduces interest rate and helps economy. Over-plus process of a company can be used to reduce taxes and other trading activities etc.

Figure 2 shows the variation of stochastic over-plus process of company claims of the one who insures and reinsures. This result has the same explanation as in Figure 1. It attests adequacy of both deterministic and stochastic systems in respect to over-plus process claims of insurance company.

Figure 3 shows the variation of deterministic risky asset with return's rate. It is clear that an in return's rate leads to increase in deterministic risky asset. This is physically consistent because the more an investor continues to enlarge its portfolio of investment shows equally that its expected return's rate will also increase. Risky assets have a higher expected return's rate for higher risk. This result is line with that of [1, 20]

Figure 4 shows the differences of stochastic risky asset with return's rate. It is clear that the parameter changes did not affect the direction of the trend; it also leads to a rise in the return's rate. All parameter changes move towards extinction of a common trading activities due limited goods and services. This result is in consonance with the results of [17-19].

Figure 5 shows the variation of deterministic risk-less asset. The result showed increase in return's rate leads to a decrease in deterministic risk free asset. This quite expected because risky-less assets have low return's rate because there is no-risk involve in the trading business. The same explanation holds for Figure 6 which is variation of stochastic risk-less asset.

Conclusion

In this paper, a combination of deterministic and stochastic systems with its random parameters in the models are considered. The analytical solution to the problem has been presented and was verified graphically. The graphical results showed (i) an increase in return's rate decreases over-plus process of insurance company in both deterministic and stochastic systems. (ii) an increase in return's rate leads to an increase in risky asset for both systems but the stochastic system captured the randomness and converges almost surely to zero. (iii) an increase in return's rate leads to a decrease in the risk free assets for the models. This is to show that risk free asset is opposite of risky assets. Finally, the stochastic structure between the over-plus process claims, risky assets and risk free assets for the one who insures and reinsures may be another interest area of further study.

References

[1]- Okoroafor , A.C and Osu, B.O(2009). An empirical optimal portfolio selection model. African journal of Mathematics and Computer science research,vol.2(1) ,1-5.

[2]- .Osu, B.O. (2010). A Stochastic Model of the Variation of the Capital Market Price. International journal of of Trade,Economics and Finance,1,3, 297-302.

[3]- Browne,S.(1995). Optimal investment policies for a firm with a random risk process exponential utility and minimizing the probability of ruin. Mathematics of operations research, vol.20,4,937-958.

[4]- Bai,I. and Guo, J.(2008). Optimal proportional reinsurance and investment with multiple risky assets and no-shorting constraint. Insurance Mathematics and Economics,vol.42,3,968-975.

[5]- Liang ,Z and Bayraktar ,E.(2014). Optimal reinsurance and investment with unobservable claim size and intensity, Insurance Mathematics and Economics,vol.55,156-166.

[6]- Ihedioha,S.A and Osu, B.O(2015). Optimal Probability of Survival of an insurer and a reinsurer under proportional reinsurance and power utility preference. International journal of innovation in science and Mathematics. Vol.3 ,issue 6,2347-9051.

[7]- Zhao, H, Rong X. and Zhao, Y(2013). Optimal excess of loss reinsurance and investment problem for an insurer with jump-diffusion risk process under the Heston model, Insurance Mathematics and Economics ,vol.53,3,504-514.

[8]- Gu, A. ,Guo X and Li, Z.(2012). Optimal control of excess of loss reinsurance and investment for insurer under a CEV model, Insurance Mathematics and Economics vol.51,3, 674-684.

[9]- Lin ,X and li. Y.(2012). Optimal reinsurance and investment for a jump diffusion risk process under the CEV model. North American journal, vol.15,3, 417-431.

[10]- .Osu, B.O. and Okoroafor, A.C.(2007).On the measurement of random behavior of stock price changes. Journal of mathematical science Dattapukur,18(2),131-141.

[11]- .Davies, I. Amadi, I.U and Ndu, R.I(2019).Stability Analysis of Stochastic Model for Stock Market Prices. International journal of Mathematics and Computational Methods ,4,79-86.

[12]- .Osu, B.O, Okoroafor,A.C. and Olunkwa, C.(2009). Stability Analysis of Stochastic model of Stock market price. African journal of Mathematics and Computer Science 2(6),98-103

[13]- Adeosun,M.E., Edeki,S.O. and Ugbebor,O.O(2015).Stochastic Analysis of Stock Market Price Models: A case Study of the Nigerian Stock Exchange(NSE). WSEAS transactions on Mathematics, 14,353-363.

[14]- .Ofomata,A.I.O.,Inyama,S.C.,Umana,R.A,and,Omane,A.O(2017).A stochastic Model of the Dynamics of Stock Price for Forecasting. Journal of Advances in Mathematics and Computer Science.25(6),1-24.

[15]- .Osu, B.O. (2010). A Stochastic Model of the Variation of the Capital Market Price. International journal of of Trade,Economics and Finance,1,3, 297-302.

[16]- .Ugbebor ,O. O ,Onah,S.E. and Ojowo, O.(2001). An Empirical Stochastic Model of Stock Price Changes. Journal Nigerian Mathematical Society,20,95-101.

[17]- Liang,Z., Yuen,K. C., Guo, J. and Cheung, C.(2012). Optimal OrnsteinUhlenbeck investment problem in a constant elasticity of variance stock market for jump-diffusion risk model. Insurance applied stochastic models in business and industry vol.28 ,6,585-597.

[18]- Adeeb, N.,Ahmed, B. Redhwan, N, and Abdullah ,.A.(2020).Analysis of stochastic population with random parameters. Entropy, 22,562-578.

[19]- Amadi ,I.U. Eleki, A.G . and Onwugbta, G.(2017). Application of nonlinear first order ordinary differential equations on stock market prices. International journal of pure and applied sciences, vol.1, issue 1,2635-3393.

[20]- Giet ,J.Vallois,P. Wantz-Meziers,S.(2015). Logistic Stochastic Differential Equation. Theory of stochastic process,20,28-62.