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Singular and Bright soliton solutions for the coupled system corresponding to Gerdjikov-Ivanov equation with four-wave mixing terms

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ABSTRACT

The extended trial function method is utilized in birefringent fibers to generate optical soliton solutions to the coupled system that correspond to the Gerdjikov-Ivanov equation with four-wave mixing terms. The approach yields singular and bright soliton solutions, and in addition extremely important singular and singular periodic soliton solutions are extracted by taken the limit of the modulus of ellipticity of the Jacobi's elliptic function. The conditions under which the soliton solution exist are also provided.

الحلول الفردية و الامعة غير المتغيرة زمنيا لمعادلة جيرجيكوف ايفانوف مع تداخل الموجات الأربعة في الألياف ثنائية الانكسار باستخدام طريقة الدالة التجريبية الممتدة

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الكلمات المفتاحية:
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الحلول البصرية

الملخص

تم استخدام طريقة الدالة التجريبية الممتدة للحصول على حلول ضوئية غير متغيرة زمنيا للنظام المقترن المقابل لمعادلة جيرجيكوف ايفانوف مع تداخل الموجات الأربع في ألياف ثنائية الانكسار. تكشف الطريقة عن حلول غير متغيرة زمنيا فردية وحلول للامعة، كما تظهر حلول غاية في الأهمية بصيغة دالة جاكوبي الإهليجية، وفي حالة النهاية للمعامل الإهليجي نحصل علي حلول غير متغيرة زمنيا فردية و فردية دورية جنبًا إلى جنب مع معايير وجودهم.

Introduction

The Gerdjikov-Ivanov (GI) model with four-wave mixing terms (FWM) in birefringent fibers, has an important place to govern parallel transmission of pulses via optical fibers, this model is one of variey models that study the progression of optical soliton propagation for transmission technology, optical fibers, data transfer across intercontinental and transatlantic distances, and telecommunications industry. This model has been studied for polarization-preserving fibers along with strategic algorithms like as the csch method, the extended $\tanh-\coth$ method, $\frac{G'}{G^2}$-expansion method, sine-cosine method, Hirota's method, sine-Gordon equation method, first integral method, and the $\exp(-(\phi))$-expansion method $[1-7]$. The model also studied in [8] without FWM. Though there are a lot of advances, the solitons only along one portion of the model have been considered. For more improvements, the extended trial function scheme has been utilized with the coupled GI model with FWM given in two parts forms which gives rise to improve the model further and strategic singular and bright soliton solutions are obtained. Also, extremely important singular and singular periodic soliton solutions which are extracted by taken the limit of the modulus of ellipticity of the Jacobi's elliptic function. The conditions under which the soliton solution exist are also provided.

Governing model

The considered GI equation [1-8] is represented as

$$i\psi_t + a\psi_{xx} + b|\psi|^4\psi + ic\psi^2\psi_x^* = 0. \quad (1)$$

The initial term represents the pulses' temporal progression while the group velocity dispersion is supplied by the factor of a. The complex valued function $\psi(x,t)$ referred to the wave profile. The b coefficient is named as the nonlinear term that signifies quintic nonlinearity. Once and for all, the type of dispersive phenomenon is ensured by the c coefficient.

The coupled system corresponding to GI model with FWM in birefringent fibers [7] is described by

$$i\psi_t + a_1\psi_{xx} + (b_1|\psi|^4 + c_1|\psi|^2|\phi|^2 + d_1|\phi|^4)\psi +$$

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$$i(\beta_1\psi^2+\gamma_1\phi^2)\psi_x^*+(\delta_1(\psi\phi^*)_{xx}+\sigma_1(\psi^*\phi)_{xx})\psi=0,$$
$$i\phi_t+a_2\phi_{xx}+(b_2|\phi|^4+c_2|\phi|^2|\psi|^2+d_2|\psi|^4)\phi+$$
$$i(\beta_2\phi^2+\gamma_2\psi^2)\phi_x^*+(\delta_2(\phi\psi^*)_{xx}+\sigma_2(\phi^*\psi)_{xx})\phi=0 \tag{2}$$

The indices of a_j indicates group velocity dispersion while the indices of b_j originate from self-phase modulation. Finally, the indices of c_j and d_j indicates cross-phase modulation whilst the indices of β_j, γ_j account for various kinds of dispersive phenomena, in addition the indices of δ_j, σ_j mean four-wave mixing along with j=1, 2.

Mathematical preliminaries

For solving the considered coupled system we start with

$$\psi(x,t)=w_1(\zeta(x,t))e^{i\theta(x,t)}, \tag{3}$$
$$\phi(x,t)=w_2(\zeta(x,t))e^{i\theta(x,t)}, \tag{4}$$

where w_j for $j=1,2$ indicate the amplitude of the soliton, ζ represent the wave variable, and θ represents the phase of the soliton that is described as

$$\zeta(x,t)=k_1x-\nu t, \tag{5}$$
$$\theta(x,t)=-k_2x+\mu t+k_3. \tag{6}$$

Here, the soliton's velocity is represented by ν, k_2 indecate the frequency of the solitons in both components while μ is the soliton wave number and k_3 is the phase constant. Putting (5) and (6) into (3) and (4) and then into (2) we get

$$-(\mu+a_1k_2^2)w_1+a_1k_1^2w_1''+b_1w_1^5+c_1w_1^3w_2^2+$$
$$d_1w_1w_2^4-k_2\beta_1w_1^3-k_2\gamma_1w_2^2w_1+$$
$$k_1^2(\delta_1+\sigma_1)(2w_1'w_1w_2'+w_1^2w_2''+w_1w_1''w_2)+i(-\nu-$$
$$2a_1k_1k_2+k_1\beta_1w_1^2+k_1\gamma_1w_2^2)w_1'=0, \tag{7}$$
$$-(\mu+a_2k_2^2)w_2+a_2k_1^2w_2''+b_2w_2^5+c_2w_2^3w_1^2+$$
$$d_2w_1^4w_2-k_2\beta_2w_2^3-k_2\gamma_2w_1^2w_2+$$
$$k_1^2(\delta_1+\sigma_1)(2w_2'w_2w_1'+w_2^2w_1''+w_2w_2''w_1)+$$
$$i(-\nu-2a_2k_1k_2+k_1\beta_2w_2^2+k_1\gamma_2w_1^2)w_2'=0. \tag{8}$$

Equation (7) and (8) can be gathered as

$$-\left(\mu+a_jk_2^2\right)w_j+a_jk_1^2w_j''+b_jw_j^5+$$
$$c_jw_j^3w_l^2+d_jw_jw_l^4-k_2\beta_jw_j^3-k_2\beta_jw_j^3-k_2\gamma_jw_l^2w_j+$$
$$k_1^2\left(\delta_j+\sigma_j\right)\left(2w_j'w_jw_l'+w_j^2w_l''+w_jw_j''w_l\right)+$$
$$i\left(-\nu-2a_jk_1k_2+k_1\beta_jw_j^2+k_1\gamma_jw_l^2\right)w_j'=0 \tag{9}$$

where $j=1,2$ and $l=3-j$, using the balancing principle we get $w_j=w_l$

$$-\left(\mu+a_jk_2^2\right)w_j+a_jk_1^2w_j''+\left(b_j+c_j+d_j\right)w_j^5-$$
$$k_2\left(\beta_j+\gamma_j\right)w_j^3+2k_1^2\left(\delta_j+\sigma_j\right)\left(w_j\left(w_j'\right)^2+w_j^2w_j''\right)+i\left(-\nu-\right.$$
$$\left.2a_jk_1k_2+k_1\left(\beta_j+\gamma_j\right)w_j^2\right)w_j'=0, \tag{10}$$

separating the real and the imaginary portions yields

$$-(\mu+a_jk_2^2)w_j+a_jk_1^2w_j''+(b_j+c_j+d_j)w_j^5-k_2(\beta_j+\gamma_j)w_j^3+$$
$$2k_1^2(\delta_j+\sigma_j)(w_j(w_j')^2+w_j^2w_j'')=0 \tag{11}$$
$$-\nu-2a_jk_1k_2+k_1(\beta_j+\gamma_j)w_j^2=0, \tag{12}$$

from (12) the velocity of the soliton solution is

$$\nu=2\,a_j\,k_1\,k_2, \tag{13}$$

and we deduce the condition $\beta_j=-\gamma_j$. So equation (11) reduce to

$$\alpha_{(1,j)}w_j+\alpha_{(2,j)}w_j''+\alpha_{(3,j)}w_j^5+\alpha_{(4,j)}(w_j(w_j')^2+w_j^2w_j'')=0 \tag{14}$$

Where

$$\alpha_{(1,j)}=-(\mu+a_jk_2^2),\ \alpha_{(2,j)}=a_jk_1^2,$$
$$\alpha_{(3,j)}=(b_j+c_j+d_j),\ \alpha_{(4,j)}=2k_1^2(\delta_j+\sigma_j).$$

Application of the extended trial equation scheme

Assuming the solution for the given nonlinear partial differential equation of the form

$$\varphi_j=\sum_{i=0}^{N}A_{i,j}u^i,\quad j=1,2, \tag{15}$$

where

$$(u')^2=\Gamma(u)=\frac{\Theta(u)}{\Upsilon(u)}=\frac{\sum_{i=0}^{\tau}\lambda_iu^i}{\sum_{i=0}^{\rho}\chi_iu^i}, \tag{16}$$

where $\lambda_i,\ \chi_i,\ A_{i,j}$ are constants and $\lambda_\tau,\ \chi_\rho,\ A_{N,j}$ are non-zero. Equation (15) can be written as

$$\pm(\zeta-\zeta_0)=\int\frac{du}{\sqrt{\Gamma(u)}}=\int\sqrt{\frac{\Upsilon(u)}{\Theta(u)}}du, \tag{17}$$

The balancing principle applied to (14) implies

$$\tau=\rho+2N+2, \tag{18}$$

Assuming $\rho=0$ and $N=1$ we get $\tau=4$ consequently from (15) we have

$$\varphi_j=A_{0,j}+A_{1,j}u, \tag{19}$$
$$(\varphi_j')^2=\frac{(A_{1,j})^2\sum_{i=0}^{4}\lambda_iu^i}{\chi_0}, \tag{20}$$
$$\varphi_j''=\frac{(A_{1,j})\sum_{i=0}^{4}i\lambda_iu^{i-1}}{2\chi_0}, \tag{21}$$

where $\lambda_4\neq0$ and $\chi_0\neq0$. Substituting Eqs. (19) – (21) into Eq. (14), we obtain a system of algebraic equations, solving the system, we get:

$$A_{0,j}=A_{0,j},\ \chi_0=\chi_0,\ \mu=A_{0,j}^4D_2-a_jk_2^2$$

$$\lambda_0=\frac{4\sqrt{3}A_{0,j}^2a_j^2\chi_0D_2^2\,y_1}{9k_1^2D_1(D_2(y_2)^2)},$$

$$\lambda_1=-\frac{2A_{0,j}a_j(b_j+c_j+d_j)(A_{0,j}a_j\chi_0(b_j+c_j+d_j))}{3k_1^2D_1\,y_2},$$

$$\lambda_2=-\frac{9A_{0,j}^2\chi_0D_1D_2-a_j\chi_0D_2}{6k_1^2D_1^2},$$

$$\lambda_3=-\frac{\chi_0\,y_2}{12a_jk_1^2D_1^2},$$

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$$\lambda_4=-\frac{\chi_0\,y_2}{48A_{0,j}^2a_j^2k_1^2D_1^3D_2},$$

$$A_{1,j}=\pm\sqrt{-\frac{3k_1^2D_1\lambda_4}{\chi_0D_2}},$$

where $D_1=(\delta_1+\sigma_j),\ D_2=(b_j+c_j+d_j),$
$D_3=(\delta_j^2+\sigma_j^2),\ D_4=(\delta_j^3+\sigma_j^3).$

$$y_1=\sqrt{3}D_2\left(24A_{0,j}^4D_3-a_j^2+9A_{0,j}^2a_jD_1+48A_{0,j}^4\delta_j\sigma_j\right)-$$
$$60\sqrt{3}A_{0,j}^4k_1^2D_4-180\sqrt{3}A_{0,j}^4k_1^2\delta_j\sigma_jD_1$$

$$y_2=48A_{0,j}^4D_3-3a_j^2+16A_{0,j}^2a_jD_1+96A_{0,j}^4\delta_j\sigma_j$$
$$-120A_{0,j}^4k_1^2D_4-360A_{0,j}^4k_1^2\delta_j\sigma_jD_1$$

Substitute into (16) and (17), we get

$$\pm(\zeta-\zeta_0)=Q\int\frac{du}{\sqrt{\Gamma(u)}}, \tag{22}$$

where $Q=\sqrt{\frac{\chi_0}{\lambda_4}},\ \ \Gamma(u)=\sum_{i=0}^{4}\frac{\lambda_i}{\lambda_4}u^i.$

Therefore the solutions to Eq.(2) are:

When $\Gamma(u)=(u-\vartheta_1)^4$

$$\psi(x,t)=\left(A_{0,1}+A_{1,1}\vartheta_1\pm\frac{A_{1,1}Q}{k_1x-2a_1k_1k_2t-\zeta_0}\right)$$
$$e^{i(-k_2x+\mu t+k_3)}, \tag{23}$$

$$\phi(x,t)=\left(A_{0,2}+A_{1,2}\vartheta_1\pm\frac{A_{1,2}Q}{k_1x-2a_2k_1k_2t-\zeta_0}\right)$$
$$e^{i(-k_2x+\mu t+k_3)}. \tag{24}$$

When $\Gamma(u)=(u-\vartheta_1)^3(u-\vartheta_2)$, and $\vartheta_2>\vartheta_1$

$$\psi(x,t)=(A_{0,1}+A_{1,1}\vartheta_1+$$
$$\frac{4A_{1,1}Q^2(\vartheta_2-\vartheta_1)}{4Q^2-[(\vartheta_1-\vartheta_2)(k_1x-2a_1k_1k_2t-\zeta_0)]^2})e^{i(-k_2x+\mu t+k_3)}, \tag{25}$$

$$\phi(x,t)=(A_{0,2}+A_{1,2}\vartheta_1+$$
$$\frac{4A_{1,2}Q^2(\vartheta_2-\vartheta_1)}{4Q^2-[(\vartheta_1-\vartheta_2)(k_1x-2a_2k_1k_2t-\zeta_0)]^2})e^{i(-k_2x+\mu t+k_3)}. \tag{26}$$

When $(u-\vartheta_1)^2(u-\vartheta_2)^2$

$$\psi(x,t)=(A_{0,1}+A_{1,1}\vartheta_L+$$
$$\frac{(-1)^{L+1}A_{1,1}(\vartheta_1-\vartheta_2)}{\exp(\frac{(\vartheta_1-\vartheta_2)}{Q}(k_1x-2a_1k_1k_2t-\zeta_0))-1})e^{i(-k_2x+\mu t+k_3)}, \tag{27}$$

$$\phi(x,t)=(A_{0,2}+A_{1,2}\vartheta_L+$$
$$\frac{(-1)^{L+1}A_{1,2}(\vartheta_1-\vartheta_2)}{\exp(\frac{(\vartheta_1-\vartheta_2)}{Q}(k_1x-2a_2k_1k_2t-\zeta_0))-1})e^{i(-k_2x+\mu t+k_3)}, \tag{28}$$

Where $L=1,2$.

When $\Gamma=(u-\vartheta_1)^2(u-\vartheta_2)(u-\vartheta_3)$, and $\vartheta_1>\vartheta_2>\vartheta_3$

$$\psi(x,t)=(A_{0,1}+A_{1,1}\vartheta_1-\frac{2A_{1,1}(\vartheta_1-\vartheta_2)(\vartheta_1-\vartheta_3)}{2\vartheta_1-\vartheta_2-\vartheta_3+(\vartheta_3-\vartheta_2)\cosh\left(\frac{k_1\sqrt{(\vartheta_1-\vartheta_2)(\vartheta_1-\vartheta_3)}}{Q(k_1x-2a_1k_1k_2t)}\right)})$$

e^$(i(-k$_$2\, x+\mu t+k$_$3))$, (29)

$\phi(x,t)=(A_{0,2}+A_{1,2}\vartheta_1-\frac{2A_{1,2}(\vartheta_1-\vartheta_2)(\vartheta_1-\vartheta_3)}{2\vartheta_1-\vartheta_2-\vartheta_3+(\vartheta_3-\vartheta_2)\cosh(\frac{k_1\sqrt{(\vartheta_1-\vartheta_2)(\vartheta_1-\vartheta_3)}}{Q}(k_1x-2a_2k_1k_2t)}) e^{i(-k_2x+\mu t+k_3)}.$ (30)

When $\Gamma=(u-\vartheta_1)(u-\vartheta_2)(u-\vartheta_3)(u-\vartheta_4)$, and $\vartheta_1>\vartheta_2>\vartheta_3>\vartheta_4$

$\psi(x,t)=\left(A_{0,1}+A_{1,1}\vartheta_2+\frac{2A_{1,1}(\vartheta_1-\vartheta_2)(\vartheta_4-\vartheta_2)}{\vartheta_4-\vartheta_2+(\vartheta_1-\vartheta_4)sn^2\left(\pm\frac{\sqrt{(\vartheta_1-\vartheta_3)(\vartheta_2-\vartheta_4)}}{2Q}(k_1x-2a_1k_1k_2t-\zeta_0),m\right)}\right)e^{i(-k_2x+\mu t+k_3)},$ (31)

$\phi(x,t)=(A_{0,2}+A_{1,2}\vartheta_2+\frac{2A_{1,2}(\vartheta_1-\vartheta_2)(\vartheta_4-\vartheta_2)}{\vartheta_4-\vartheta_2+(\vartheta_1-\vartheta_4)sn^2(\pm\frac{\sqrt{(\vartheta_1-\vartheta_3)(\vartheta_2-\vartheta_4)}}{2Q}(k_1x-2a_2k_1k_2t-\zeta_0),m)})e^{i(-k_2x+\mu t+k_3)}.$ (32)

Where $m^2=\frac{(\vartheta_2-\vartheta_3)(\vartheta_1-\vartheta_4)}{(\vartheta_1-\vartheta_3)(\vartheta_2-\vartheta_4)}$.

Not that ϑ_i for $i=1,\dots,4$ are the root of $\Gamma(u)=0$.

When $A_{0,j}=-A_{1,j}\vartheta_1$ and $\zeta_0=0$, the solutions $(23)-(32)$ are reduced to the following plane wave solutions

$\psi(x,t)=(\pm\frac{A_{1,1}Q}{k_1x-2a_1k_1k_2t})e^{i(-k_2x+\mu t+k_3)},$ (33)

$\phi(x,t)=(\pm\frac{A_{1,2}Q}{k_1x-2a_2k_1k_2t})e^{i(-k_2x+\mu t+k_3)},$ (34)

$\psi(x,t)=(\frac{4A_{1,1}Q^2(\vartheta_2-\vartheta_1)}{4Q^2-[(\vartheta_1-\vartheta_2)(k_1x-2a_1k_1k_2t)]^2})e^{i(-k_2x+\mu t+k_3)},$ (35)

$\phi(x,t)=(\frac{4A_{1,2}Q^2(\vartheta_2-\vartheta_1)}{4Q^2-[(\vartheta_1-\vartheta_2)(k_1x-2a_2k_1k_2t)]^2})e^{i(-k_2x+\mu t+k_3)},$ (36)

singular soliton solutions

$\psi(x,t)=(\frac{A_{1,1}(\vartheta_2-\vartheta_1)}{2}(1\mp\coth(\frac{(\vartheta_1-\vartheta_2)}{2Q}(k_1x-2a_1k_1k_2t))))e^{i(-k_2x+\mu t+k_3)},$ (37)

$\phi(x,t)=(\frac{A_{1,2}(\vartheta_2-\vartheta_1)}{2}(1\mp\coth(\frac{(\vartheta_1-\vartheta_2)}{2Q}(k_1x-2a_2k_1k_2t))))e^{i(-k_2x+\mu t+k_3)},$ (38)

and bright soliton solutions

$\psi(x,t)=\left(\frac{D}{\sqrt{C+\cosh(B(k_1x-2a_1k_1k_2t))}}\right)e^{i(-k_2x+\mu t+k_3)},$ (39)

$\phi(x,t)=\left(\frac{D}{\sqrt{C+\cosh(B(k_1x-2a_2k_1k_2t))}}\right)e^{i(-k_2x+\mu t+k_3)},$ (40)

where $D=\sqrt{\frac{2A_{1,j}(\vartheta_1-\vartheta_2)(\vartheta_1-\vartheta_3)}{(\vartheta_3-\vartheta_2)}}$, $B=\frac{\sqrt{(\vartheta_1-\vartheta_2)(\vartheta_1-\vartheta_3)}}{Q}$,

$C=\frac{2\vartheta_1-\vartheta_2-\vartheta_3}{\vartheta_3-\vartheta_2}$, $j=1,2$.

The soliton's amplitude is provided by D where the inverse width of the soliton is provided by B. These solutions exists for $A_{1,j}<0$. Furthermore, when

$A_{0,j}=-A_{1,j}$ and $\zeta_0=0$, Jacobi's elliptic function solutions $(31),(32)$ are written as

$\psi(x,t)=\left(\frac{D_1}{\sqrt{C_1+sn^2(B_L(k_1x-2a_1k_1k_2t))}}\right)e^{i(-k_2x+\mu t+k_3)},$ (41)

$\phi(x,t)=\left(\frac{D_1}{\sqrt{C_1+sn^2(B_L(k_1x-2a_2k_1k_2t))}}\right)e^{i(-k_2x+\mu t+k_3)},$ (42)

where $D_1=\sqrt{\frac{A_{1,j}(\vartheta_1-\vartheta_2)(\vartheta_4-\vartheta_2)}{(\vartheta_1-\vartheta_4)}}$,

$B_L=\frac{(-1)^L\sqrt{(\vartheta_1-\vartheta_3)(\vartheta_2-\vartheta_4)}}{2Q}$, $C_1=\frac{2\vartheta_4-\vartheta_2}{\vartheta_1-\vartheta_4}$, $L=1,2$.

Remark-1: When $m\to1$, singular optical soliton solutions obtained as

$\psi(x,t)=\left(\frac{D_1}{\sqrt{C_1+\tanh^2(B_L(k_1x-2a_1k_1k_2t))}}\right)e^{i(-k_2x+\mu t+k_3)},$ (43)

$\phi(x,t)=\left(\frac{D_1}{\sqrt{C_1+\tanh^2(B_L(k_1x-2a_2k_1k_2t))}}\right)e^{i(-k_2x+\mu t+k_3)},$ (44)

where $\vartheta_3=\vartheta_4$.

Remark-2: When $m\to0$, periodic singular solutions obtained as

$\psi(x,t)=\left(\frac{D_1}{\sqrt{C_1+\sin^2(B_L(k_1x-2a_1k_1k_2t))}}\right)e^{i(-k_2x+\mu t+k_3)},$ (45)

$\phi(x,t)=\left(\frac{D_1}{\sqrt{C_1+\sin^2(B_L(k_1x-2a_1k_1k_2t))}}\right)e^{i(-k_2x+\mu t+k_3)},$ (46)

where $\vartheta_2=\vartheta_3$.

Conclusion

The coupled system corresponds to GI equation with FWM in birefringent fibres was considered on account of acquiring optical soliton solutions. Bright and singular soliton solutions were presented by the proposed method. Additional solutions which are singular and singular-periodic soliton solutions were extracted using the limiting of the modulus of ellipticity of the Jacobi's elliptic function.

References

[1]- Kadkhoda, N., Jafari, H., (2017), Analytical solutions for the Gerdjikov-Ivanov equation by using exp (− φ (ξ))-expansion method, Optik, Rev., **139**, 72-76. DOI: 10.1016/j.ijleo.2017.03.078

[2]- Kudryashov, N. A. (2020). Traveling wave solutions of the generalized Gerdjikov-Ivanov quation. Optik, Rev.,165193. DOI:10.1016/j.ijleo.2020.165193

[3]- Saleh .M. H, Altwaty. A. A.., (2020), Optical Solitons of The Extended Gerdjikov-Ivanov Equation In DWDM System By Extended Simplest Equation Method, Appl. Math. Inf. Sci. Rev.,**14**, No. 5, 1-7. DOI:10.18576/amis/paper

[4]- Altwaty. A.A , Saleh. M. H, Dumitru B., (2020), Soliton and wave solutions to the extended Gerdjikov-Ivanov equation in DWDM system with auxiliary equation method, Math. Sci. Lett. Rev**., 9**, No. 3, 57-63. DOI:10.18576/msl/090302

[5]- Biswas. A, Yildirim. Y, Yasar. E, Zhou. Q, Alshomrani. A.S, Moshokoa. S.P, Belic. M., (2018), Solitons for perturbed Gerdjikov-Ivanov equation in optical fibers and PCF by extended Kudryashov's method, Opt. Quant Electron.Rev.,**50**(3),149.DOI:10.1007/s11082-018-1417-0

[6]- Altwaty. A.A , Saleh. M. H, Bader R.K. M., (2021). Optical solitons with fractional temporal evolution in fiber bragg gratings with generalized anti-cubic nonlinearity by the fractional Riccati method, Results in Physics, Rev., **22**(20):103872. DOI:10.1016/j.rinp.2021.103872

[7]- Ding, C.-C., Gao, Y.-T., & Li, L.-Q. (2019). Breathers and rogue waves on the periodic background for the Gerdjikov-Ivanov equation for the Alfvén waves in an astrophysical plasma. Chaos, Solitons & Fractals, Rev., **120**, 259–265. DOI:10.1016/j.chaos.2019.01.007

[8]- Emad E. M. Mikael, Altwaty .A.A., Masry. B. R. K., (2021), Optical Soliton Solutions to Gerdjikov-Ivanov Equation Without Four-Wave Mixing Terms in Birefringent Fibers by Extended Trial Function Scheme Rev., **36** (1), 67-72. DOI: 10.54172/mjsc.v36i1.23