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Some Characteristics Of Ng-Groups

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ABSTRACT

The This paper aims to consider and study some new properties in NG groups that consist of non-bijective transformations that are not a subset of symmetric groups. The regularity of these groups presents such as new results. Moreover, a new definition of anti-inverse NG groups is given. Some new results and properties are studied.

بعض خصائص مجموعات NG

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الكلمات المفتاحية:
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الملخص

تهدف هذه الورقة إلى دراسة بعض الخصائص الجديدة في الزمر NG التي تتكون من تحويلات غير تقابليه والتي لا يمكن أن تكون فئة جزئية من الزمرة المتماثلة. الانتظام في هذه الزمر يقدم كنتائج جديدة. علاوة على ذلك، تم تقديم التعريف الجديد لزمرNG المضادة للانعكاس. بعض النتائج والخصائص الجديدة درست.

Introduction

The theory of the transformation group is one of the parts of Mathematics [1]. The NG group was presented by Abdunabi [2] as a group consisting of non-bijective transformations from X to itself with respect to compositions mapping on a non-empty set X. These groups were problem 1.4 in [3]. In [4], Y. Wu and X.Wei present the conditions of the groups generated by non-bijective transformations on a set. Authors in [5], introduce and study the regularity of these groups such as new results. In this paper, we introduce anti-inverse NG groups as a new result in NG groups. Moreover, some new results and properties are studied.

Preliminary:

In this section, recall some basic notations and study some properties of a non-empty and finite group that was used in our paper. For more detail, lots of abstract algebra and finite group theory can see [6],[7] and would be good supplementary sources for the theory needed here. Through this paper, P(X) denoted the set of all its transforms, the image of f is Im(f) for any $f \in$P(X).

Definition 2.1. Suppose that A is a non-empty set. A binary relation ~ is an equivalence relation if it satisfies the following:

1) $a \sim a$, for all $a \in A$;
2) $If\ a \sim b\ then\ b \sim a$ for all $a, b \in A$;
3) $If\ a \sim b\ , b \sim c\ then\ a \sim c. for\ all\ \ a, b, c \in A.$

Definition 2.2. The equivalence class of an equivalence relation on $X\ is\ [x]_{\sim} = \{x \in X | x_{\sim}\}\ ,\ and\ X/\sim = \{[x]_{\sim}\ |x \in X\}$ is said to be the quotient set of X relative to the equivalence relation ~.

Propstion2.1 [2]. Suppose that NG is a group. For any $f \in NG$ and the e the identity element of NG, $\sim_e\ =\ \sim_f$.

Proof. Let X= NG, for any $x \in X$, our goal is to show that $[x]_f = [x]_e$. On one hand, if $a \in [x]_f, i.e. f(a) = f(x)$. We know X is a group with identity element e, there is a transformation $f \in X$ such that $f'f = e = f'f$. Therefore, $e(a)\ =\ f'(f(a))\ =\ f'(f(x)) = e(x)$, Which yields that $a \in [x]_e$.

$(\Leftarrow)\ \ If\ y \in [x]_e\ i.e. e(a)e(y). Hence, f(a) = (fe)(a) = f(e(y)) = (fe)(y) = f(y) \Rightarrow y \in [x]_f$. It follows that $[x]_e = [x]_e$ for any $x \in X = NG,$ as wanted.

Remark 2.1. For proposition 2.1, $\sim_f = \sim_g$ for any element $f, g \in NG$.

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Propstion2.2.[2] Suppose that f is an element (P(X)) and $\hat{f}$ is the induced transformation of f on $X/\sim_f$, $i.e\ \hat{f}: X/\sim_f \longrightarrow X/\sim_f, [x]_f \mapsto [f(x)]_f$. Then there exists groups $NG \subseteq P(A)$ containing f as the identity element iff $f^2=f$. And, there is a group $NG \subseteq P(X)$ containing f as the identity element iff $\hat{f}$ is bijective on $X/\sim_f$.

Proposition 2.3. Suppose that X is a non-empty set and $NG \subseteq P(X)$ is a group that is not a subset of S_n. Set $NG = \{\hat{f} \mid f \in NG\}$; then $\hat{}NG$ is a symmetric group on $X/\sim$ and $\rho: NG \longrightarrow \hat{}NG, f \mapsto \hat{f}$, is an isomorphism.

Proof. Suppose that f, $g \in NG$ and for any $[x] \in X/\sim$, we have $\rho(fg)([x]) = [(fg)(x)] = [f(g(x))] = \rho(f)([g(x)]) = (\rho(f)\rho(g))([x]) \Rightarrow \rho(fg) = \rho(f)\,\rho(g) \Rightarrow \rho$ is a homomorphism. By the definition of $\hat{}NG$, it is obvious that ρ is surjective.

Now, for any two elements f, $g \in NG$, put $\rho(f) = \rho(g), i.e. [f(x)] = [g(x)], \forall x \in X$:

Suppose that e is the identity element of NG, then $[f(x)]_e = [g(x)]_e$; $\forall x \in X$. It follows that $e(f(x)) = e(g(x))$; $\forall x \in X$. Hence, $f(x) = (ef)(x) = e(f(x)) = e(g(x)) = g(x), \forall x \in X$. Therefore, f=g. Conclude that ρ is injective. Therefore, ρ is an isomorphism.

3- New results

In this section, new concepts in NG groups particularly on a set have three or four elements are introduce.

Let consider a set A ={1,2,3}, there are 27 transformations maps from A ={1,2,3} to itself.

Trans(A) as: {(1,1,1), (1,1,2), (1,1,3), (1,2,1), (1,2,2), (1,2,3), (1,3,1), (1,3,2), (1,3,3), (2,1,1), (2,1,2), (2,1,3), (2,2,1), (2,2,2), (2,2,3), (2,3,1), (2,3,2), (2,3,3), (3,1,1), (3,1,2), (3,1,3), (3,2,1), (3,2,2), (3,2,3), (3,3,1), (3,3,2), (3,3,3)} and S3 ={(1,2,3), (2,3,1), (3,1,2), (1,3,2), (3,2,1), (2,1,3)}. Some groups are subsets of Trans(X), but not subsets of the S_3. The groups of order 2 are : NG1={(1,1,3),(3,3,1)}, NG2={(1,2,1),(2,1,2)},NG3={(1,2,2),(2,1,1)},NG4={(1,3,3),(3,1,1)},NG5={(2,2,3),(3,3,2)},andNG6={(2,3,2),(3,2,3)}.

Proposition 3-1[5]: Suppose that NG_1 and NG_2 are two NG-groups that are a not subset of S_3, then the union and intersection of NG1 and NG2 are not necessary to be NG-groups.

Definition 3-1: An element f of NG is anti- inverse of NG if there exists g∈ NG such that $f = g$ and $gfg = f$.

Definition 3-2. The NG-groups are anti-inverse semigroups if for every $f \in$NG, there exists anti- inverse element $g \in$NG. By $\mathfrak{B}$ to denote the class of anti-invers semi groups.

Remark 3.1. $\mathfrak{X}_{m,n}$ denotes to the class of NG groups for which holds $(\forall\, g \in NG)(\exists f \in NG)(f^m = g^m\ )\ (f^m = (fg)^m (f^n = f)$.

Definition3.3. The NG-groups is called quasi-seperative if $for\ any\ f, g \in NG, f^2 = fg = g^2 \Rightarrow f = g$.

Example 3-1: Consider NG= {(1,1,3),(3,3,1)} ; If f=(1,1,3) and g=(3,3,1), then fgf= (1,1,3) (3,3,1) (1,1,3)= (3,3,1) =g and gfg= (3,3,1) (1,1,3) (3,3,1) = (1,1,3)=f. By definition 3-1, then NG is anti-inverse smigroup

Remark 3.2- All NG groups that not subsets of S3 are anti-inverse semigroup.

Definition 3.3. The NG groups is called weakly seperative if $f^2 = fg = gf = g^2 \Rightarrow x = y\ for\ all\ f, g\ in\ NG$.

Definition3.4. A NG groups is called seperative if $f^2 = fg\ and\ g^2 = gf \Rightarrow f = g\ f^2 = gf\ and\ g^2 = fg \Rightarrow f = g$.

Proposition 3.2: Suppose that $NG \in \mathfrak{B}$, g∈ NG ⇔(($\forall\, g \in NG)(\exists f \in NG)(f^2 = g^2)$

Proof:Suppose that $NG \in \mathfrak{B}$, For all $f \in NG$, there exit anti element $g \in NG$, $f^2 = f$ from prpostion $2-2$. since f is invesr element , then $(gfg) = f, fgf = g$. Thus, $f^2 = (gfg)f = g(fgf) = gg = g^2$.

Conversely, suppose that $f^2 = g^2$, from prpostion $2-2$, $g \in$ NG ,

Proposition 3.3: Suppose that $NG \in \mathfrak{B}$ and $f \in$ NG, then g∈ NG,

1) $((\forall\, g \in NG)(\exists f \in NG)(gf = f^3 g)$.
2) $(\exists f \in NG)(f = f^5)$.

Proof:

1) Suppose $NG \in \mathfrak{B}$, then g∈ NG. $gf = (fgf)(gfg) = f(gfg)fg = f(ff)g = f^3 g$, from proposition 3.2.
2) $f = gfg = (f^3 g)g\ from 1, then\ f^3 g^2 = f^3 f^2\ from\ propstion\ 3.2, f = f^5$.

Proposition 3.4: Suppose that $NG \in \mathfrak{B}$ and $f \in$ NG, then g∈ NG , $((\forall\, g \in NG)(\exists f \in NG)((fg)^2 = f^2)$.

Proof: from proposition 2-2 $(((\forall\, g \in NG)(\exists f \in NG)(f^2 = g^2)$. Since $(\exists f \in NG)(f = f^5\ )$ from proposition 3.3. $(fg)^2 = (fg)(fg) = f(gfg) = ff = f^2$.

Proposition 3.5: Every NG from $\mathfrak{X}_{1,n}(n \in N(nutral\ numbers)$.

Proof: from remark 3.1, if m=1 $\mathfrak{X}_{1,n}$ is $(\forall\, g \in NG)(\exists f \in NG)(f^1 = g^1)(f^1 = (fg)^1 (f^n = f)$. From which have $(\forall\, g \in NG)(\exists f \in NG)((f^2 = f)$.

Conclusion

The semi group is anti-invers if for every element x in it, then there exists anti- inverse element y such that xyx=y and yxy=x. In this paper, we define the groups that not subset of symmetric groups as an anti-inverse santi-inverse semigroupsproperties of these groups by a new definition have studied.

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