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On faintly g- cleavability (Splittability)

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Abstract Some properties and basic definitions of a new class of functions called faintly g-continuous functions are introduced [7]. In this paper we studied the concept of cleavability over some special topological spaces as θ-T_k, g-T_k spaces(k=1,2) , GO- compact space, , θ - compact space , GO-connected space and θ - connected space.

Keywords: faintly g- point wise cleavability, faintly g- cleavability, faintly g-absolutely cleavability.

حول قابلية انشطار (انشقاق)-faintly g

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الملخص بعض الخواص والتعريفات الاساسية لفصل جديد من الدوال يسمى: faintly g -continuous functions. و قد تم تقديمه و دراسته [7] و في هذا البحث قمنا بدارسة مفهوم الانشطار باستخدام هذه الدوال على بعض الفضاءات التبولوجية الخاصة التالية: θ-T_k , g-T_k spaces (k=1,2) , GO- compact space, , θ - compact space , GO- connected space and θ - connected space.

الكلمات المفتاحية: انشطار- faintly g النقطي ، انشطار -faintly g ، انشطار- faintly g المطلق .

1- Introduction and Preliminaries:

Different types of cleavability (originally named splitability) of topological spaces where introduced by Arhangel'skii [1]. as following :
A topological space X is said to be cleavable over a class of topological spaces $\mathcal{P}$ if for $A \subset X$ there exists a continuous mapping $f: X \to Y \in \mathcal{P}$ such that
$f^{-1}f(A) = A$, $f(X)=Y$.
Throughout this paper (X, τ) and (Y,σ) represent topological spaces on which no separation axioms are assumed unless otherwise mentioned. . The complement of g-closed set is called g-open. The family of all g-open sets of (X, τ) is denoted by GO(X). A point $x \in$ X is called a θ-cluster point of A if Cl(A) ∩ A $\neq \emptyset$ for every open set A of X containing x. The set of all θ -cluster points of A is called the θ -closure of A and is denoted by Cl_θ (A). If A = Cl_θ (A), then A is said to be θ-closed The complement of θ-closed set is said to be θ-open.

Definition 1.1
A topological space (X, τ) is said to be:
(i) g -T_1 [5] (resp. θ-T_1) if for each pair of distinct points x and y of X, there exists g-open (resp. θ - open) sets U and V containing x and y, respectively such that y $\notin$ U and x $\notin$ V .
(ii) g -T_2 [4] (resp. θ-T_2 [7]) if for each pair of distinct points x and y in X, there exists disjoint g -open (resp. θ -open) sets U and V in X such that x $\in$ U and y $\in$ V.

Definition 1.2
A topological space (X, τ) is said to be GO-compact[7] (resp. θ – compact [2]) if each cover of X by g -open (resp θ -open) has a finite subcover .

Definition 1.3[3]
A topological space (X, τ) is said to be GO-connected
if X cannot be written as a disjoint union of two nonempty g-open sets.

Definition 1.4[6]
A function f : (X, τ) → (Y, σ) is said to be:
faintly g - continuous if f^{-1}(V) is g -open in X for every θ -open set V of Y.

Theorem 1.1[6]
For a function f : (X, τ) → (Y, σ), the following statements are equivalent:
(1) f is faintly g-continuous;
(2) f^{-1}(F) is g -closed in X for every θ-closed subset F of Y ;
(3) f : (X, τ) → (Y, σ) is g-continuous.

Theorem 1.2[6]
Every g-continuous function is faintly g-continuous.

2- faintly g – cleavability

Definition 2.1

A topological space X is said to be faintly g-cleavable over a class of spaces $\mathcal{P}$ if for any subset A of X, there exists a faintly g -continuous mapping $f:X \longrightarrow Y \in \mathcal{P}$, such that $f^{-1}f(A)=A$ and $f(X) = Y$.

Definition 2.2

A topological spaces X is said to be a pointwise faintly g -cleavable over a class of spaces $\mathcal{P}$. if for every point $x \in X$ there exists a faintly g continuous mapping $f:X \rightarrow Y \in \mathcal{P}$, such that $f^{-1} f(x)=\{x\}$.

Definition 2-3

The faintly g- cont. func. $f:X \rightarrow Y \in \mathcal{P}$ Is to be faintly g- open (closed) point wise cleavability if f is an injective and open(closed) respectively.

Definition 2.4

A topological space X is said to be absolutely faintly g- cleavable over a class of spaces $\mathcal{P}$, if for any subset A of X, there exists an injective faintly g -continuous mapping $f:X \longrightarrow Y \in \mathcal{P}$, such that $f^{-1}f(A)=A$. and if $\mathcal{P}$ is the class of all spaces, we shall say that X is absolutely faintly g -cleavable over $\mathcal{P}$. If f is an open (closed)faintly g continuous mapping, we shall say that X is open (closed) absolutely faintly g cleavable over $\mathcal{P}$ respectively.

Proposition 2.1

Let X be an open faintly g – point wise cleavable over a class of θ-T_1 spaces $\mathcal{P}$, then X is g - T_1-space.

Proof:

Let $x \in X$, then there exists a θ -T_1-space Y and a faintly g - continuous mapping $f:X\rightarrow Y \in \mathcal{P}$ such that $f^{-1}f(x) =\{x\}$,$f^{-1}f(x)=\{x\}$. This implies mapping $f:X\rightarrow Y \in \mathcal{P}$ such that $f^{-1}f(x) =\{x\}$, $f^{-1}f(x)=\{x\}$ .This implies that for every $y \in X$ with $x \neq y$, we. have $f(x) \neq f(y)$. Since Y is θ - T_1-space , so there exist two θ - open sets U and V such that $f(x) \in U$, $f(y) \notin U$ and $f(y) \in V$, $f(x) \notin V$. then $f^{-1}f(x) \in f^{-1}(U)$,
$f^{-1}f(x) \notin f^{-1}(V)$. $f^{-1}f(y) \notin f^{-1}(U)$ and
$f^{-1}f(y) \in f^{-1}f(V)$, This implies that $x \in f^{-1}(U)$, $y \notin f^{-1}(U)$ and $y \in f^{-1}(V)$, $x \notin f^{-1}(V)$ By a faintly g -continuity of f, $f^{-1}(U)$, $f^{-1}(V)$ are g- open sub sets in X. Hence X is g- T_1-space.

Proposition 2.2.

Let X be an open faintly g - pointwise cleavable over a class of θ-T_2 spaces $\mathcal{P}$, then X is g -T_2-space.

Proof:

Let $x \in X$, then there exists a θ-T_2 space Y and an open faintly g -continuous mapping $f: X\rightarrow Y \in \mathcal{P}$ such that

$f^{-1}f(x) = \{x\}$. This implies that for every $y \in Y$ with $x \neq y$,we have $f(x) \neq f(y)$. Since Y is a θ-T_2 space , so there exist two disjoint θ-open sets U and V such that $f(x) \in U$, $f(y) \in V$ then $f^{-1}f(x) \in f^{-1}(U)$, $f^{-1} f(y) \in f^{-1}(V)$, this implies that $x \in f^{-1}(U)$, $y \in f^{-1}(V)$, since f is a faintly g -continuous , so $f^{-1}(U)$, $f^{-1}(V)$ are g- open sets of X and

$f^{-1}(U) \bigcap f^{-1}(V)=f^{-1}(U \bigcap V)= f^{-1}(\emptyset)= \emptyset$

then X is g -T_2-space.

Proposition 2.3.

Let X be a GO- compact space is an open faintly g- absolutely cleavable space over a class of spaces $\mathcal{P}$, then Y is θ - compact space.

Proof:

Suppose $\{V_i\}_{i\in I}$ be any θ -open cover of Y, since X is an open faintly g- cleavable, so there exists an open faintly g - continuous mapping $f:X \longrightarrow Y \in \mathcal{P}$, such that $f^{-1}f\{f^{-1}\{V_i\}_{i\in I}\} = f^{-1}\{V_i\}_{i\in I}$ since f is an open faintly g - continuous , then $f^{-1}\{V_i\}_{i\in I}$ is a g -open cover of X but X is g- compact , so there exists a finite sub cover$\{f^{-1}(V_1), \ldots, f^{-1}(V_n)\}$ of X, such that

$X \subset \bigcup_{i=1}^{n} \{f^{-1}(V_i)\}$, since $f f^{-1}(V_i) = V_i$,

So $\{V_1, \ldots, V_n\}$ is a finite sub cover of Y. Therefore Y is a θ compact space.

Proposition 2.4.

Let X be a a GO- connected space is an open faintly g- cleavable space over a class of spaces $\mathcal{P}$, then Y is a connected space.

Proof:

Suppose Y is not connected space , then $Y = V_1 \bigcup V_2$, where V_1, V_2 are disjoint non empty open sets of Y, then there exists an open faintly g - continuous mapping $f:X \longrightarrow Y \in \mathcal{P}$ such that $f^{-1}f\{f^{-1}(V_1)\} = f^{-1}(V_1)$, $f^{-1}f\{f^{-1}(V_2)\} = f^{-1}(V_2)$, since $Y = V_1 \bigcup V_2$,

then $f^{-1}(Y)= f^{-1}(V_1 \bigcup V_2) \Rightarrow X = f^{-1}(V_1) \bigcup f^{-1}(V_2)$

,and $f^{-1}(V_1)$, $f^{-1}(V_2)$ are disjoint nonempty

subsets of X. Since V_i is open and closed, V_i is θ-open sets for each i = 1, 2, since f is a faintly g - continuous, then
$f^{-1}(V_i) \in GO(X)$ .There for X is not GO-connected. . This is a contradiction and hence Y is connected space

conclusion
In this paper we have studied the following three cases :
1) If $\mathcal{P}$ is a class of θ-T_k spaces with, certain properties and if X is a faintly g - pointwise cleavable over $\mathcal{P}$, then X is $g - T_k$-space (k=1 , 2) ,i.e ($X \notin \mathcal{P}$)

2) If $\mathcal{P}$ is a class of a GO- compact space with certain properties and if X is a faintly g-absolutely cleavable over $\mathcal{P}$, then Y is θ - compact space. i .e ($Y \notin \mathcal{P}$)

3) If $\mathcal{P}$ is a class of a GO- connected space with certain properties and if X is a faintly g- cleavable over $\mathcal{P}$, then Y is θ - connected space. i .e ($Y \notin \mathcal{P}$)

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