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Nonlinear Finite Element Analysis of Soil Structure Interaction in Multi -Story Building

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Keywords:
Soil – Structure Interaction.
Finite Element Modelling.
Material Non-Linearity.
Hyperbolic Model.
Foundation System.

ABSTRACT

The effects of Soil-Structure Interaction (SSI) have become a crucial consideration in structural engineering, particularly with the emergence of large-scale constructions on soft soils, including buildings, bridges, tunnels, and underground structures. Therefore, recent design codes incorporate requirements to account for SSI to enable realistic structural modelling. This study sought to simulate the intricate interactions among the various elements of the structure. The need for a simple and accurate model that realistically represents the actual behaviour of the structure and foundation system remains a major concern. This research presents the development of a two-dimensional finite element model of a reinforced concrete frame and pile foundation system, explicitly integrating nonlinear soil response. The soil nonlinearity was modeled using the Duncan and Chang method, which is commonly applied in the hyperbolic model introduced by Kondner and Zelasko. The accuracy of this model was verified through its application in analyzing a multi-story building. The analysis is performed on a numerical model of an eight-story reinforced concrete frame subjected to combined loading of dead, live, and wind loads for two cases: linear elastic analysis and nonlinear elastic analysis. The results show the influence of soil behaviour on the overall response of the structure. A comparison of linear and nonlinear analysis results is carried out to study the effect of soil nonlinearity on the structural response.

النمذجة غير الخطية للعناصر المحددة (FEA) لتفاعل التربة والمنشأ في المباني متعددة الطوابق

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الملخص

تُعَدّ تأثيرات التداخل بين التربة والمنشأ (SSI) جانبًا مهمًا في الهندسة الإنشائية بعد التوسع في تنفيذ المنشآت الضخمة على الترب الناعمة مثل المباني والجسور والأنفاق والمنشآت تحت الأرض. وبناءً على ذلك، فقد تضمنت أكواد التصميم الحديثة متطلباتٍ تأخذ في الاعتبار تأثير التفاعل بين التربة والمنشأ بهدف تحقيق نمذجة إنشائية واقعية. تهدف هذه الدراسة إلى نمذجة التفاعل المعقد بين الاجزاء المختلفة للمنشأ، إذ تظل الحاجة قائمة إلى نموذج رياضي بسيط ودقيق يمكنه تمثيل السلوك الفعلي لمنظومة المنشأ والأساسات بصورة واقعية. قدمت هذه الدراسة طريقة تطوير نموذج ثنائي الأبعاد بالعناصر المحددة لهيكل خرساني مسلح مع نظام أساسات من الركائز (الخوازيق)، ياخذ سلوك التربة غير الخطية في الحساب. وقد تم تمثيل لاخطية التربة باستخدام طريقة (Duncan and Chang)، وهي طريقة شائعة الاستخدام في النموذج (hyperbolic model) الذي اقترحه كل من (Kondner and Zelasko).. تم التحقق من صلاحية النموذج من خلال تحليل مبنى متعدد الطوابق. حيث أجري التحليل على نموذج عددي لمبنى خرساني مسلح مكوّن من ثمانية طوابق خاضع لأحمال ميتة وحية ورياح، وذلك في حالتين: التحليل المرن الخطي والتحليل المرن غير الخطي. أظهرت النتائج تأثير سلوك التربة على السلوك العام للمنشأ، كما أُجري تحليل مقارن بين نتائج التحليل الخطي وغير الخطي لدراسة تأثير اللاخطية التربة على السلوك الإنشائي الكلي.

الكلمات المفتاحية:
التداخل بين التربة و المنشأ.
طريقة العناصر المحددة (*FEM*).
سلوك التربة الغير خطي.
النموذج الزائدي.
نظام الأساسات.

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1. Introduction

Soil-Structure Interaction (SSI) analysis examines the combined behaviour of three interconnected components: the structure itself, its foundation, and the soil both beneath and around the foundation. The SSI phenomenon has become a significant aspect of structural engineering, especially with the rise of large-scale constructions on soft soils, as it tends to increase the overall flexibility of structures. Classical structural analysis techniques assume a rigid foundation, which is valid only for structures supported by rock or very stiff soil. However, for structures constructed on soft soils (flexible base structures), this assumption is no longer valid. Therefore, for reliable analysis, soil structure interaction has to be considered in the analysis, where SSI work reduces stiffness of the structure and increases the system's natural period. [1].

Sunny and Mathai [2] defined SSI as a mutual response where the soil affects the motion of the structure, and simultaneously, the structure's behavior impacts the soil's response. The analysis of piled foundations is particularly challenging because the load transfer involves multiple interactions: not only between the soil and the piles themselves, but also between the foundation structure and the superstructure above it.

Julio A. Garcia [3] concluded that considering the soil in structural analysis yields stresses and deformations that better reflect the real behavior of the structure compared to a fixed-base analysis. Moreover, accounting for soil effects in the analysis and design can lead to more cost-effective structural solutions.

Ravishankar and Satyam [4] proposed a finite Element model for a high-rise building and its surrounding unbounded soil, inherently accounting for the radiation conditions related to wave propagation. In presented work, dynamic analysis was carried out based on ground motion data from the 2001 earthquake with a magnitude of 7.7, assuming a homogeneous soil stratum. Studies on soil-structure interaction (SSI) comparing pile and raft foundation systems have revealed that for the same soil strata, displacements and stresses in pile foundation systems were generally lower than those in raft foundation systems.

Finite-element analysis was carried out to model the behavior of buried steel pipes during different stages of construction. The proposed model was validated using results from four different experimental soil box tests. In this work, nonlinear material and contact algorithms were incorporated in the analysis for both soil and steel pipe materials. The FE analysis results were shown to accurately model and replicate the observed test outcomes [5].

Santisi and Caballero [6] proposed a finite element modelling technique that accounts for the effects of soil-structure interaction (SSI). Their study developed a one-dimensional model of a nonlinear, multi-layered soil profile coupled with a multi-story, multi-span frame, incorporating soil-structure interaction for a rigid shallow foundation while neglecting rocking effects. This model allowed for the analysis of structural dynamic behavior, seismic wave propagation, and the impact of local soil stratigraphy on surface ground motion. It effectively captured the anticipated responses of layered soils exhibiting increasing nonlinearity and varying inertia distribution within the frame.

Han et al. [7] have examined how soil nonlinearity influences dynamic SSI. A two-dimensional numerical model was developed for two superstructures placed on rigid foundations embedded within layered soil overlying elastic bedrock. The model was constructed using the finite element method coupled with an indirect boundary element (IBE) approach. Both the near-field and far-field soil regions were represented, allowing the soil to exhibit nonlinear behaviour. A series of parametric studies was performed to evaluate the contribution of soil nonlinearity to the overall SSI response. The findings indicate that soil nonlinearity substantially reduces relative shear displacement amplitudes, while it significantly increases foundation rotation.

Based on the above literature review, it is evident that continuous efforts are required to develop realistic numerical models for SSI. The key conclusions drawn from the review are: (1) considering the effect of SSI leads to a considerable difference in the structural response, and (2) the nonlinearity of the soil significantly influences the structure's behavior. These insights emphasize the importance of incorporating accurate soil behavior in SSI analysis for more reliable and effective structural design. The primary objectives of the present research work are to propose a numerical procedure using finite elements to simulate the structure, foundations, and underlying soils, and to assess the effect of soil nonlinearity on the overall response of the structure.

2. Direct Formulation of Finite Element Characteristics

The stress strain relation for isotropic materials is expressed as;

$$\{\sigma\} = [D]\ ([B]\{\delta\}^e - \{\varepsilon_0\}) + \{\sigma_0\} \quad (1)$$

Where $\{\varepsilon_0\}$ is the initial strain vector, $\{\sigma_0\}$is the initial stress vector, and [D] is the element material matrix [8]. For linear Elastic Constitutive Relationship [D] is taken as;

$$[D] = \frac{E}{(1+v)(1-2v)} \begin{bmatrix} 1-v & v & 0 \\ . & 1-v & 0 \\ Symmetric & . & \frac{1-2v}{2} \end{bmatrix} \quad (2)$$

Where *v* and *E* denote the Poisson's ratio and Young's modulus of the material, respectively.

3. Non-linear Elastic Constitutive Relationship

One of the most widely used constitutive models in finite element analysis (FEA) is the hyperbolic model introduced by Duncan in 1970. Its popularity stems from its applicability to both drained and undrained soil conditions and the availability of extensive parameter databases.

A typical stress–strain curve obtained from a triaxial compression test under constant confining pressure is illustrated by the dashed line in Figure 1. This behavior is commonly represented mathematically by a hyperbolic function, shown as the solid line in Figure 1, and expressed in the following form:

$$\sigma_1 - \sigma_3 = \frac{\varepsilon}{\frac{1}{E_i} + \frac{\varepsilon}{(\sigma_1 - \sigma_3)_u}} \quad (3)$$

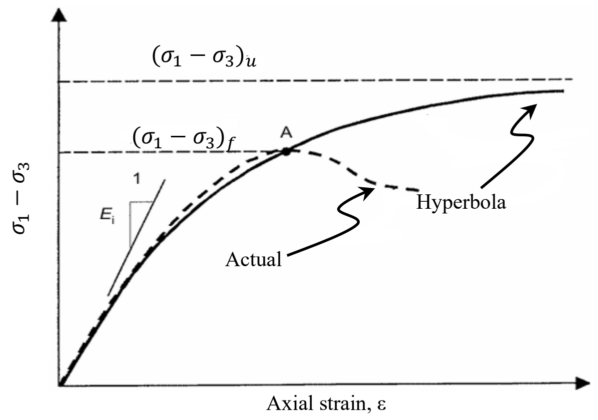

Fig. 1: Stress- Strain Curve

The model effectively captures the nonlinear stress-strain behavior of soils through a hyperbolic formulation, making it applicable to a range of soil-structure interaction problems [8]. It is formulated using two variable elastic parameters—the tangent Young's modulus (E_t) and the tangent bulk modulus (E_i)—which can be expressed using the following relationships:

$$E_T = \left[1 - \frac{R_F\,(1-\sin\emptyset)(\sigma_1-\sigma_3)}{2(C\cos\emptyset + \sigma_3\sin\emptyset)}\right]^2 E_i \quad (4)$$

Unloading is modelled as linear and elastic. The unloading-reloading modulus is a function only of the confining stress as [9]

$$E_i = K\,P_a \left(\frac{\sigma_3}{P_a}\right)^n \quad (5)$$

where R_f = Failure ratio, P_a = Atmospheric pressure, n = exponent determining the rate of variation of E_t with σ_3, K = a modulus number, C = cohesion, = the soil friction angle, σ_1 = maximum principal stresses, σ_3 = minimum principal stress.

4. Proposed Physical Model

A frame structure comprises various components, each with distinct structural behavior. Additionally, the foundation systems and soil conditions must be modeled using appropriate geometric elements. Therefore, to accurately represent such a system, different isoparametric elements should be employed for each specific component.

Figure 2 shows a three-node isoparametric beam bending element with three degrees of freedom per node, which is used to represent the beam, column, and pile. This element accounts for the interaction between axial and flexural behavior, providing a more accurate structural model [10].

An eight-node conventional parabolic finite element shown in figure 3 is commonly used to represent the surrounding soil mass in finite element modelling. This type of element allows for accurate geometric representation and simulates the behavior of the interacting soil medium effectively [11].

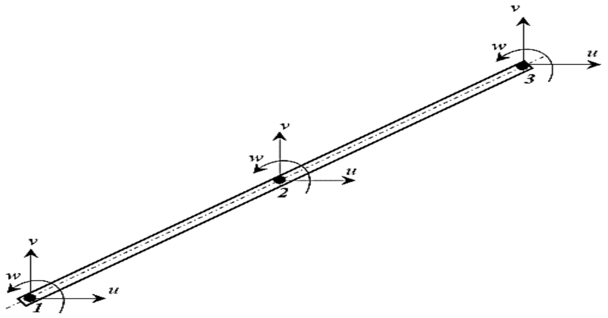

Fig. 2: Parabolic isoperimetric beam element

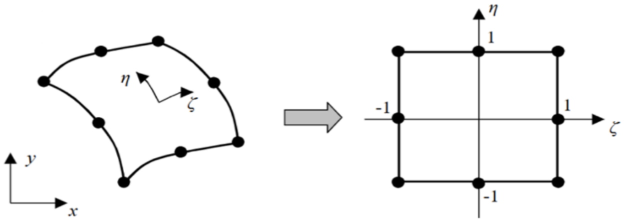

Fig. 3: Eight node conventional parabolic finite element

The elements global stiffness matrix in general is evaluated as

$$[K] = \sum \int [B]^T\,[D][B]\,\partial v \qquad (6)$$

Where [D] and [B] are the element material and strain matrix, respectively. [B] is defined based on the shape functions presented in Table 1[10]

$$[B] = \begin{bmatrix} \frac{\partial N}{\partial x} & 0 & 0 \\ 0 & \frac{\partial N}{\partial y} & 0 \\ 0 & 0 & \frac{\partial N}{\partial z} \\ \frac{\partial N}{\partial y} & \frac{\partial N}{\partial x} & 0 \\ 0 & \frac{\partial N}{\partial z} & \frac{\partial N}{\partial y} \\ \frac{\partial N}{\partial z} & 0 & \frac{\partial N}{\partial x} \end{bmatrix} \qquad (7)$$

Table 1. Shape functions for elements were used in idealization of the structure (ξ, η, and η are the local coordinates) [10]

Isoperimetric element	Shape functions
Eight node parabolic finite element	Corner Nodes: $N_i = \frac{1}{4}(1+\xi\xi_i)(1+\eta\eta_i)(\xi\xi_i+\eta\eta_i-1)$ Midside nodes: $N_i = \frac{\xi_i^2}{2}(1+\xi\xi_i)(1+\eta^2) + \frac{\eta_i^2}{2}(1+\eta\eta_i)(1+\xi^2)$
Parabolic isoperimetric beam element	$N_1 = -\frac{1}{2}\xi(1-\xi)$ $N_2 = (1-\xi^2)$ $N_3 = \frac{1}{2}\xi(1+\xi)$

5. Non-Linear Solution Algorithm

The incremental–iterative methodology is a numerical technique used to address problems with nonlinear relationships between loads and displacements. In this method, loading is applied gradually through small increments. Within each increment, an iterative process ensures equilibrium between internal and external forces. The stiffness matrix is updated based on the stress values accumulated at the end of the previous load increment;

i. Using the finite element method equilibrium equation:

$$[K_n]\{\Delta\delta_n\} = \{\Delta P_n\} \qquad (8)$$

where [K] is the global stiffness matrix, {Δδ}is the column matrix of nodal displacements, and {ΔP} is the load increment column matrix.

ii. Calculate the tangent modulus ET of the soil using Equation (4) and update the constitutive matrix [D].

iii. Determine the column matrix of nodal stresses at the n^th^ step as;

$$\{\Delta\sigma_n\} = [D]\{\Delta\varepsilon\}_n \qquad (9)$$

iv. Then, compute the equilibrated forces and residual forces.

$$\{\Delta f\}_n = \int [B]\{\sigma_n\}\partial v \qquad (10)$$

$$\{R\} = \{\Delta P_n\} - \{\Delta f\}_n \qquad (11)$$

v. At the end of each iteration, solve the system of equations using these residual forces to reach equilibrium displacement;

$$\{\delta_n\} = \{\delta_{n-1}\} + \{\Delta\delta_n\} \qquad (12)$$

6. Development of the Computer Code

In this study, the finite element program developed by Noorzaei et al. [10] was employed, based on the proposed physical and material models. The current version of the program includes a one-dimensional beam isoparametric element with three degrees of freedom per node (u, υ, θ), as well as two-dimensional finite elements within its library. The program is capable of accounting for the nonlinear stress-strain behavior of soil. This nonlinearity is managed using incremental, iterative, or a combined incremental-iterative approach. The flow chart of computational procedures is shown in figure 4.

7. Verification of the Proposed Model

To verify the validity of the proposed methodology and the two-dimensional finite element formulation, a thick circular cylinder subjected to internal pressure (as illustrated in Figure 5) was analyzed. The model was discretized using two-dimensional isoparametric quadrilateral elements under plane strain conditions. The obtained numerical results showed a good agreement with those reported by Hinton and Owen [13] as shown Figure 6, thereby confirming the accuracy and reliability of the proposed formulation.

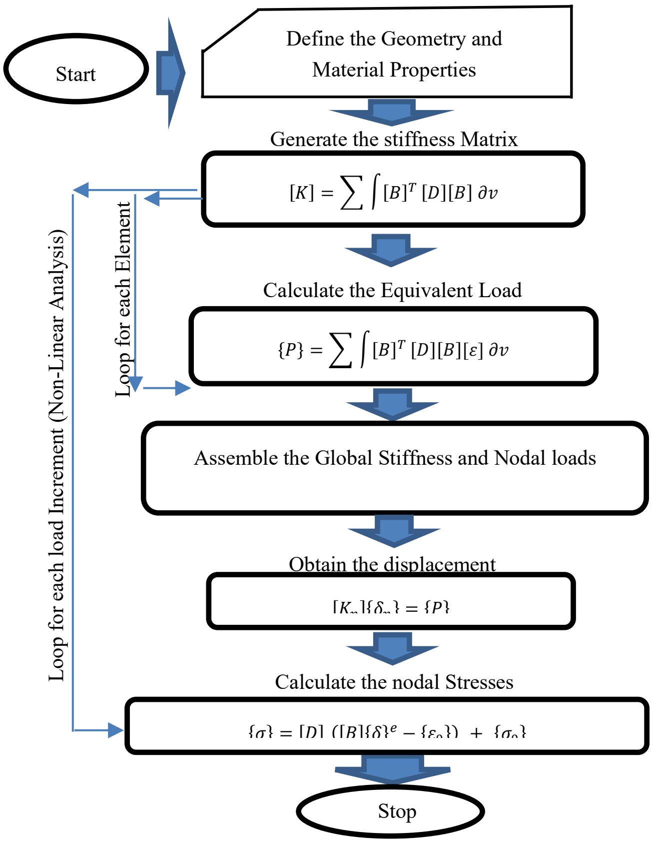

Fig. 4: Flow Chart for Computational Procedures of Finite Element Method

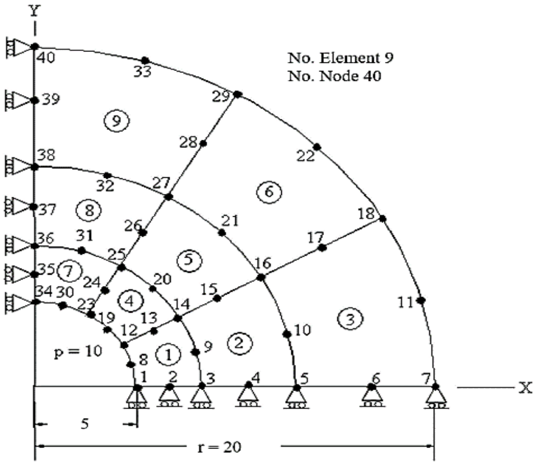

Fig. 5: Finite element mesh of thick circular cylinder [13]

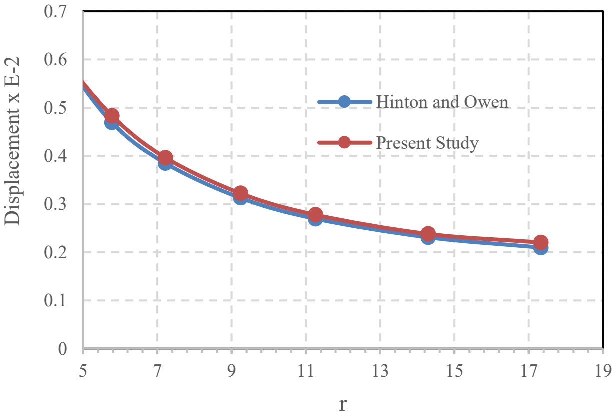

Fig.6: Displacement in X- Direction

8. Structural Idealization

The proposed physical and constitutive model was demonstrated through the analysis of an eight-story framed structure supported by a pile foundation, with a story height of 3.5 m. The overall plan dimension is 12 x 8 square meters. Figure 7 shows the typical structural layout. Beam is 500 x 300 mm for all stories. The columns 1, and 2 are rectangular 500 x 300, and 400 x 300 mm respectively.

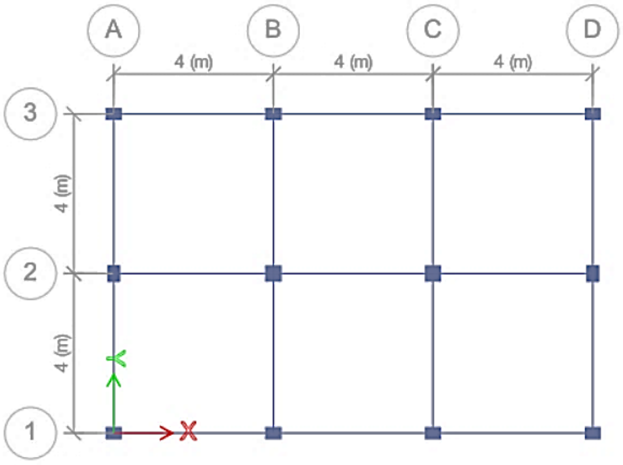

Fig. 7: Structural Layout

9. Material Properties

Due to the fact that, the soil is a nonlinear material, the nonlinearity of the soil will be taken into consideration by using Duncan models. The soil experimental data which reported in the literature for different soil layers has been adopted in the present study. These data tabulated in Table 2 [10]. This date obtained from the laboratory trial test for various type of soil. The soil profile of the presented case is based on Benghazi's soil profile as shown in figure 8.

Table 2: Soil Properties [10]

Description	Dense Sand	Sand Clay	limestone
Modules of elasticity	15000 kN/m^2	45000 kN/m^2	1.5×10^9 kN/m²
K modulus number	198	200	400
n, Exponent	0.82	0.995	0.45
Rf, Failure ratio	0.855	0.88	0.7
C, cohesion	70 KN / m²	22 KN / m²	25 KN / m²
Φ, Angle of friction	22	19	40

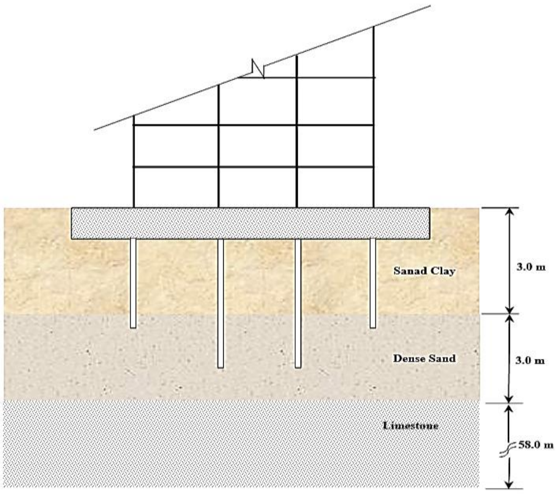

Fig. 8: Benghazi's soil profile

10. Wind Load Characteristics in Libya

Libya is divided into four zones based on the design wind speeds corresponding to a 50-year return period, as illustrated in figure 9[12]. Zone 4, which represents the highest design wind speed, was selected to simulate the worst-case scenario in the analysis.

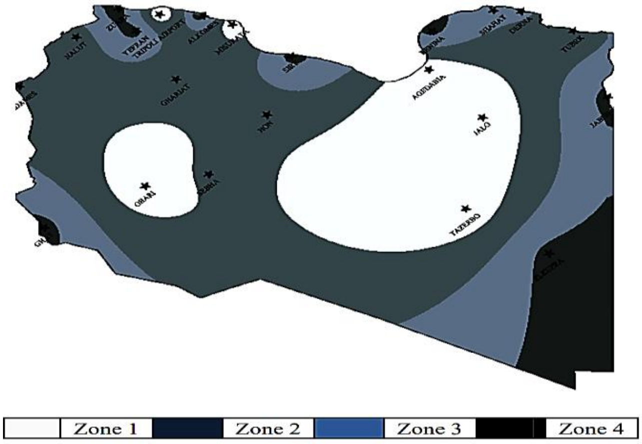

Fig. 9: Design wind speed map of Libya (50-year return period, 10 m height) [12].

11. Finite-Infinite Element Model

The two-dimensional finite element model of the structure-foundation- pile and soil system are shown in figure 10. Eight node element is used for the purpose of discretization of the mat foundation and the soil. The superstructure members have been represented by three degrees of freedom per node to take into account shear deformation as well as the bonding.

12. Results of Analysis

Soil shows complex nonlinear behavior influenced by many factors; assuming linear elasticity can cause significant errors in displacement and stress predictions. Duncan hyperbolic model is used to capture this behavior, with various parameters like modulus number (k), exponent (n), failure ratio (R_f), and initial tangent modulus (E_i) based on lab and field data. The structure will be analyzed using the strength load combinations prescribed in ACI 318-19 [14], including the following cases: (i) factored dead and live loads, (ii) factored dead and wind loads, and (iii) the combined effect of factored dead, live, and wind loads.

Figure 11 shows the lateral displacement of the building with and without the consideration of soil–structure interaction (SSI). It can be observed that the displacement obtained when soil effects are included is nearly twice that of the fixed-base condition. This increase in lateral displacement is attributed to the SSI effect, which introduces additional flexibility to the structure—especially in the case of large buildings resting on soft soil.

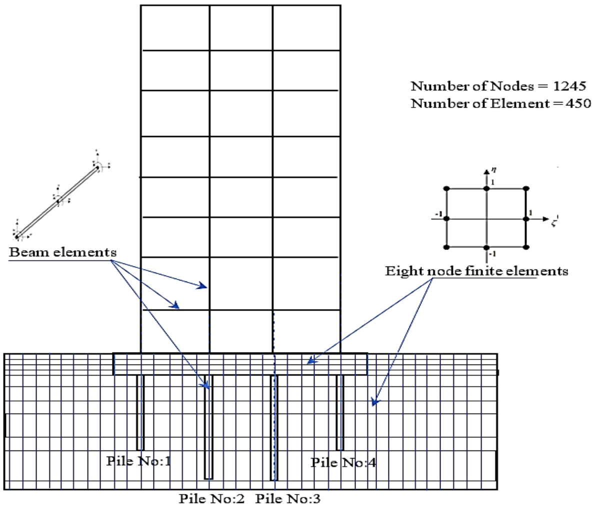

Fig. 10: Finite element discretization of structure- foundation- pile and soil system

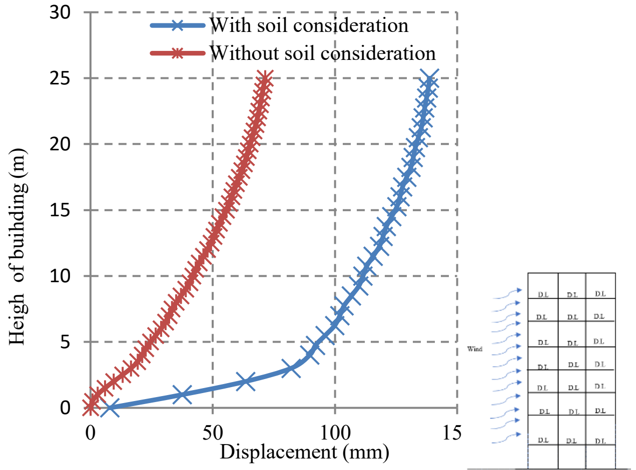

Fig. 11: Lateral displacement (mm) of building (2nd load Comb.)

Figure 12 illustrates the lateral displacement profiles of the piles along their height under the second load combination, which is the most critical load case influenced by wind forces. The deformation of the piles decreases progressively with depth, showing maximum displacement at the top and minimal movement at the bottom. The analysis indicates that the maximum lateral displacement is **8.0 mm**, while the minimum displacement approaches zero, primarily due to the restraining effect of the surrounding soil.

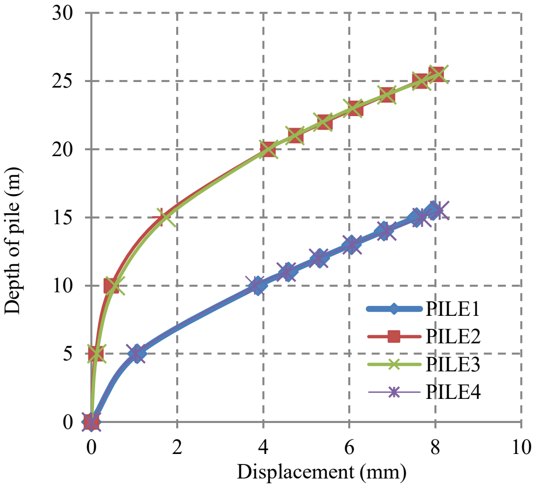

Fig. 12: Lateral Movement of Piles (2nd Combination)

The finite element analysis of the mat foundation in this study relies on several assumptions, primarily because a linearly elastic approach is adopted. This is justified by the substantially greater stiffness of reinforced concrete compared to the underlying soil. Figure 13 (a) and (b) show the distribution of the normal stress s_y. It is clear from this plot that the maximum tensile stress forms at the lower part between the piles, with a maximum value of 1.62×10^{-3} N/mm². However, the maximum compressive stress is located at the connection points of the pile, reaching a maximum of 1.19 N/mm².

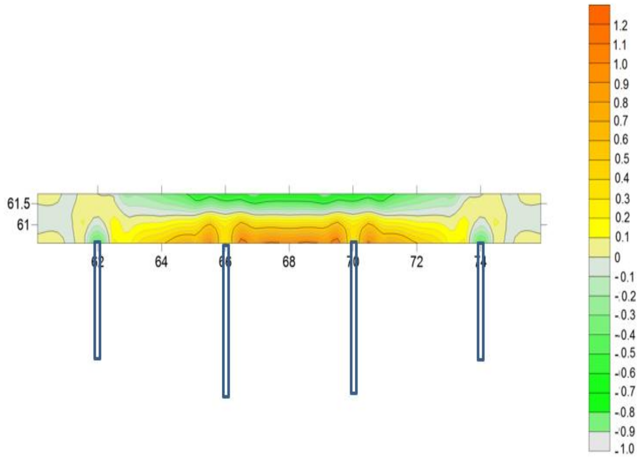

Fig.13 (a): S_y N/mm^2 distribution in the mat foundation (1st combination)

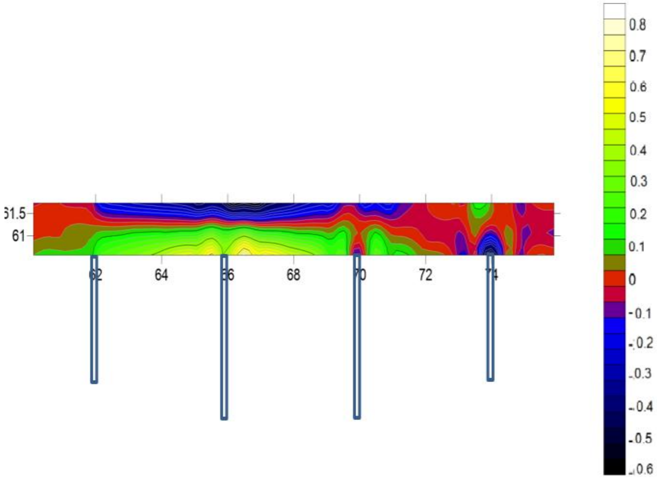

Fig.13 (b): S_y N/mm^2 distribution in the mat foundation (2nd combination)

Figure 14 (a) and (b) illustrate the soil movement beneath the structure for both load combinations. The plots reveal the uplift pressure effect in the first case. In the second case, the soil movement corresponds to the building's response to wind load, caused by the horizontal pushing pressure attempting to overturn the structure. These results and figures offer valuable insight into the soil behaviour beneath the building and illustrate the influence of the structure on nearby constructions.

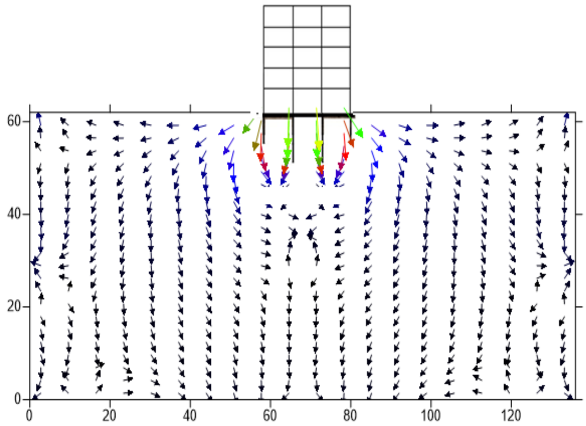

Fig.14 (a): Soil Deformation Patterns (1st combination)

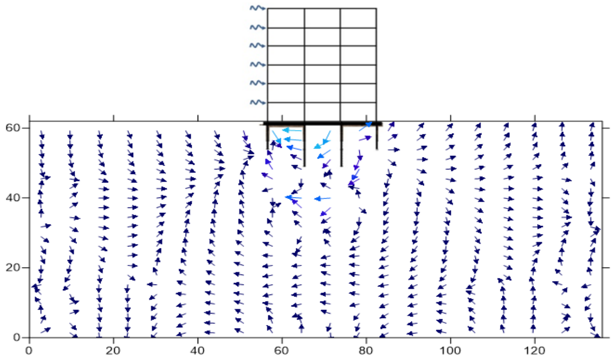

Fig.14 (b): Soil Deformation Patterns (2^{nd} combination)

Soil nonlinearity greatly impacts the computed in-structure response. The core assumption is that soil nonlinearities can affect the soil's elastic moduli. Finite element analyses were conducted for both linear and nonlinear cases. Figure 15 compares the lateral displacement of piles along their height under various load combinations, showing that displacement increased by up to 20% in the nonlinear analysis.

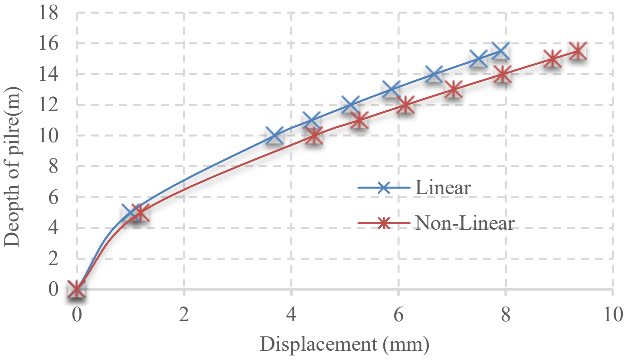

Fig.15: Lateral displacement of pile No: 1

The nonlinear analysis indicates an increase in predicted displacements of up to 20% compared with results obtained from linear soil modelling. This increase is attributed to strain-dependent modulus degradation at elevated shear strain levels, thereby underscoring the importance of incorporating soil nonlinearity to achieve reliable and accurate performance assessment. To evaluate the accuracy and reliability of the present study, the obtained conclusions were examined in light of previously published investigations. Although a direct comparison was not conducted due to differences in the studied cases, the overall findings are consistent with those reported by Sunny and Mathai [2], Han et al. [7], and Noorzaei et al. [10].

13. Conclusions

The research develops a nonlinear finite element model using the Duncan and Chang hyperbolic soil model to realistically simulate soil-structure interaction (SSI) in multi-storey reinforced concrete buildings with pile foundations. The study analyses an eight-story building subjected to combined loads with both linear and nonlinear elastic methods. Results show that considering soil nonlinearity significantly increases lateral displacements and alters stress distribution compared to fixed-base assumptions, highlighting the critical role of accurate soil behaviour modelling for reliable and economical structural design. Neglecting SSI effects risks unsafe and costly designs, whereas nonlinear modelling offers improved prediction of real structural performance on soft soils. Although the present two-dimensional (2D) model provides valuable insight into nonlinear soil–structure interaction behavior and yields result consistent with previously published studies, a fully three-dimensional analysis may be necessary when dealing with complex geometries, irregular structural configurations, or highly non-uniform soil conditions, where significant spatial effects and three-dimensional load redistribution mechanisms cannot be neglected.

The basic premise is that soil nonlinearity can significantly influence its elastic modulus, leading to variations in the soil–structure interaction response. A comparison between linear and nonlinear analyses of the pile lateral displacement along its height under different load combinations shows that the displacement increases by up to 20% in the nonlinear case. The increase in displacement can be attributed to a reduction in the soil's tangent (or secant) elastic modulus as strain levels rise, which effectively softens the surrounding soil and decreases its lateral confinement: as soil stiffness drops the resistance offered by the soil to lateral pile movement reduces, thus yielding larger displacements. For example, Gunawan et al. found that modelling the non-linear modulus around laterally loaded piles produced surface deflections 10 %–14 % higher than a linear modulus assumption for the first 10 % of pile length [15].

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