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(g^*-Pre Regular and g^*-Pre Normal) –Cleavability

*Ghazeel .Almahdi. Jalalah[1] , Nazha Emhimed Alhaj[2]
[1]Department of Mathematics, Faculty of Education , Sirte University, Libya
[2]Department of Mathematics, Faculty of Sciences, Sebha University, Libya
*Corresponding author: gjellala@yahoo.com

Abstract T. D. Rayanagoudar and P. G. Patil [8] introduced two new classes of spaces, called g^*-pre regular and g^*-pre normal spaces.

In this paper we studied the concept of cleavability over these spaces: (g^*-Pre Regular and g^*-Pre Normal) as following:

1- If $\mathcal{P}$ is a class of topological spaces with certain properties and if X is cleavable over $\mathcal{P}$, then $X \in \mathcal{P}$.

2- If $\mathcal{P}$ is a class of topological spaces with certain properties and if Y is cleavable over $\mathcal{P}$, then $Y \in \mathcal{P}$.

Keywords:, g^*p(pre)-irresolute M-pre-open (M-pre-closed) absolutely cleavability , g^*p(pre)-irresolute, M-pre-open(M-pre-closed) absolutely double cleavability.

(g^*-Pre Regular and g^*-Pre Normal) Cleavability (انشطار) قابلية انشقاق

*غزيل المهدي جلاله[1] و نزهة امحمد الحاج[2]
[1]قسم الرياضيات–كلية التربية–جامعة سرت، ليبيا
[2]قسم الرياضيات–كلية العلوم–جامعة سبها، ليبيا
للمراسلة:gjellala@yahoo.com

في هذا البحث درسنا حالة الانشقاق أو (الانشطار)باستخدام دوال خاصة على بعض الفضاءات الطوبولوجية الخاصة :
(g^*-pre regular and g^*-pre normal spaces) كالتالي:

1− إذا كان $\mathcal{P}$ فصل من الفضاءات الطبولوجية بخصائص معينة و كانت X قابلة للانشطار باستخدام الدالة
(g^*p(pre)-irresolute , M-pre-open(M- preclosed) على $\mathcal{P}$ فإن $X \in \mathcal{P}$

2_إذا كان $\mathcal{P}$ فصل من الفضاءات الطبولوجية بخصائص معينة و كانت Y قابلة للانشطار باستخدام الدالة
(g^*p(pre)-irresolute , M-pre-open(M- preclosed) على $\mathcal{P}$ فإن $Y \in \mathcal{P}$

الكلمات المفتاحية: الانشطار المطلق g^*p(pre) -irresolute , M-pre-open(M- preclosed)
الانشطار المطلق المضاعف g^*p(pre)-irresolute , M-pre-open(M- preclosed)

1- Introduction

In 1985 Arhangl' Skii [1] introduced different types of cleavability (originally named splitability) as following :

A topological space X is said to be cleavable over a class of spaces $\mathcal{P}$ if for $A \subset X$ there exists a continuous mapping $f: X \to Y \in \mathcal{P}$ such that $f^{-1}f(A) = A$, $f(X) = Y$. Throughout this paper, X and Y denote the topological spaces (X,τ) and (Y,σ) respectively , Let A be a subset of the space X. The interior and closure of a set A in X are denoted by $int(A)$and $cl(A)$respectively. The complement of A is denoted by $(X - A)$ or A^c.

2-Preliminaries:

Definition 2.1.[8]

A subset A of a topological space (X,τ) is called pre-open set if$A \subseteq int(cl(A))$.

The complement of pre-open set is called. pre-closed set

Definition 2.2. [7, 2]

Let $A \subseteq X$. The intersection of all pre-closed sets containing A is called pre-closure of A and is denoted by $pcl(A)$.

Definition 2.3.

A subset A of a topological space (X,τ) is called

1) g -closed **[6]** if $cl(A) \subseteq G$ whenever $A \subseteq G$and G is open in (X,τ). The complement of g -closed set is called g -open.

2) g^*p -closed **[5]** if $pcl(A) \subseteq G$ whenever $A \subseteq G$ and G is g -open in (X,τ). The complement of g^*p -closed set is called g^*p -open.

Definition 2.4.

A map $f: X \longrightarrow Y$ is called **:**

1) M-pre-open (resp. M-pre-closed) **[3]** if $f(V)$is pre-open (resp. pre-closed) set in Y for every pre-open (resp. pre-closed) set V of X.

2) g^*p -irresolute **[5]** if $f^{-1}(F)$ is g^*p -closed in X for every g^*p -closed set F in Y .

3) pre-irresolute **[4]** if $f^{-1}(F)$ is pre-open in X for every pre-open set F in Y .

Definition 3.5.[8]

A space (X,τ) is said to be g^*-pre regular (briefly g^*p -regular) if for every g^*p -closed set F and a point $x \notin F$, there exist disjoint pre-open sets U and V such that $F \subseteq U$ and $x \in V$.

Definition 3.6[8]

. A topological space (X,τ) is said to be g^*-pre-normal (g^*p -normal) if for any pair of disjoint g^*p -closed sets A and B, there exist disjoint pre-open sets U and V such that $A \subseteq U$ and $B \subseteq V$.

3- g^*-Pre Regular – cleavability

Definition 3.1

A topological space X is said to be absolutely g^*p -irresolute, M-pre-open (resp-pre-closed) cleavable over a class of spaces $\mathcal{P}$, if

$A \subset X$ and there exists an injective g^*p -irresolute, M-preopen(resp-pre closed) continuous mapping $f: X \longrightarrow Y \in \mathcal{P}$, such that $f^{-1}f(A) = A$.

Definition 3.2

A topological space X is said to be absolutely pre-irresolute, M-preopen(resp-pre closed) cleavable over a class of spaces $\mathcal{P}$, if

$A \subset X$ and there exists an injective pre-irresolute, M-preopen(resp-pre closed) continuous mapping $f: X \longrightarrow Y \in \mathcal{P}$, such that $f^{-1}f(A) = A$.

Remark3.1

By g^*p -irresolute, M-preopen(resp-pre closed) cleavable ,we mean that g^*p -irresolute, M-preopen (resp-M-pre closed) - continuous function $f: X \longrightarrow Y \in \mathcal{P}$ is an injective g^*p -irresolute, pre-open(pre-closed) respectively.

Remark3.2

By pre-irresolute, M-preopen(resp-pre closed) absolutely cleavable ,we mean that pre-irresolute, M-preopen (resp-M-pre closed) function $f: X \longrightarrow Y \in \mathcal{P}$ is an injective and pre-irresolute, M-preopen (M-pre-closed) respectively

Theorem 3.1.[8]

Let (X,τ) be a topological space. Then the following statements are equivalent:

(i) $(X,\tau)$is g^*p -regular.

(ii) For each point $x \in X$ and for each g^*p -open neighbourhood W of x, there exists a pre- open set U of x such that $pcl(U) \subseteq W$.

(iii) For each point X and for each g^*p -closed set F not containing x, there exists a pre-open set V of x such that $pcl(V) \cap F = \emptyset$.

Theorem 3. 2.[8]

A topological space $(X,\tau)$is g^*p -regular if and only if for each g^*p -closed set F of (X,τ) and each $x \in F^c$, there exist pre-open sets U and V of (X,τ) such that $x \in U$ and $F \subseteq V$ and $pcl(U) \cap pcl(U) = \emptyset$.

Proposition 3.1.

Let space X be a pre-irresolute, M-pre-closed absolutely cleavable space over a class $\mathcal{P}$ of g^*p -regular spaces Y, then X is g^*p -regular spaces

Proof:

Let x be any point in X and a g^*p -closed subset F of X with $x \notin$ F,since X is pre-irresolute M-pre-closed, absolutely cleavable , so there exists an injective pre-irresolute M-pre-closed mapping $f: X \longrightarrow Y \in \mathcal{P}$ such that $f^{-1}f(F) = F$, and for every $y \in$ Y there exists $x \in X$ such that $y = f(x) \Leftrightarrow f^{-1}(y) = x$, since f is M-pre-closed map so $f(F)$ is a g^*p closed subset of **Y**, such that $\mathbf{y = f(x) \notin f(F)}$.

Since**Y** is g^*p - regular, so there exist two pre-open sets G and H of **Y** with $\mathbf{y = f(x)} \in G$, $f(F) \subset H$, $G \cap H = \emptyset$, then $f^{-1}(y) = x \in f^{-1}(G)$ $\mathbf{f^{-1}}(F) \subset f^{-1}(H)$this implies that $x \in f^{-1}(G)$, $F \subset f^{-1}(H)$, since f is a pre-irresolute , then $\mathbf{f^{-1}}(G)$, $\mathbf{f^{-1}}(H)$ are pre- open sets of X, $f^{-1}(G) \cap f^{-1}(H) = f^{-1}(G \cap H) = f^{-1}(\emptyset) = \emptyset$

,

Therefore X is g^*p -regular space. Hence $X \in \mathcal{P}$

Proposition 3.2

Let X be a g^*p -regular space is a g^*p -irresolute and M-pre-open cleavable over a class $\mathcal{P}$, then Y is g^*p -regular , hence $Y \in \mathcal{P}$

Proof.:

suppose y be any point in Y and E be any g^*p -closed subset of Y with $y \in E$, there exists $\mathbf{x \in X}$ with $y = f(x) \Leftrightarrow f^{-1}(y) = x$,and g^*p -irresolute and M- pre-open Injective continuous mapping $f: X \longrightarrow Y$ such that $f^{-1}f(f^{-1}(E) = f^{-1}(E))$, since f is a g^*p -irresolute , then $f^{-1}(E)$ is g^*p - closed set in X this implies that$f^{-1}(y) \notin f^{-1}(E)$, then$x \notin f^{-1}(E)$ in X,

since X is a g^*p -regular space , so there exist pre-open sets U ,Vsuch that $x \in U$ and $f^{-1}(E) \subset V$,this implies that $f(x) = y \in f(U)$, and $ff^{-1}(E) \subset f(V)$, this implies that $E \subset f(V)$, since f is M-pre-open and bijective , so $f(U)$, $f(V)$ are pre-open sets of $Y$and $f(U) \cap f(V) = f(U \cap V) = f(\emptyset) = \emptyset$

Then Y is g^*p -regular space .Hence $Y \in \mathcal{P}$.

4- g^* Pre Normal – Cleavability

Definition 4.1

A topological space X is said to be double pre-irresolute, M-pre -open (pre-closed) cleavable over a class of spaces $\mathcal{P}$, if for any subsets $A \subset X$ and $B \subset X$, there exists a pre-irresolute, M-preopen (pre-closed) mapping $f: X \longrightarrow Y$ such that $f^{-1}f(A) = A$ and $f^{-1}f(B) = B$

Definition 3.2
A topological space X is said to be double g^*p -irresolute, M-pre -open (pre-closed) cleavable over a class of spaces $\mathcal{P}$, if for any subsets $A \subset X$ and $B \subset X$, there exists a g^*p -irresolute, M-preopen (pre-closed) mapping $f: X \longrightarrow Y$ such that $f^{-1}f(A) = A$ and $f^{-1}f(B) = B$.

Theorem 4.3

Let (X,τ)be a topological space. Then the following statements
are equivalent.
(i) (X,τ) is g^*p -normal.
(ii) For each g^*p -closed F and for each g^*p -open set U containing F, there exists a pre-open set V containing F such that $pcl(V) \subseteq U$.
(iii) For each pair of disjoint g^*p -closed sets A and B in (X,τ),
there exists a pre-open set U containing A such that $pcl(U) \cap B = \emptyset$.
(iv) For each pair of disjoint g^*p -closed sets A and B in (X,τ), there exist a pre-open sets U and V such that$A \subseteq U, B \subseteq V$ and$pcl(A) \cap pcl(B) = \emptyset$.

Proposition 4.1
Let X be a pre-irresolute, M-pre-closed, absolutely double cleavable space over a class $\mathcal{P}$ of g^*p normal spaces , then X is normal .Hence $X \in \mathcal{P}$.

Proof:
Suppose F_1, F_2be two disjoint closed g^*p -closed sets of X, then there exists an injection pre-irresolute, M-pre-closed, mapping $f: X \longrightarrow Y \in \mathcal{P}$ such that $f^{-1}f(F_1) = F_1, f^{-1}f(F_2) = F_2$ .Since f is M-pre-closed , then , $f(F_1), f(F_2)$, are two disjoint g^*p closed sets of Y , since Y is g^*p normal space , so there exist two pre- open sets U,V such that $F_1 \subset f^{-1}(U), F_2 \subset f^{-1}(V)$
,Since f is pre- irresolute then $f^{-1}(U), f^{-1}(V)$ are pre- open sets of X and
$f^{-1}(U) \cap f^{-1}(V) = f^{-1}(U \cap V) = f^{-1}(\emptyset) = \emptyset$
Then X is a g^*p normal space .Hence $X \in \mathcal{P}$

Proposition 4.2
Let X be g^*p - normal space is a g^*p -irresolute, , M-preopen absolutely double cleavable over a class of spaces Y, then Y is g^*p - normal space.

Proof:
Let E_1, E_2, be disjoint g^*p - closed subset of Y, then there exists an injective g^*p -irresolute
, M-preopen mapping $f: X \longrightarrow Y$ such that
$f^{-1}f\{f^{-1}(E_1)\} = f^{-1}(E_1), f^{-1}f\{f^{-1}(E_2)\} = f^{-1}(E_2)$
Since f is g^*p -irresolute ,so $f^{-1}(E_1), f^{-1}(E_2)$ are disjoint g^*p - closed sets of X , since X is g^*p -normal , so there exist pre- open sets G, H **such that** $f^{-1}(E_1) \subset G, f^{-1}(E_2) \subset H, G \cap H = \emptyset$ **and** $ff^{-1}(E_1) \subset f(G), ff^{-1}(E_2) \subset f(H)$ this implies that $E_1 \subset f(G)$, $E_2 \subset f(H)$, since f is pre-open ,then $f(G), f(H)$, are pre- open sets of Y and
$f(G) \cap f(H) = f(G \cap H) =$
$f(\emptyset) = \emptyset$
Therefore Y is g^*p - normal space . Hence $Y \in \mathcal{P}$.

5-conclusion:
In this paper we have studied and proved these cases:
1) If $\mathcal{P}$ is a class of g^*p -regular spaces with certain properties and if X is pre-irresolute, M-pre-closed absolutely cleavable over $\mathcal{P}$, then $X \in \mathcal{P}$,also if $\mathcal{P}$ is a class of g^*p -regular spaces with certain properties and if X is pre-irresolute, M-pre-closed spaces with certain properties and if X is a g^*p –irresolute, M-pre-open absolutely cleavable over $\mathcal{P}$, then $Y \in \mathcal{P}$.
2) If $\mathcal{P}$ is a class of g^*p - normal spaces with certain properties and if X is a pre-irresolute, M-pre-closed, absolutely double cleavable over $\mathcal{P}$, then $X \in \mathcal{P}$, also If $\mathcal{P}$ is a class of g^*p - normal spaces with certain properties and if X is a g^*p – irresolute, M-pre-open absolutely double cleavable over $\mathcal{P}$, then $X \in \mathcal{P}$.

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