

## Comparison of 4 transmit antennas OFDM-STBC Systems based on channel feedback

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**Abstract** Space time block coding (STBC) with transmit beamsteering can achieve significant diversity and array gain in multi-antenna wireless systems which is a MIMO transmit strategy supporting the most recently wireless applications. However, it exploits perfect channel information (CSI) state at the transmitter, which is practically difficult to be achieved due to the limitation of a feedback channel, therefore quantized channel information can be used with a minimum feedback overhead. Linear precoding with a certain matrix size can be used to 4 transmit antennas system which can be combined with STBC. Alamouti scheme for 2 transmit antennas can be also extended to 4 transmit antennas. In addition to its diversity gain, it can achieve array gain based on generation of two beamsteering phase angles. A comparison of precoded STBC scheme versus extended STBC is made and simulation results show that the performance of extended Alamouti is to have a significant advantage over precoded STBC scheme and can reduce feedback signalling overhead.

**Keywords:** CSI, MIMO Transmit diversity, OFDM, STBC.

### مقارنة أنظمة اتصال 4 هوائيات إرسال بتقنية OFDM-STBC مع قناة تغذية خلفية راجعة

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**المخلص** يمكن أن يحقق تشفير فضاء الزمكان (STBC) مع حزمة إرسال الإرسال تنوعاً كبيراً وكسب صفييف في الأنظمة اللاسلكية متعددة الهوائيات، وهي استراتيجية إرسال MIMO تدعم التطبيقات اللاسلكية الحديثة. ومع ذلك، فإنه يستغل حالة معلومات القناة المثالية (CSI) في جهاز الإرسال، وهو أمر يصعب تحقيقه عملياً بسبب قصر قناة التغذية المرتدة، وبالتالي يمكن استخدام معلومات القناة الكمية مع الحد الأدنى. يمكن استخدام التشفير المسبق الخطي مع حجم مصفوفة معين ل 4 هوائيات الإرسال التي يمكن دمجها مع STBC. يمكن أيضاً توسيع مخطط Alamouti الخاص بهوائيين للإرسال إلى 4 هوائيات للإرسال. بالإضافة إلى كسب التنوع، يمكن أن يحقق كسب الصفييف استناداً إلى توليد زاويتين لطور الحزمة. تم إجراء مقارنة بين مخطط STBC المسبق مقابل STBC الممتد وأظهرت نتائج المحاكاة أن أداء Alamouti الممتد هو أن يكون له ميزة كبيرة على مخطط STBC المسبق ويمكن أن يقلل من الإشارة العامة المرتدة).

**الكلمات المفتاحية:** معلومات القناة، الإرسال المتعدد، تقنية تشفير الزمكان، تقنية الإرسال المتعدد بتقسيم التردد.

### Introduction

Wireless LANs and personal area networks are just a few examples of widely used wireless networks. However, wireless devices are range and data rate limited. For frequency-selective channel wireless applications, it is generally advantage to combine with multicarrier methods such as orthogonal frequency division multiplexing (OFDM) in order to operate narrowband STBC schemes in decoupled subcarriers which are free of inter-symbol (ISI) and inter-carrier interference (ICI). The basic multicarrier transceiver features that OFDM possesses when it is applied to booth a single-antenna and multi-antenna wireless system [1] [2][3].

Linear precoding with different matrix sizes can be combined with STBC to efficiently improve the diversity order of the multi-antenna wireless

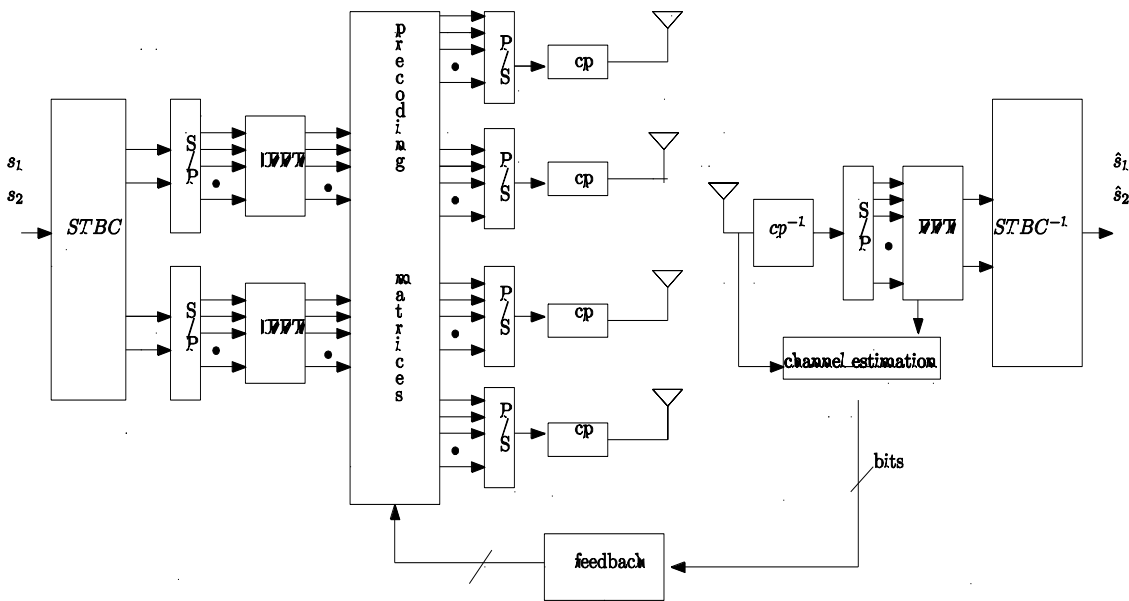
system [4]. Many studies are carried out to find out low rate feedback method [5][6]. Since then, One particular design method is to use unitary linear precoding matrices such as the Fourier, the Vandermonde and the complex Hadamard matrices. The first code word matrix only to be determined, then, the remaining code words is obtained by multiplying the first code word with unitary diagonal matrix of which the entry variables are determined such that the minimum chordal distance is maximized [7].

Alamouti scheme for two transmit antennas has low decoding complexity, and with complex constellation can achieve full diversity gain at a code rate equal to one. STBC codes for more than two transmit antennas referred to an extended orthogonal STBC, which Alamouti extends to more

than two antennas while preserving full diversity benefits, full data rate, and low decoding complexity[8][9][10]. A simple beamsteering scheme based on phase angles feedback are widely used in different STBC such as in order to obtain full diversity gain. Phase feedback way has been suggested as a way to combine STBCs over groups of antennas to ensure the full diversity advantage from arbitrary number of transmit antenna at full rate. Such beamsteering can provide a significant improvement in BER performance of space-time coded systems [11]. Therefore, we aim to evaluate and compare OFDM-STBC system based on precoded and extended Alamuti scheme to achieve better system performance, and less quantised information with

fewer computations. In ASTBC system, a finite reverse link can be used to return appropriate phases for each antenna. Hence simple adaptive 4 transmit diversity schemes need 6 bits of feedback information. However, for instance, extended Alamouti for 4 transmit antennas and one receive antenna needs only 2 bits per quantised phase angle, which can be made overhead in OFDM systems, but frequency correlation among subcarriers can be exploit to reduce this overhead. In this paper, we compare both systems performance in order to achieve maximum diversity gain. This paper is organized as follows. Sec. 1 is Introduction. In Sec. 2 we give system model, Sec. 3 compares performances, and Sec. 4 conclusion.

**System model**



**Fig. 1:** System model

System model which can represent precoded or extended Alamouti based on precoding matrices, is shown in Fig 1. The coded OFDM symbols are modulated to N subcarriers. Which denote the number of active subcarriers. Therefore data is divided into N substreams with each substream processing in blocks at both the transmitter and the receiver. Each block comprises of orthogonal signals, and then each OFDM symbol is preceded by a cyclic prefix (CP) of length cp. Finally the OFDM symbols are transmitted from the transmitter to the receiver. At the recitative, the CP is removed first for each OFDM symbol. Then the received signals are demodulated by N-point FFT.

Consider two consecutive symbols on subcarrier k, denoted by  $X_{k,1}$  and  $X_{k,2}$  (in the frequency domain). According to the Alamouti code structure,  $X_{k,1}$  and  $X_{k,2}$  are transmitted during the first symbol period, on subcarrier k, and  $-X_{k,2}^*$  and  $X_{k,1}^*$  are transmitted during the second symbol period, on subcarrier k, . We assume that the channel coefficients are constant during two OFDM symbol intervals. Thus, at the receiver, after the CP removal and FFT processing the

received signal can be expressed in matrix notation as  $\mathbf{R}_{k,n} = [R_{k,1} \ R_{k,2}]^T$  , under the assumption of perfect timing and frequency synchronizations.

OFDM-STBC can be extended to 4 transmit antennas through precoding matrices as shown in Fig 1, where rate one and maximum diversity order are achieved due to the use of additional beamsteering based on feedback of channel state information (CSI) [9]. Precoded Alamouti which utilize the linear precoding unitary matrices and extended Alamouti also can use beamsteering matrices which are simply set as two phase angles per subcarrier.

**1. Precoded Alamouti**

Alamouti uses linear precoding matrices called precoded Alamouti. Precoder can be combined with Alamouti code word to improve the space-time diversity order of the multiple antenna system, as the code word matrix is multiplied by an matrix and sent over transmit antennas. In the precoded Alamouti systems, the space-time code word[6][7][8].

$$\mathbf{S}_{k,n} = \begin{bmatrix} X_{k,1} & -X_{k,2}^* \\ X_{k,2} & X_{k,1}^* \end{bmatrix} \quad (1)$$

over  $n = 2$  time slots, is multiplied by a precoding matrix  $\mathbf{W}_k$  for  $N_T = 4$ ,  $\mathbf{W}_k \in \mathbb{C}^{N_T \times N_T}$ . Such precoding matrix is chosen from the codebook  $\mathbf{C}_k = \{\mathbf{W}_0, \mathbf{W}_1, \dots, \mathbf{W}_{L-1}\}$  with feedback bits  $B = \log_2(L)$  bits. The received signal over two time slots for a given subcarrier channel  $\mathbf{H}_k = [H_1 \ H_2 \ H_3 \ H_4]$  can be expressed as

$$\mathbf{R}_{k,n} = \mathbf{H}_k \mathbf{W}_k \mathbf{S}_{k,n} + \mathbf{V}_{k,n} \quad (2)$$

The objective is to select an appropriate code word that improves the overall system performance. Using the orthogonal property of Alamouti code, selection criterion can be implemented by computing a matrix multiplication and Frobenius norm for each of the  $L$  codebook matrices. This leads us to the following choose the precoder according to

$$\mathbf{W}_k^{opt} = \arg \min_{\mathbf{W} \in \mathbf{C}} \|\mathbf{H}_k \mathbf{W}_k\|_F^2 \quad (3)$$

The adaptive transmit is the antennas signal include a modification according to whereby the matrices

$$\Phi_k^{opt} = \text{diag}\{\mathbf{W}_{k,1}^{opt} \dots \mathbf{W}_{k,N}^{opt}\} \quad (4)$$

### 1.1 Precoding Matrices

In practical design cases, a suboptimal method for an arbitrary  $N_T$  transmit antennas, code word length  $n = 2$ , and code-book size  $L$  is to practical codebook designs use unitary linear precoding matrices, constructed with Discrete Fourier Transform matrices (DFT matrices) [1], given by considering codebooks of the form

$$\mathbf{C} = \{\mathbf{W}_{\text{DFT}}, \Theta \mathbf{W}_{\text{DFT}}, \dots, \Theta^{L-1} \mathbf{W}_{\text{DFT}}\} \quad (5)$$

The first code word WDFT is obtained by selecting  $M$  columns from the DFT matrix  $F$ . Further,  $\Theta$  is unitary diagonal matrix. The diagonal matrix  $\Theta$  given by

$$\Theta = \text{diag}\{e^{j2u_1/L}, e^{j2u_2/L}, \dots, e^{j2u_{N_T}/L}\}, \quad (6)$$

where variables  $\{u_i\}_{i=0}^{N_T-1}$ ; referred to the rotation vector. Given the first code word  $\mathbf{W}_{\text{DFT}}$  the remaining  $(L - 1)$  code words are obtained by multiplying WDFT by  $\Theta^i$ ,  $i = 0, 2, \dots, L-1$ . The values of rotation vector are to be determined such that the minimum chordal distance is maximized, that is

$$\mathbf{U} = \arg \min_{\{u_i\}_{i=0}^{N_T-1}} \min_{l=1,2,\dots,L-1} d(\{\mathbf{W}_{\text{DFT}}, \Theta^l \mathbf{W}_{\text{DFT}}\}) \quad (7)$$

Thus, there are different  $L^{N_T}$  possibilities for  $\mathbf{U}$  that must be checked. Clearly, this method requires the feedback using  $B = N_T \log_2(L)$  bits.

For every subcarrier transmission Alamouti can be extended into 4 transmit antennas, the space-time code word becomes [2][3][4].

$$\mathbf{S}_{k,n} = \begin{bmatrix} X_{k,1} & X_{k,1} & -X_{k,2}^* & -X_{k,2}^* \\ X_{k,2} & X_{k,2} & X_{k,1}^* & X_{k,1}^* \end{bmatrix} \quad (8)$$

the channels between transmitter and receiver node in frequency domain is

$\mathbf{H}_k = [H_1 \ H_2 \ H_3 \ H_4]$ . Hence, the received signal  $\mathbf{R}_{k,n}$  is characterized as

$$\mathbf{R}_{k,n} = \begin{bmatrix} R_{k,1} \\ R_{k,2} \end{bmatrix} = \mathbf{H}_{k,n} \begin{bmatrix} X_{k,1} \\ X_{k,2} \end{bmatrix} + \begin{bmatrix} V_{k,1} \\ V_{k,2} \end{bmatrix} \quad (9)$$

where  $\mathbf{V}_{k,n} = [V_{k,1}, V_{k,2}]^T$  is equivalent noise vector, and the space-time equivalent transmission channel matrix  $\mathbf{H}_{k,n}$ , which can be formulated as

$$\mathbf{H}_{k,n} = \begin{bmatrix} H_{k,1} + H_{k,2} & H_{k,3} + H_{k,4} \\ H_{k,3}^* + H_{k,4}^* & -H_{k,1}^* - H_{k,2}^* \end{bmatrix} \quad (10)$$

where  $\mathbf{H}_{k,n}$  contains channel coefficients corresponding to  $H_{k,n}$ . Decoding are performed through a simple matched filtering such as  $[\mathbf{H}_{k,n}^H \ \mathbf{H}_{k,n}]$ . In space-time block coded systems, the channels are normally hold to remain block stationary. Writing out the diagonal elements in full we come to

$$[\mathbf{H}_{k,n}^H \ \mathbf{H}_{k,n}] = \frac{1}{4} \mathcal{E}(\alpha_k + \beta_k) \quad (11)$$

$$\alpha_k = \sum_{m=1}^4 |H_{k,m}|^2 \quad (12)$$

$$\beta_k = \sum_{m=1}^4 2\Re\{H_{k,1}H_{k,2}^* + H_{k,3}H_{k,4}^*\} \quad (13)$$

where  $|\cdot|^2$  denotes the modulus squared of a complex number and  $\Re\{\cdot\}$  denotes real part of complex number. It is clear that interference between channels introduced by  $\beta$  can be forced to be positive with optimum angles feedback based on phase rotation at the transmitter in which a significant array gain can be achieved [10].

### 3. Beamsteering phase angles

We provide a simple two phase rotation angles to demonstrate how beamsteering techniques can be applied to 4 transmitter antenna system. Note that the actual effect under two phase shift angles  $\varphi_{1,k}$  and  $\varphi_{2,k}$  can be attained the maximum gain in equation (13), where the phase rotation on the channel coefficients is equivalent to rotating the phases of the corresponding transmitted symbols, emitted from antennas 1 and 3. Hence the space-time equivalent transmission channel matrix  $\mathbf{H}_{k,n}$  with beamsteering becomes

$$\mathbf{H}_{k,n} = \begin{bmatrix} e^{-j\varphi_{1,k}} H_{k,1} + H_{k,2} & e^{-j\varphi_{2,k}} H_{k,3} + H_{k,4} \\ e^{j\varphi_{2,k}} H_{k,3}^* + H_{k,4}^* & -e^{j\varphi_{1,k}} H_{k,1}^* - H_{k,2}^* \end{bmatrix} \quad (14)$$

It is possible to force the  $\beta$  term to be positive and maximum magnitude, by ensuring that the angles  $\varphi_{1,k}, \varphi_{2,k}$  are applied some beam-steering without increasing the transmit power, the signals are rotated from the first and third antennas, the only cost of this technique is the requirement of feeding back the two phase angles to the

transmitter, the maximum value of the factor  $\beta$  can be written as,

$$\beta_k = \sum_{m=1}^4 2\Re \{ H_{k,1} H_{k,2}^* e^{j\varphi_{1,k}} + H_{k,3} H_{k,4}^* e^{j\varphi_{2,k}} \} \quad (15)$$

where the phase rotations for antenna one and three essentially result in a new set of channel coefficients. According to [15], the optimum steering parameters are selected and appropriately fed back to the transmitter, e.g. via a quantised feedback link

$$\varphi_{1,k} = \angle -H_{k,1} H_{k,2}^* \quad (16)$$

$$\varphi_{1,k} = \angle -H_{k,3} H_{k,4}^* \quad (17)$$

The beamsteering is the first and third antenna signal include a modification for subcarriers  $k = 1 \dots N$  according to the matrices.

$$\Phi_{1,k} = \text{diag} \{ e^{j\varphi_{1,1}}, \dots, e^{j\varphi_{1,k}} \} \quad (18)$$

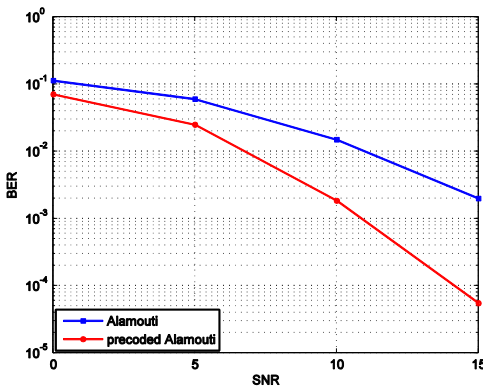
$$\Phi_{2,k} = \text{diag} \{ e^{j\varphi_{2,1}}, \dots, e^{j\varphi_{2,k}} \}, \quad (19)$$

where  $\Phi_{1,k}$  and  $\Phi_{2,k}$  are unitary diagonal matrices applied to antenna 1 and antenna 3 respectively.

**Results**

Using an OFDM based FFT with  $N = 64$  subcarriers, simulations are performed over 104 randomly drawn channels. The channel impulse responses have a length of  $L = 4$ , with Rayleigh fading coefficients, which can be modelled as a tapped delay line model with fixed tap spacing [11].

**1-BER Performance of Precoded Alamouti**



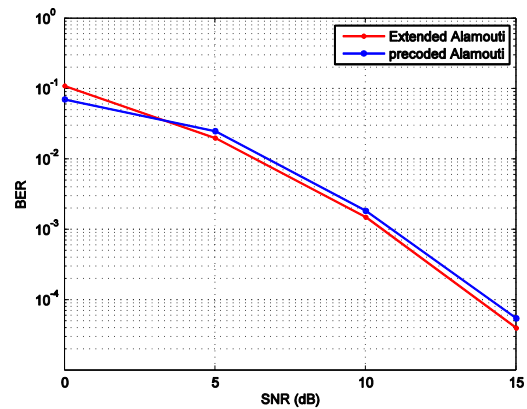
**Fig. 2:** BER performance of Alamouti scheme with and without precoding

Fig.2 shows BER performance of precoded Alamouti using the design method in equation (5) with  $NT = 4$ ,  $M = 2$ , and  $L = 64$  as given in [7]. Fig.2 compares the performance of Alamouti [8] to precoded Alamouti. It can be shown that the precoded Alamouti scheme outperforms the traditional Alamouti scheme without increasing transmitted power or increasing spectral bandwidth. However, it may require completely or partially CSI at the transmitter side, which is not easily achievable in practice mainly due to the limited bandwidth of the feedback link. It is desired to maximize diversity gain while reducing the number of feedback bits. Therefore extended

Alamouti has nearly the same performance as will be shown next, but with much less feedback information and less computations.

**2-BER Performance of extended Alamouti**

We also show the BER performances of the precoded Alamouti against an extended Alamouti scheme for 4 transmit antennas systems. The BER curve at low SNR shows a little bit difference in performance for precoded Alamouti, however, high SNR result shows significantly small performance improvement of extended Alamouti scheme over precoded Alamouti. It is important to notice that extended Alamouti beamsteering (precoding matrices) is justified with a simple method compared with precoded Alamouti.



**Fig. 3:** Comparison of average BER performance.

**Conclusion**

Precoding and Beamsteering based on a finite feedback scheme exploiting CSI at the transmitter systems such as Precoded Alamouti and extended Alamouti can be used for 4 transmit antennas. Both systems are compared in term of their performances. Precoded Alamouti with the expense of increased feedback overhead can provide a significant reliable efficiency, which is practically used in wireless applications. Then we compare its performance to extended Alamouti scheme with less computations feedback method, which offers advantages such as linear decoding complexity and also provide array gain provided a scalar parameter can be maximized via an optimum feedback. Simulation results demonstrate that extended Alamouti with a simple beamsteering not only can outperform precoded Alamouti, but also has less feedback overhead. Sometimes we cannot be decided which systems can make advantage until we study.

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