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Certain Subclasses of k - Uniformly Starlike and Convex Functions Defined by Convolution with Varying Arguments of Coefficients

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Abstract In this paper we introduce and study the subclass $VU_{\theta,\gamma}(f,g,\alpha,\beta,k)$, which represent the k -uniformly analytic functions of order α with varying argument of coefficients. Moreover, we give coefficient estimates, growth distortion bounds and radii of starlikeness and convexity.

Keywords: Analytic functions, k -uniformly convex, k -uniformly starlike , varying arguments.

فصول جزئية معينة للدوال التحليلية النجمية والمحدبة وحيدة التكافؤ وذات المعاملات متغيرة السعة والمعرفة بواسطة الالتفاف

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الملخص في هذا البحث عرفنا الفصل $VU_{\theta,\gamma}(f,g,\alpha,\beta,k)$ للدوال التحليلية وحيدة التكافؤ من رتبة α وذات المعاملات متغيرة السعة والمعرفة على قرص الوحدة $\mathbb{U}=\{z\in\mathbb{C}:0<|z|<1\}$، تحصلنا عل نظرية المعاملات لهذا الفصل ودرسنا ة بعض الخواص لدوال هذا الفصل وهي – التشوه – وأنصاف أقطار التحدب.)، وكل النتائج التي حصلنا عليها قاطعة.

الكلمات المفتاحية: الدوال التحليلية النجمية أحادية التكافؤ– الدوال التحليلية المحدبة أحادية التكافؤ– سعات متغيرة.

Introduction

The class of analytic and univalent functions in the open unit disk $\mathbb{U}=\{z\in\mathbb{C}:0<|z|<1\}=$ and has the form:

$$f(z)=z+\sum_{n=1}^{\infty}a_nz^n \qquad (1.1)$$

is denoted by S For $f(z)\in S$ and $g(z)\in S$ of the form:

$$g(z)=z+\sum_{n=2}^{\infty}b_nz^n, \qquad (1.2)$$

the Hadamard product (or convolution) $(f*g)(z)$ of $f(z)$and $g(z)$ is defined (as usual) by

$$(f*g)(z)=z+\sum_{n=2}^{\infty}a_nb_nz^n, \qquad (1.3)$$

Using the convolution, we define a unified subclass of univalent function class S as follows:

Definition 1. *For $(0\le\alpha<\beta\le1, 0\le\gamma<1, k(1-\beta)<(1-\alpha)$ and $g(z)$ of the form (1.2) , let $U_\gamma(f,g,\alpha,\beta,k)$ denote the subclass of S consisting of functions $f(z)$ of the form (1.1) and satisfy the following inequality:*

$$Re\left\{\frac{z(f*g)'(z)+(1+2\gamma)z^2(f*g)''(z)+\gamma z^3(f*g)'''(z)}{z(f*g)'(z)+\gamma z^2(f*g)''(z)}-\alpha\right\}\ge k\left|\frac{z(f*g)'(z)+(1+2\gamma)z^2(f*g)''(z)+\gamma z^3(f*g)'''(z)}{z(f*g)'(z)+\gamma z^2(f*g)''(z)}-\beta\right|. \qquad (1.4)$$

For different choices of parameters α,β,γ,k and the function $g(z)$ in (1.4) we obtain many subclasses of the class $U_\gamma(f,g,\alpha,\beta,k)$ studied earlier see (for example)([1], [2], [3], [4], [5], [6], [7], [8], [9]), as well as new classes defined by differentiable and integrable operators. Also we obtain the following new subclass as follows:

$U_0(f,g,\alpha,\beta,k)=k-UCV(f,g,\alpha,\beta)$

$$\left\{\begin{matrix} f\epsilon S: Re\left\{1+\frac{z(f*g)''(z)}{(f*g)'(z)}-\alpha\right\}>k\left|1+\frac{z(f*g)''(z)}{(f*g)'(z)}-\beta\right| \\ (0\le\alpha<\beta\le1, k(1-\beta)<(1-\alpha), z\in\mathbb{U})\end{matrix}\right\}. \qquad (1.5)$$

Let V_θ be the class of functions $f(z)\in S$ of the form (1.1), for which $arg(a_n)\equiv\pi+(n+1)\theta, n\ge2.$ We note that for $\theta=0,$ we obtain the familiar class T of functions with negative coefficients [10]. Moreover, we define $V=\cup_{\theta\epsilon R}V_\theta$. The class V was introduced by Sliverman [11].

Further, we define the class $VU_{\theta,\gamma}(f,g,\alpha,\beta,k)$ by

$$VU_{\theta,\gamma}(f,g,\alpha,\beta,k)=U_\gamma(f,g,\alpha,\beta,k)\cap V_\theta. \qquad (1.6)$$

For different choices of parameters α,β,γ,k and the function $g(z)$ in (1.6) we obtain many subclasses of the class $VU_{\theta,\gamma}(f,g,\alpha,\beta,k)$ studied earlier see (for example) ([2] and [7]), as well as new subclasses with varying arguments of coefficients defined by differentiable and integrable operators. Also we obtain a new subclass with varying arguments of coefficients to the subclass defined in (1.5) as follows:

$$VU_{\theta,0}(f,g,\alpha,\beta,k) = VU_{\theta}(f,g,\alpha,\beta,k) = U(f,g,\alpha,\beta,k)\cap V_{\theta} = k - VUCV(f,g,\alpha,\beta).$$

In this paper we study some properties of the function of the form (1.1) and belongs to the class $VU_{\theta,\gamma}(f,g,\alpha,\beta,k)$.

Coefficiente stimates.

Unless otherwise mentioned, we assume throughout our present paper that: $g(z)$is defined by (1.2) with $b_n > 0 (n \geq 2)$, $0 \leq \alpha < \beta \leq 1, 0 \leq \gamma < 1, k(1-\beta) < (1-\alpha)$, $z \in \mathbb{U}$ and

$$\Psi_n = n[1+\gamma(n-1)] \qquad (2.1)$$

Theorem 1. A function $f(z) \in U_{\gamma}(f,g,\alpha,\beta,k)$, if

$$\sum_{n=2}^{\infty}[k(n-\beta)+(n-\alpha)]\Psi_n b_n|a_n| \leq (1-\alpha)-k(1-\beta) \qquad (2.2)$$

Proof. It is sufficient to show that inequality (1.4) holds true. Using the fact that

$$Re\{w-\alpha\} > k|w-\beta| \Leftrightarrow Re\{(1+ke^{i\theta})w - \beta ke^{i\theta}\} > \alpha, \qquad (2.3)$$

then inequality (1.4) may be written as

$$Re\left\{(1+ke^{i\theta})\frac{z(f*g)'(z)+(1+2\gamma)z^2(f*g)''(z)+\gamma z^3(f*g)'''(z)}{z(f*g)'(z)+\gamma z^2(f*g)''(z)} - \beta ke^{i\theta}\right\} \geq \alpha, \qquad (2.4)$$

or

$$Re\left\{\frac{A(z)}{B(z)}\right\} > \alpha, \qquad (2.5)$$

Where

$$A(z) = (1+ke^{i\theta})[z(f*g)'(z)+(1+2\gamma)z^2(f*g)''(z)+\gamma z^3(f*g)'''(z)] - \beta ke^{i\theta}[z(f*g)'(z)+\gamma z^2(f*g)''(z)],$$

and

$$B(z) = z(f*g)'(z)+\gamma z^2(f*g)''(z).$$

We have

$$|A(z)+(1-\alpha)B(z)| - |A(z)+(1+\alpha)B(z)| \geq 0. \qquad (2.6)$$

Note that

$$|A(z)+(1-\alpha)B(z)| = \left|[(1-\beta)ke^{i\theta}+2-\alpha]z - \sum_{n=2}^{\infty}[(n-\beta)ke^{i\theta}-1+\alpha]\Psi_n b_n a_n z^n\right| \geq [(1-\beta)k+2-\alpha]|z| - \sum_{n=2}^{\infty}[(n-\beta)k-1+\alpha]\Psi_n b_n|a_n||z^n| \qquad (2.7)$$

and

$$|A(z)-(1+\alpha)B(z)| = \left|[(1-\beta)ke^{i\theta}+2-\alpha]z + \sum_{n=2}^{\infty}[(n-\beta)ke^{i\theta}-1-\alpha+n]\Psi_n b_n a_n z^n\right| \leq [(1-\beta)k+\alpha]|z| - \sum_{n=2}^{\infty}[(n-\beta)k-1\alpha+n]\Psi_n b_n|a_n||z^n|. \qquad (2.8)$$

Using (2.7) and (2.8), we obtain the following inequality:

$$|A(z)+(1-\alpha)B(z)| + |A(z)-(1+\alpha)B(z)| \geq 2[(1-\alpha)-k(1-\beta)] - 2\sum_{n=2}^{\infty}[(n-\beta)(n-\alpha)]\Psi_n b_n|a_n||z^n| \qquad (2.9)$$

The expression $|A(z)+(1-\alpha)B(z)| + |A(z)-(1+\alpha)B(z)|$ is bounded below by if

$$\sum_{n=2}^{\infty}[(n-\beta)k+(n-\alpha)]\Psi_n b_n|a_n| \leq (1-\alpha)-k(1-\beta). \qquad (2.10)$$

Hence the proof is completed■.

In the following theorem, we find the necessary condition for functions of the form (1.1) to be in the subclass $VU_{\theta,\gamma}(f,g,\alpha,\beta,k)$

Theorem 2. A function $f(z) \in VU_{\theta,\gamma}(f,g,\alpha,\beta,k)$ if and only if

$$\sum_{n=2}^{\infty}[(n-\beta)k+(n-\alpha)]\Psi_n b_n|a_n| \leq (1-\alpha)-k(1-\beta), \qquad (2.11)$$

where $\Psi_n$are given by (2.1).

Proof. In view of Theorem **1**, we need only to show that $f(z) \in VU_{\theta,\gamma}(f,g,\alpha,\beta,k)$ satisfies the coefficient inequality (2.11). If $f(z) \in VU_{\theta,\gamma}(f,g,\alpha,\beta,k)$, then from (1.4), we have

$$Re\left\{\frac{(1-\alpha)+\sum_{n=2}^{\infty}(n-\alpha)\Psi_n b_n a_n z^{n-1}}{1+\sum_{n=2}^{\infty}\Psi_n b_n a_n z^{n-1}}\right\} \geq k\left|\frac{(1-\beta)+\sum_{n=2}^{\infty}(n-\alpha)\Psi_n b_n a_n z^{n-1}}{1+\sum_{n=2}^{\infty}\Psi_n b_n a_n z^{n-1}}\right|. \qquad (2.12)$$

Since $f(z)$ is of the form (1.1) with the argument property given in the class $V(\theta_n)$ and setting $z = re^{i[\theta_n+(n-1)\delta]}$ in the above inequality, we have

$$Re\left\{\frac{(1-\alpha)+\sum_{n=2}^{\infty}(n-\alpha)\Psi_n b_n|a_n|e^{i[\theta_n+(n-1)\delta]}z^{n-1}}{1+\sum_{n=2}^{\infty}\Psi_n b_n|a_n|e^{i[\theta_n+(n-1)\delta]}z^{n-1}}\right\} \geq k\left|\frac{(1-\beta)+\sum_{n=2}^{\infty}(n-\alpha)\Psi_n b_n|a_n|e^{i[\theta_n+(n-1)\delta]}z^{n-1}}{1+\sum_{n=2}^{\infty}\Psi_n b_n|a_n|e^{i[\theta_n+(n-1)\delta]}z^{n-1}}\right|.$$

Since Re{w(z)}<|w(z)|<1, we get

$$\frac{(1-\alpha)-\sum_{n=2}^{\infty}(n-\alpha)\Psi_n b_n|a_n|r^{n-1}}{1-\sum_{n=2}^{\infty}\Psi_n b_n|a_n|r^{n-1}} > k\left[\frac{(1-\beta)\sum_{n=2}^{\infty}(n-\alpha)\Psi_n b_n|a_n|r^{n-1}}{1-\sum_{n=2}^{\infty}\Psi_n b_n|a_n|r^{n-1}}\right]. \qquad (2.13)$$

Letting $r \to 1^-$, then we have the inequality (2.11).■

Corollary 1. If $f(z) \in VU_{\theta,\gamma}(f,g,\alpha,\beta,k)$ then

$$|a_n| \leq \frac{(1-\alpha)-k(1-\beta)}{[(n-\beta)k+(n-\alpha)]\Psi_n b_n} (n \geq 2), \qquad (2.14)$$

and equality holds for

$$f(z) = z + \frac{(1-\alpha)-k(1-\beta)}{[(n-\beta)k+(n-\alpha)]\Psi_n b_n} e^{i\theta_n} z^n \; (n \geq 2; z \in \mathbb{U}). \quad (2.15)$$

3. Growth and distortion theorems.

Theorem 3. Let $f(z) \in VU_{\theta,\gamma}(f, g, \alpha, \beta, k)$ Then for $|z| < r = 1$

$$r - \frac{(1-\alpha)-k(1-\beta)}{[(2-\beta)k+(2-\alpha)]\Psi_2 b_2} r^2 \leq |f(z)| \leq r + \frac{(1-\alpha)-k(1-\beta)}{[(2-\beta)k+(2-\alpha)]\Psi_2 b_2} r^2. \quad (3.1)$$

Sharppeness holds for

$$f(z) = z + \frac{(1-\alpha)-k(1-\beta)}{[(2-\beta)k+(2-\alpha)]\Psi_2 b_2} e^{i\theta} z^2. \quad (3.2)$$

Proof. From (2.11), we find that

$$[(2-\beta)k + (2-\alpha)]\Psi_2 b_2 \sum_{n=2}^{\infty} |a_n| \leq \sum_{n=2}^{\infty} [(n-\beta)k + (n-\alpha)]\Psi_n b_n |a_n| \leq (1-\alpha) - k(1-\beta),$$

which yields

$$\sum_{n=2}^{\infty} |a_n| \leq \frac{(1-\alpha)-k(1-\beta)}{[(2-\beta)k + (2-\alpha)]\Psi_2 b_2}.$$

Thus

$$|f(z)| \leq |z| + |z|^2 \sum_{n=2}^{\infty} |a_n| \leq r + r^2 \sum_{n=2}^{\infty} |a_n|$$

$$\leq r + \frac{(1-\alpha)-k(1-\beta)}{[(2-\beta)k + (2-\alpha)]\Psi_2 b_2} r^2.$$

Similarly,

$$|f(z)| \geq |z| - |z|^2 \sum_{n=2}^{\infty} |a_n| \leq r - r^2 \sum_{n=2}^{\infty} |a_n|$$

$$\leq r - \frac{(1-\alpha)-k(1-\beta)}{[(2-\beta)k + (2-\alpha)]\Psi_2 b_2} r^2.$$

This completes the proof of Theorem 3.■

Theorem 4. Let $f(z) \in VU_{\theta,\gamma}(f, g, \alpha, \beta, k)$. Then for $|z| < r = 1$

$$1 - \frac{2[(1-\alpha)-k(1-\beta)]}{[(2-\beta)k+(2-\alpha)]\Psi_2 b_2} r \leq |f'(z)| \leq 1 + \frac{2[(1-\alpha)-k(1-\beta)]}{[(2-\beta)k+(2-\alpha)]\Psi_2 b_2} r. \quad (3.3)$$

Sharppeness holds for $f(z)$ given by (3.2).

Proof. We have

$$|f'(z)| \leq 1 + \sum_{n=2}^{\infty} n|a_n||z|^{n-1} \leq 1 \; r\sum_{n=2}^{\infty} n|a_n| \quad (3.4)$$

and

$$|f'(z)| \geq 1 - \sum_{n=2}^{\infty} n|a_n||z|^{n-1} \geq 1 - r\sum_{n=2}^{\infty} n|a_n|. \quad (3.5)$$

In view of (2.11), we have

$$\frac{[(2-\beta)k + (2-\alpha)]\Psi_2 b_2}{2} \sum_{n=2}^{\infty} n|a_n| \leq \sum_{n=2}^{\infty} [(\Omega_n - \beta\Psi_n) + (\Omega_n - \alpha\Psi_n)] b_n |a_n|$$

$$\leq (1-\alpha) - k(1-\beta),$$

or,

$$\sum_{n=2}^{\infty} n|a_n| \leq \frac{2\,[(1-\alpha)-k(1-\beta)]}{[(2-\beta)k+(2-\alpha)]\Psi_2 b_2} \quad (3.7)$$

A substitution of (3.7) into (3.4) and (3.5) yields the inequality (3.3).

Theorem 5. Let $f(z) \in VU_{\theta,\gamma}(f, g, \alpha, \beta, k)$, with $arg(a_n) \equiv \pi + (n+1)\theta, n \geq 2$. Let

$$f_1(z) = z$$

and

$$f_n(z) = z + \frac{[(1-\alpha) - k(1-\beta)]}{[(n-\beta)k + (n-\alpha)]\Psi_n b_n} e^{i\theta_n} z^n$$

$$(n \geq 2; 0 \leq \theta \leq 2\pi). \quad (3.8)$$

Then $f(z) \in VU_{\theta,\gamma}(f, g, \alpha, \beta, k)$, if and only if $f(z)$ can be expressed in the for

$$f(z) = \sum_{n=1}^{\infty} \nu_n f_n(z), \quad (3.9)$$

where $\nu_n \geq 0$ and $\sum_{n=1}^{\infty} \nu_n = 1$.

Proof. If $f(z)$ is given by (3.9), then

$$f(z) = \sum_{n=1}^{\infty} \nu_n f_n(z) = \nu_1 f_1(z) + \sum_{n=2}^{\infty} \nu_n f_n(z).$$

$$= (\nu_1 + \sum_{n=2}^{\infty} \nu_n)z + \sum_{n=2}^{\infty} \frac{[(1-\alpha)-k(1-\beta)]}{[(n-\beta)k+(n-\alpha)]\Psi_n b_n} \nu_n e^{i\theta_n} z^n.$$

$$z + \sum_{n=2}^{\infty} \frac{[(1-\alpha)-k(1-\beta)]}{[(n-\beta)k+(n-\alpha)]\Psi_n b_n} \nu_n e^{i\theta_n} z^n. \quad (3.10)$$

We see that

$$\sum_{n=2}^{\infty} [(n-\beta)k + (n-\alpha)]\Psi_n b_n \left| \frac{[(1-\alpha) - k(1-\beta)]}{[(n-\beta)k + (n-\alpha)]\Psi_n b_n} \nu_n e^{i\theta_n} \right|$$

$$= \sum_{n=2}^{\infty} [(1-\alpha) - k(1-\beta)]\, \nu_n$$

$$= (1-\nu_1)(1-\alpha) - k(1-\beta)$$

$$\leq (1-\alpha) - k(1-\beta).$$

Then $f(z)$ satisfies (2.11) and hence $f(z) \in VU_{\theta,\gamma}(f, g, \alpha, \beta, k)$.

Conversely, let the function $f(z)$ defined by (1.1) belonges to the class $f(z) \in VU_{\theta,\gamma}(f,g,\alpha,\beta,k)$,

$$\nu_n = \frac{[(n-\beta)k + (n-\alpha)]\Psi_n b_n}{(1-\alpha) - k(1-\beta)} a_n \ (n \geq 2),$$

$$\nu_1 = 1 - \sum_{n=2}^{\infty} \nu_n.$$

From Theorem 2, $\sum_{n=2}^{\infty} \nu_n \leq 1$ and so $\nu_1 \geq 0$. Since $\nu_n f_n(z) = \nu_n z + a_n z^n$, then

$$\sum_{n=1}^{\infty} \nu_n f_n(z) = z + \sum_{n=2}^{\infty} a_n z^n = f(z).$$

Remark 1. Putting $\gamma = 0$ and $g(z) = \frac{z}{1-z}$ or ($b_n = 1$ with $n \geq 2$) in Theorems 1, 2, 3, 4 and 5, respectively, we obtain the results obtained by El-Ashwah et al. ([2], Theorems 8, 14, 15 and 17, respectively.

4. Radii of close-to-convexity, starlikeness and convexity

Theorem 6. Let $f(z) \in VU_{\theta,\gamma}(f,g,\alpha,\beta,k)$ Then

(i) $f(z)$ is close-to-convex of order $\delta(0 \leq \delta \leq 1)$ in $|z| < r_1$,

$$r_1 = \inf_{n\geq 2} \left\{\frac{(1-\delta)\,[(n-\beta)k + (n-\alpha)]\Psi_n b_n}{n[(1-\alpha) - k(1-\beta)]}\right\}^{\frac{1}{n-1}}. \quad (4.1)$$

(ii) $f(z)$ is starlike of order $\delta(0 \leq \delta \leq 1)$ in $|z| < r_2$,

$$r_2 = \inf_{n\geq 2} \left\{\frac{(1-\delta)\,[(n-\beta)k + (n-\alpha)]\Psi_n b_n}{(k-\delta)[(1-\alpha) - k(1-\beta)]}\right\}^{\frac{1}{n-1}}, \quad (4.2)$$

(iii) $f(z)$ is convex of order $\delta(0 \leq \delta \leq 1)$ in $|z| < r_3$,

$$r_3 = \inf_{n\geq 2} \left\{\frac{(1-\delta)\,[(n-\beta)k + (n-\alpha)]\Psi_n b_n}{n(k-\delta)[(1-\alpha) - k(1-\beta)]}\right\}^{\frac{1}{n-1}}. \quad (4.3)$$

Each of these results is sharppeness for $f(z)$ given by (2.15).

Proof. We must show that

$$|f(z) - 1| \leq 1 - \delta \text{ for } |z| < r_1, \quad (4.4)$$

where r_1 is given by (4.1). Indeed we find from (1.1) that

$$|f(z) - 1| \leq \sum_{n=2}^{\infty} n|a_n||z|^{n-1}.$$

Thus

$$|f(z) - 1| \leq 1 - \delta,$$

If

$$\sum_{n=2}^{\infty} \left(\frac{n}{1-\delta}\right) |a_n||z|^{n-1} \leq 1. \quad (4.5)$$

But, by Theorem 2 and (4.5) will be true if

$$\left(\frac{n}{1-\delta}\right) |z|^{n-1} \leq \frac{[(n-\beta)k + (n-\alpha)]\Psi_n b_n}{[(1-\alpha) - k(1-\beta)]}$$

that is, if

$$|z| = \left\{\frac{(1-\delta)\,[(n-\beta)k + (n-\alpha)]\Psi_n b_n}{n[(1-\alpha) - k(1-\beta)]}\right\}^{\frac{1}{n-1}},$$

the proof (i) is completed. The proof of (ii) and (iii) is similar to (i) and will be omitted.■

Remark 2. Taking $\gamma = 0$ and $g(z) = \frac{z}{1-z}$ or ($b_n = 1$ with $n \geq 2$) in our results, we obtain the results obtained by Magesh [7].

Remark 3. Putting $\gamma = 0$ in our results, we obtain new results associated with the subclass $k - VUCV(f,g,\alpha,\beta)$, defined in the introduction.

References

[1]- M. K. Aouf and G. Murugusundaramoorthy, On a subclass of uniformly convex functions defined by the Dziok-Srivastava operator, Austral. J. Math. Anal. Appl., 5(2008), no. 1, Art. 3, 1-17.

[2]- R. M. El-Ashwah, M. K. Aouf, A. A. Hassan and A. H. Hassan, Certain new classes of analytic functions with varying arguments, J. Complex Analysis, 2013(2013), 1-5.

[3]- S. Kanas, Uniformly convex functions, Internat. J. Appl. Math., 1(1999), no. 3, 305-310.

[4]- S. Kanas and H.M. Srivastava, Linear operators associated with k-uniformly convex functions, Integral Transform. Spec. Funt., 9 (2000), 121-132.

[5]- S. Kanas and A. Wisniowska, Conic regions and k-starlike functions, Rev. Roumaine Math. Pures Appl., 45 (2000), 647-657.

[6]- S. Kanas and A. Wisniowska, Conic regions and k-uniform convexity, J. Comput. Appl. Math., 105(1999), 327-336.

[7]- N. Magesh, Certain subclasses of uniformly convex functions of order and type with varying arguments, J. Egyptian Math. Soc., (2013), 1-6.

[8]- S. Owa, Y. Polatoglu and E. Yavuz, Coefficient inequalities for classes of uniformly starlike and convex functions, J. Inequal. Pure Appl. Math., 7(2006), no. 5, Art. 160, 1-5.

[9]- Y. J. Sim, O. S. Kwon, N. E. Cho, and H. M. Srivastava, Some classes of analytic functions associated with conic regions, Taiwanese J. Math., 16(2012), no. 1, 387-408.

[10]- H. Sliverman, Univalent functions with negative coefficients, Proc. Amer. Math. Soc. 51(1975), 109-116.

[11]- H. Sliverman, Univalent functions with varying arguments, Houston J. Math.,7 (1981), no. 2, 283-287.