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Some properties of fuzzy path connected Topological spaces

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Abstract The aim of this paper is to study the some properties of fuzzy path connectedness in fuzzy topological spaces. Different way will be provided to prove that fuzzy path connected space is fuzzy connected by using the concept of fuzzy connected component and also to prove that the Cartesian product of two fuzzy path connected spaces is also fuzzy path connected.

Keywords: Fuzzy topology; Fuzzy point; Fuzzy connectedness; Fuzzy path; Fuzzy path connectedness.

بعض خصائص الفضاءات الضبابية المترابطة مساريا

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الملخص الهدف من هذه الورقة هو دراسة بعض خصائص الترابط المساري الضبابي في الفضاءات الطوبولوجية الضبابية. سيتم توفير طريقة مختلفة لإثبات ان الفضاء الضبابي المترابط مساريا يكون مترابط باستخدام مفهوم الترابط الضبابي عند نقطة وأيضا سنبرهن ان الضرب الكارتيزي لفضاءين ضبابيين مترابطين مساريا يكون فضاء ضبابي مترابط مساريا.

الكلمات المفتاحية: الطوبولوجي الضبابي؛ النقطة الضبابية؛ الترابط الضبابي؛ المسار الضبابي؛ الترابط المساري الضبابي.

1. Introduction

Fuzzy set theory offers a new angle to observe and investigate the relation sets and their elements.

In 1965, Prof. L.A.Zadeh [1], generalized the usual notion of set by introducing 'fuzzy sets'. Fuzzy subsets are the classes of objects with grades of membership ranging between the nil memberships (0) and the full memberships (1). Fuzzy sets allow us to represent vague concepts expressed in natural language. The representation depends not only on the concept, but also on the context in which it is used. Hence, their application is increasing in the field of probability theory, information theory, computer science etc.

In 1968, C. L. Chang [2], introduced the concept of fuzzy topological spaces as an application of fuzzy sets to general topological spaces. Further, R. Lowen [3] and Halijol [4] and others have developed a theory of fuzzy topological space. The special case of fuzzy topology is general topology.

The concept of fuzzy connectedness have been introduced earlier by Raja-Sethupathy and Lakshmivarahan in 1977 [5], in 2017 studied and developed this concept and properties him [6]. In 1980 pu pao-Ming and liu ying- Ming introduced the concept of fuzzy connected component[7].

One of the intrinsic characteristic of fuzzy topological space is path connectedness. In 1984 C. Y. Zheng has introduced fuzzy path and fuzzy connectedness [8] and in 1987 D. M. Ali introduced the new concept of fuzzy path connected topological spaces and studied its various characteristics [9]. In 2018 ruhul Amin and sohel Rana introduced a new definition of fuzzy path and fuzzy path connected [10].

In the present paper, We proved two theorems of some properties of fuzzy path connectedness spaces by different way and these theorems are that a fuzzy path connected space is fuzzy connected and the Cartesian product of two fuzzy path connected spaces is fuzzy path connected space. and Some examples have been given for clarification.

2. Preliminaries

Defition 2. 1:

Let X be a non-empty set. A fuzzy set A in X is characterized by its membership function μ_A :

$X \rightarrow [0,1]$ (1)

and $\mu_A(x)$ is interpreted as the degree of membership of element x in fuzzy set A, for each x exists in X. It is clear that A is completely determined by the set of tuples

$A = \{(x, \mu_A(x)) : x \in X\}$. [1]

Defition 2. 2:

The membership function $\mu_C(x)$ of the intersection is $C = A \cap B$ pointwise defined by $\mu_C(x) = \min\{\mu_A(x), \mu_B(x)\}, \quad x \in X$.[1]

Defition 2. 3:

The membership function $\mu_D(x)$ of the union $D = A \cup B$ is pointwise defined by

$\mu_D(x) = max\{\mu_A(x), \mu_B(x)\}, \quad x \in X$.[1]

Defition 2. 4:

The membership function of the complement of a normalized fuzzy set $A, \mu_{A^c}(x)$ is defined by $\mu_{A^c}(x) = \{1 - \mu_A(x)\}, \quad x \in X$.[1]

Defition 2. 5:

A fuzzy topology is a family τ of fuzzy sets in X which satisfies the following condition:

i. $0,1 \in \tau$,
ii. If $A, B \in \tau$, than $A \cap B \in \tau$,
iii. If $A_i \in \tau$ for each $i \in I$, than $\cup_i A_i \in \tau$.

τ is called a fuzzy topology for X, and the pair (X, τ) is a fuzzy topological space, or fts, for short.[2]

Defition 2. 6:

A fuzzy point p in X is a special type of fuzzy set with membership function

$$p(x) = \begin{cases} r & if \quad x = x_0 \\ 0 & if \quad x \neq x_0 \end{cases}$$

where $0 < r < 1$.[1]

Defition 2. 7:

Let (X,τ) and (Y,s) be two fuzzy topological spaces. A function $f : (X,\tau) \to (Y,s)$ is called fuzzy continuous iff $f^{-1}(v) \in \tau$ for every $v \in s$.[1]

Defition 2. 8:

Two fuzzy sets M and N separate a fuzzy topological space X, if:

i. M and N are open fuzzy subset of X;
ii. $M \neq 0$, $N \neq 0$;
iii. $M \cap N = 0$;
iv. $M \cup N = 1$.[5]

Defition 2. 9:

A fuzzy topological space X is fuzzy connected if there exists not M and N which separate X. Otherwise, it is called disconnected.[5]

Defition 2. 10:

A fuzzy subset A of a fuzzy topological space X is called a fuzzy connected set, if A is a fuzzy connected subspace of X. Otherwise, it is a fuzzy disconnected set.

Theorem 2. 1:

The continuous image of a fuzzy connected space is fuzzy connected.[5, 6]

Theorem 2. 2:

A fuzzy subset $E$of $\mathbb{R}$ is fuzzy connected iff E is an interval.

Defition 2. 11:

Let X be a fuzzy topological space and x a fuzzy point in X. The fuzzy connected component of x is defined by

$$C_x = \cup \{A \subseteq X : A \; is \; fuzzy \; connected, x \in A\} \quad (2)$$

i.e C_x is the largest fuzzy connected set contains x.[7]

Defition 2. 12:

Let X be a fuzzy topological space. A fuzzy path is a fuzzy continuous function $p: I \to X$, where $I = [0,1]$. $p(0)$ is the initial point, $p(1)$ is the terminal point. [9]

Defition 2. 13:

Let X be a fuzzy topological space. A constant fuzzy path is a fuzzy path p where $p: I \to X$; $p(t) = x_0 \;\; \forall t \in I$. x_0 is a fixed point. [4]

Defition 2. 14:

Let X be a fuzzy topological space. Let $p: I \to X$ be a fuzzy path from x to y and $q: I \to X$ be a path from y to z. The composition (product) of fuzzy paths p and q is a continuous function $p * q: I \to X$ is given by :

$$(p * q)(t) = \begin{cases} p(2t) & , \quad 0 \leq t \leq 1/2 \\ q(2t-1) & , \quad 1/2 \leq t \leq 1 \end{cases}$$

$p * q$ is a fuzzy path from x to z.[4]

Defition 2. 15:

A fuzzy topological space X is a fuzzy path connected if for every two points x and y in X there exists a fuzzy path $p: I \to X$ such that $p(0) = x$ and $p(1) = y$. [4]

Theorem 2. 3:

A fuzzy continuous image of a fuzzy path connected space is fuzzy path connected.[10]

3. Main Results

Proposition 3. 1:

Let X be a fuzzy topological space and x a fuzzy point in X. Than:

X is fuzzy connected iff $C_x = X$.

Proof:

($\Longrightarrow$) Let X is fuzzy connected space.

$C_x = \{A \subseteq X : A \; is \; fuzzy \; connected, x \in A\}$ is the largest fuzzy connected set contains x.

So $C_x = \cup X = X$.

($\Longleftarrow$) Let $C_x = X$

Such that C_x is the largest fuzzy connected set contains x.

So X is fuzzy connected space.

Remark:

We have proved the proposition 3.1 because there isn't exists in other references and we will use provide proof the following proposition.

Proposition 3. 2:

A fuzzy path connected space is fuzzy connected.

Proof:

Let X be a fuzzy path connected space.

Suppose x is a fuzzy point in X.

To show that $C_x = X$. (by proposition 3.1).

$C_x \subseteq X$ where C_x is the largest fuzzy connected set contain x.

Let y a fuzzy point in X. there exists a fuzzy path $p: I \to X$ from x to y.

I is fuzzy connected. So $p(I)$ is fuzzy connected and contain y.

$p(I) \subseteq C_x$. So $y \in C_x$. So $X \subseteq C_x$. So $C_x = X$.

So X is fuzzy connected.

Example:

indiscrete space is fuzzy path connected.

Proof:

Let x and y two fuzzy points in X

$P: I \to X$ is given by $p(t) = \begin{cases} x & if \quad 0 \leq t < \frac{1}{2} \\ y & if \quad \frac{1}{2} \leq t \leq 1 \end{cases}$

Such that $p(0) = x$ and $p(1) = y$

To show that P is fuzzy continuous map.

$p(t) = x , \forall \, t \in [0, \frac{1}{2}[$ and $p(t) = y, \forall \, t \in [\frac{1}{2}, 1]$

Such that x, y are fixed points.

So p is constant map.

So p is continuous map.

So p fuzzy continuous map.

So indiscrete space is fuzzy path connected.

Corollary:

indiscrete space is fuzzy path connected so is fuzzy connected.(by proposition 3.2)

Proof:

$\tau_0 = \{0,1\}$

So $0,1 \in \tau_0$ i.e $0,1$ are two open fuzzy sets only in τ_0.

So there are no two open fuzzy sets separating this space.

So indiscrete space is fuzzy connected.

Remark:

We have proved that A fuzzy path connected space is fuzzy connected by concept of fuzzy connected component While this theory has been proven using a concept fuzzy connected in [10].

Proposition 3. 3:

If X and Y are two fuzzy path connected spaces, the so is $X \times Y$.

Proof:

Let $(x_0, y_0), (x_1, y_1)$ two fuzzy points in $X \times Y$.

x_0, x_1 two fuzzy points in X and

y_0, y_1 two fuzzy points in Y.

So there exists two fuzzy paths $p: I \to X$ from x_0 to $, x_1$ and $q: I \to Y$ from y_0 to y_1.

Define $r: I \to X \times Y$ by $r(t) = (p(t), q(t)) \quad \forall\, t \in I$.
r is fuzzy continuous map,
Since $p_1 r = p$ & $p_2 r = q$.
Such that $p_1: X \times Y \to X$ and $p_2: X \times Y \to Y$
$r(0) = \big(p(0), q(0)\big) = (x_0, y_0)$& $r(1) = \big(p(1), q(1)\big) = (x_1, y_1)$.
So $X \times Y$ is fuzzy path connected space.

4. Conclusion

We studied some properties of fuzzy path connectedness in fuzzy topological spaces and provide proof in a different way to prove that fuzzy path connected space is fuzzy connected by concept of fuzzy connected component That is, several methods can be used to prove this theorem. and also proved that the Cartesian product of two fuzzy path connected spaces is fuzzy path connected space.

The results presented in this note indicate that many of the basic concepts path connectedness in general topology can readily be extended to fuzzy path connectedness in fuzzy topological spaces.

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