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A comparative study between new proposed Technique (LPAM) and the other existing techniques to Find IBFS for Transportation Problem

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Abstract Transportation model is a special case of linear programming problem in which the main objective is to transport raw material, or finished products are transported from warehouses to factories or vice-versa, at a total minimum cost. Finding an initial basic feasible solution is the basic requirement to acquire an optimal solution for the transportation problems. In this study, a new technique named (LPAM- Technique) (The Largest penalty and allocate the maximum- technique) is proposed to locate initial basic feasible solution for transportation problems. The performance of the (LPAM) is evaluated and compared with the existing techniques in the literature (NWCM, LCM, VAM, CVAM, iLCM), To clarify the statement, The experiments conducted show that the (LPAM) generally outperformed the other techniques when applied to the applications twenty randomly chosen an available in the literature to Obtain (IBFS) of Transportation Problem. Obtained results show by using this technique we found that (IBFS) of most of the transportation problem very close to optimal solution or same as optimal solution than using the other existing techniques which is discussed in this study, and it is observed that the performance of proposed technique is suitable for solving transportation problems.

Keywords: Transportation Problem, Transportation Cost, Optimal Solution, Initial Basic Feasible Solution (IBFS), Proposed technique (LPAM).

دراسة مقارنة بين التقنية المقترحة الجديدة (LPAM) والتقنيات الأخرى المتاحة لإيجاد IBFS لمشكلة النقل

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الملخص نموذج النقل هو حالة خاصة لمشكلة البرمجة الخطية حيث يكون الهدف الرئيسي هو نقل المواد الخام أو المنتجات النهائية من المستودعات إلى المصانع أو العكس بأقل تكلفة إجمالية. إن إيجاد حل أساسي الابتدائي هو المطلب الرئيسي للحصول على الحل الأمثل لمشاكل النقل. في هذه الدراسة، تم اقتراح طريقة جديدة تسمى (LPAM) لإيجاد الحل الاساسي الابتدائي الممكن لمشكلة النقل، يتم تقيم اداء الطريقة المقترحة(LPAM) ومقارنتها بالتقنيات الحالية المتاحة (NWCM, LCM, VAM, CVAM, iLCM) وذلك لبيان افضليتها. أظهرت التطبيقات التي أجريت أن التقنية المقترحة (LPAM) تفوقت بشكل عام على التقنيات الاخرى عند تطبيقها على العشرين تطبيقا المختارة عشوائيا والمتاحة في المرجعيات لإيجاد (IBFS) لمشكلة النقل. أظهرت النتائج التي تم الحصول عليها باستخدام هذه التقنية أن (IBFS) لمعظم مشكلة النقل قريبة جدا من الحل الأمثل أو نفس الحل الأمثل من استخدام التقنيات الأخرى التي تمت مناقشتها في هذه الدراسة، ويلاحظ أن أداء التقنية المقترحة مناسب لحل مشاكل النقل.

الكلمات المفتاحية: مشكلة النقل – تكلفة النقل – الحل الامثل – الحل الاساسي الابتدائي – التقنية المقترحة (LPAM).

Introduction:

Transportation cost has significant effect at the cost and the pricing of raw materials and finished products. Supplier and factory try to control the cost of transportation. Transportation Problem is a specific part of linear programming problem that is implemented in actual life. Some well-known methods to find the minimum transportation cost are North West Corner Method (NWCM), The Least Cost Method (LCM) & Vogel's Approximation Method (VAM) are considered to provide the better Initial Basic Feasible Solution (IBFS) and for optimality check we can use MODI method [1].

Besides the conventional techniques many researchers has offer many techniques to discover a better IBFS of a TP. Some of the important associated works the current research has deal with are: A. R. Khan, A. Vilcu, N. Sultana and S. S. Ahmed (2015), Determination of Initial Basic Feasible Solution of a Transportation Problem: A TOCM-SUM Approach [2]. Md Sharif Uddin , Aminur Rahman Khan , Chowdhury Golam Kibria , Iliyana Raeva, 2016 , Improved Least Cost Method (iLCM) to Obtain a Better IBFS to the Transportation Problem[3]. M.M.Ahmed, A.R.Khan, M.S.Uddin and F.Ahmed(2016), proposed Customized Vogel's Approximation Method (CVAM) for solving transportation problems[4] . Duraphe S and Raigar S (2017) obtain" A new approach to solve transportation problems with the max-min total opportunity cost method" [5]. S.M.Abul Kalaam Aazad, Md. Bellel

Hosain,Md .M Rahman (2017)" An Algorithmic Approach to solve Transportation Problems with the Average Total Opportunity cost method" [6]. S.M.Abul Kalaam Aazad, Md. Bellel Hosain (2017) developed a new method for solving transportation problems considering average penalty [7]. Swati V. Kamble, Bhausaheb G. Kore ,2019 , proposed a New Method to Obtain an Initial Basic Feasible Solution of Transportation Problem with the Average Opportunity Cost Method [8].

In this paper we introduce technique for solving transportation problem which is very simple, easy to understand with lesser number of iterations and very easy computations, to find the IBFS to solve transportation problems, and a comparative study is also carried out by solving that (LPAM) gives better result in comparison to the other existing techniques available in the literature. We also coded the proposed heuristic by using MATLAB and the code is tested via many randomly generated applications of different size to compare the solution obtained manually and using MATLAB code in order to prove the correctness of the code. Based on the results we show that both the result has the same value when solving the transportation problem.

Mathematical Formulation [4]**:**

The transportation problem is shown as a linear transportation model as below.

$$\textbf{Minimize} \quad z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}\ x_{ij}$$

$$\textbf{Subject to}: \sum_{j=1}^{n} x_{ij} \le S_i \;\; ; \;\; i = 1, 2, \ldots., \ldots, m$$

$$\sum_{i=1}^{m} x_{ij} \ge d_j \;\; ; \;\; j = 1, 2, \ldots \ldots .. n$$

$$\textbf{and} \qquad x_{ij} \ge 0 \;\; ; \;\; \textbf{for all i and j} .$$

Where, Z = Total transportation cost to be minimized .
X_{ij} = The quantity to be shipped from source i to destination j ,
C_{ij} = Unit transportation cost from source i to destination j .
m = Total number of sources /origins .
n = Total number of destinatios .

A specially designed table is constructed to solve a TP systematically, which is called transportation table.

Factory / Origin		Destinations / Warehouse				Available products/ Supply
		1	2	.	n	
Sources	1	X_{11} C_{11}	X_{12} C_{12}	.	X_{1n} C_{1n}	S_1
	2	X_{21} C_{21}	X_{22} C_{22}	.	X_{2n} C_{2n}	S_2
	⋮	......	....	.	....	....
	m	X_{m1} C_{m1}	X_{m2} C_{m2}	.	X_{mn} C_{mn}	S_m
Demand		d_1	d_2	.	d_n	Balanced model $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$

Algorithm of Proposed Technique (LPAM):

In this study, we proposed a new solving technique (LPAM) for transportation problems. The proposed technique must operate the as following:

Step 1:- Examine whether the transportation problem is balanced or not. If it is balanced then go to next step.

Step 2:- Subtract the smallest cost from every element of every row of transportation table and place them on the right-top of corresponding elements.

Step 3:- Apply the same operation (Step 2) on each of the columns and place them on the right-bottom of the corresponding element.

Step 4:- Place Row Penalty (RP) and Column Penalty (CP) just after and below the supply and demand amount respectively, which are the difference between smallest & next smallest value of the right-top elements of each row and the right-bottom elements of each column respectively of the transportation table.

Step 5:- Identify the largest penalty and allocate the maximum possible quantity to that cell having minimum value of element in corresponding row or column.

Step 6:- Repeat step 4 to step 6 until the rim requirement is satisfied.

Step 7:- Put these allocated values in original transportation table in corresponding cell.

Step 8: − Calculate $\boldsymbol{z} = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}\ x_{ij}$, z being the minimum transportation cost and $\boldsymbol{c_{ij}}$ are the cost elements of the transportation table.

Numerical Applications

Twenty numerical applications for transportation problem of different sizes, selected at random from some reputed journals published by several authors mentioned in the Table 1 to illustrate the proposed technique. We also use these applications to compare the solution obtained by our proposed technique (LPAM) with the well-known NWCM, LCM, VAM, CVAM, iLCM.

Applications 1

Solution of the balanced transportation problem applications 1:

Factory / Origin		Destinations /Warehouse			Available products/ Supply
		1	2	3	
Sources	1	6	4	1	50
	2	3	8	7	40
	3	4	4	2	60
Demand		20	95	35	

Solution

Balance the transportation problem.

Factory / Origin		Destinations / Warehouse			Available products / Supply
		1	2	3	
Sources	1	6	4	1	50
	2	3	8	7	40
	3	4	4	2	60
Demand		20	95	35	150 / 150

The row differences and column differences are:

Balance the transportation problem.

Factory / Origin		Destinations / Warehouse			Available products/ Supply
		1	2	3	
Sources	1	6_3^5	4_0^3	1_0^0	50
	2	3_0^0	8_4^5	7_6^4	40
	3	4_1^2	4_0^2	2_1^0	60
Demand		20	95	35	

The allocations with the help of RP and CP are:

		D1	D2	D3	Supply	Row Penalty	
Factory	1	6_3^5	4_0^3 15	1_0^0 35	50-15-0	3	3
	2	3_0^0 20	8_4^5 20	7_6^4	40-20-0	4	1
	3	4_1^2	4_0^2 60	2_1^0	60-0	2	2
Demand		20-0	95-35-20-0	35-0			
Column Penalty		1	4	1			
		-	4	1			
		-	4	6			

The transportation cost is $\boldsymbol{z} = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}\ x_{ij} \ = 60 + 35 + 60 + 160 + 240 \ = \ 555 \ DL$

Table1: Numerical Applications of transportation problems

Application number	Application Capacity	Sources (References)	Data of transportation Problems
1	3x3	12	$[c_{ij}] = [6\ 4\ 1;\ 3\ 8\ 7;\ 4\ 4\ 2]$; $[s_i] = [50, 40, 60]$; $[d_j] = [20, 95, 35]$
2	3x3	17	$[c_{ij}] = [4\ 3\ 5;\ 6\ 5\ 4;\ 8\ 10\ 7]$; $[s_i] = [90, 80, 100]$; $[d_j] = [70, 120, 80]$
3	3x3	17	$[c_{ij}] = [15\ 7\ 25;\ 8\ 12\ 14;\ 17\ 19\ 21]$; $[s_i] = [12, 17, 7]$; $[d_j] = [12, 10, 14]$
4	3x4	18	$[c_{ij}] = [6\ 3\ 5\ 4;\ 5\ 9\ 2\ 7;\ 5\ 7\ 8\ 6]$; $[s_i] = [22, 15, 8]$; $[d_j] = [7, 12, 17, 9]$
5	3x4	9	$[c_{ij}] = [1\ 2\ 1\ 4;\ 4\ 2\ 5\ 9;\ 20\ 40\ 30\ 10]$; $[s_i] = [30, 50, 20]$;$[d_j] = [20, 40, 30, 10]$
6	3x4	11	$[c_{ij}] = [6\ 1\ 9\ 3;\ 11\ 5\ 2\ 8;\ 10\ 12\ 4\ 7]$; $[s_i] = [70, 55, 90]$; $[d_j] = [85, 35, 50, 45]$
7	3x4	17	$[c_{ij}] = [3\ 1\ 7\ 4;\ 2\ 6\ 5\ 9;\ 8\ 3\ 3\ 2]$; $[s_i] = [300, 400, 500]$; $[d_j] = [250, 350, 400, 200]$
8	3x4	14	$[c_{ij}] = [19\ 30\ 50\ 12;\ 70\ 30\ 40\ 60;\ 40\ 10\ 60\ 20]$;$[s_i] = [7, 10, 18]$;$[d_j] = [5, 8, 7, 15]$
9	3x5	12	$[c_{ij}] = [4\ 1\ 2\ 4\ 4;\ 2\ 3\ 2\ 2\ 3;\ 3\ 5\ 2\ 4\ 4]$; $[s_i] = [60, 35, 40]$; $[d_j] = [22, 45, 20, 18, 30]$
10	3x5	18	$[c_{ij}] = [5\ 7\ 10\ 5\ 3;\ 8\ 6\ 9\ 12\ 14;\ 10\ 9\ 8\ 10\ 15]$; $[s_i] = [5, 10, 10]$; $[d_j] = [3, 3, 10, 5, 4]$
11	4x3	16	$[c_{ij}] = [2\ 7\ 4;\ 3\ 3\ 1;\ 5\ 4\ 7;\ 1\ 6\ 2]$; $[s_i] = [5, 8, 7, 14]$; $[d_j] = [7, 9, 18]$
12	4x4	14	$[c_{ij}] = [7\ 5\ 9\ 11;\ 4\ 3\ 8\ 6;\ 3\ 8\ 10\ 5;\ 2\ 6\ 7\ 3]$; $[s_i] = [30, 25, 20, 15]$ $[d_j] = [30, 30, 20, 10]$
13	4x4	15	$[c_{ij}] = [5\ 3\ 6\ 10;\ 6\ 8\ 10\ 7;\ 3\ 1\ 6\ 7;\ 8\ 2\ 10\ 12]$; $[s_i] = [30, 10, 20, 10]$ $[d_j] = [20, 25, 15, 10]$
14	4x6	10	$[c_{ij}] = [7\ 10\ 7\ 4\ 7\ 8;\ 5\ 1\ 5\ 5\ 3\ 3;\ 4\ 3\ 7\ 9\ 1\ 9;\ 4\ 6\ 9\ 0\ 0\ 8]$; $[s_i] = [5, 6, 2, 9]$ $[d_j] = [4, 4, 6, 2, 4, 2]$
15	4x6	12	$[c_{ij}] = [9\ 12\ 9\ 6\ 9\ 10;\ 7\ 3\ 7\ 7\ 5\ 5;\ 6\ 5\ 9\ 11\ 3\ 11;\ 6\ 8\ 11\ 2\ 2\ 10]$; $[s_i] = [5, 6, 2, 9]$; $[d_j] = [4, 4, 6, 2, 4, 2]$
16	5x5	21	$[c_{ij}] = [8\ 8\ 2\ 10\ 2;\ 11\ 4\ 10\ 9\ 4;\ 5\ 2\ 2\ 11\ 10;\ 10\ 6\ 6\ 5\ 2; 8\ 11\ 8\ 6\ 4]$ $[s_i] = [40, 70, 35, 90, 85]$; $[d_j] = [80, 55, 60, 80, 45]$
17	5x5	13	$[c_{ij}] = [73\ 40\ 9\ 79\ 20;\ 62\ 93\ 96\ 8\ 13;\ 96\ 65\ 80\ 50\ 65; 57\ 58\ 29\ 12\ 87;\ 56\ 23\ 87\ 18\ 12]$; $[s_i] = [8, 7, 9, 3, 5]$; $[d_j] = [6, 8, 10, 4, 4]$
18	5x6	19	$[c_{ij}] = [5\ 3\ 7\ 3\ 8\ 5;\ 5\ 6\ 12\ 5\ 7\ 11;\ 2\ 8\ 3\ 4\ 8\ 2;\ 9\ 6\ 10\ 5\ 10\ 9; 5\ 3\ 7\ 3\ 8\ 5]$; $[s_i] = [3, 4, 2, 8, 3]$; $[d_j] = [3, 4, 6, 2, 1, 4]$

19	5x7	20	$[c_{ij}] = [12\ 7\ 3\ 8\ 10\ 6\ 6;\ 6\ 9\ 7\ 12\ 8\ 12\ 4;\ 10\ 12\ 8\ 4\ 9\ 9\ 3;\ 8\ 5\ 11\ 6\ 7\ 9\ 3;\ 7\ 6\ 8\ 11\ 9\ 5\ 6];\ [s_i] = [60, 80, 70, 100, 90];\ [d_j] = [20, 30, 40, 70, 60, 80, 100]$
20	6x6	2	$[c_{ij}] = [12\ 4\ 13\ 18\ 9\ 2;\ 9\ 16\ 10\ 7\ 15\ 11;\ 4\ 9\ 10\ 8\ 9\ 7;\ 9\ 3\ 12\ 6\ 4\ 5;\ 7\ 11\ 5\ 18\ 2\ 7; 16\ 8\ 4\ 5\ 1\ 10]\ ;\ [s_i] = [120, 80, 50, 90, 100, 60]$ $[d_j] = [75, 85, 140, 40, 95, 65]$

Comparative Study and Analysis

We study the IBFS obtained various methods including the proposed technique (LPAM). We also carry out a comparative study in different ways to analyze the perfectness and to justify the usefulness of the proposed technique (LPAM). It is to be mentioned that MATLAB software is used to obtain the solution of existing methods and also for the optimal solution. These comparisons are shown in the following tables and charts:

Table 2: A Comparative Study of Initial Basic Feasible Solution

Application number	Techniques of solution						Optimal Solution
	NWCM	LCM	VAM	CVAM	iLCM	proposed technique (LPAM)	
1	730	555	555	555	555	555	555
2	1500	1450	1500	1390	1390	1390	1390
3	545	433	425	425	425	425	425
4	176	153	149	149	149	149	149
5	600	560	450	450	450	450	450
6	1265	1165	1220	1165	1165	1165	1160
7	4400	2900	2850	2850	2850	2850	2850
8	975	894	859	859	868	859	799
9	363	305	290	290	295	290	290
10	234	191	187	183	183	183	183
11	102	83	80	80	80	80	76
12	540	435	470	415	435	415	410
13	425	310	285	285	310	285	285
14	95	70	68	68	68	68	68
15	139	114	112	112	112	112	112
16	1870	1685	1505	1475	1565	1475	1475
17	1994	1123	1104	1104	1102	1104	1102
18	129	134	116	126	121	126	116
19	3180	2080	1930	1920	1900	1900	1900
20	4285	2455	2310	2220	2325	2220	2170
	0%	5%	50%	60%	55%	65%	

Table 2, represents the IBFS obtained by various methods. Performance of the IBFS is also shown in this table. It is observed that in 65%, 60%, 55%, 50%, 5% and 0% cases IBFS obtained respectively by proposed technique (LPAM), CVAM, iLCM, VAM, LCM and NWCM is numerically equal to optimal result. This data speaks the better performance of the proposed technique. The graphical representation of IBFS is the mirror of this performance, displayed in Chart 1.

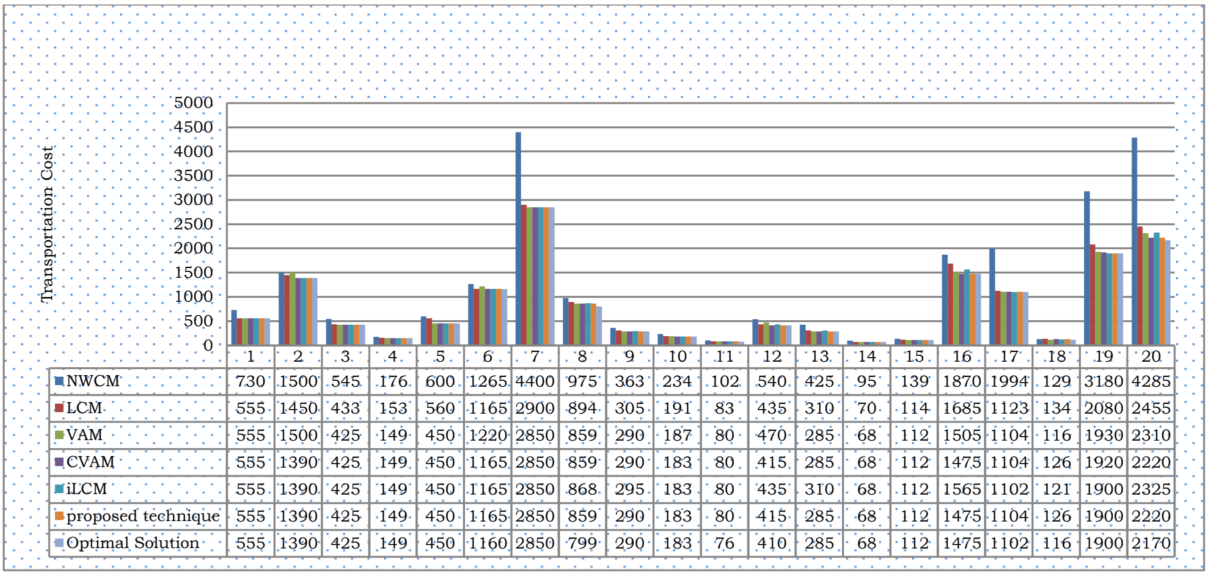

Chart-1: Graphical representation of IBFS.

Table 3: Percentage of near optimality

Application number	Techniques of solution					
	NWCM	LCM	VAM	CVAM	iLCM	proposed technique (LPAM)
1	76.03	100	100	100	100	100
2	92.67	95.86	92.67	100	100	100
3	77.98	98.15	100	100	100	100
4	84.66	97.39	100	100	100	100
5	75	80.36	100	100	100	100
6	91.70	99.57	95.08	99.5	99.5	99.57
7	64.77	98.28	100	100	100	100
8	81.95	89.37	93.(	93.(	92.05	93.02
9	79.89	95.08	100	100	98.31	100
10	78.21	95.81	97.86	100	100	100
11	74.51	91.57	95	95	95	95
12	75.93	94.25	87.23	98.80	94.25	98.80
13	67.06	91.94	100	100	91.94	100
14	71.58	97.14	100	100	100	100
15	80.58	98.25	100	100	100	100
16	78.50	87.54	98	100	94.25	100
17	55.27	98.13	99.82	99.82	100	99.82
18	89.92	86.57	100	92.06	95.87	92.06
19	59.75	91.35	98.45	98.96	100	100
20	50.64	88.39	93.94	97.75	93.33	97.75
	75.33	93.75	98.05	98.75	97.73	98.80

Table 3 is constructed to observe the near optimally status of various methods for finding an initial basic feasible solution. If the percentage is 100, indicates that the obtained result is numerically equal to the optimal solution. Shows proposed technique is directly yielding optimal result for balanced transportation problems in 65% cases.

This data analysis ensures the better performance of the proposed method.
In this study we have also calculated the average percentage of near optimally to see the overall performance of various methods which is shown in the Table 4.

Table 4: Average percentage of near optimality

NWCM	LCM	VAM	CVAM	iLCM	proposed technique (LPAM)	Optimal Solution
75.33	93.75	98.05	98.75	97.73	98.80	100

This performance is also shown in Chart 2.

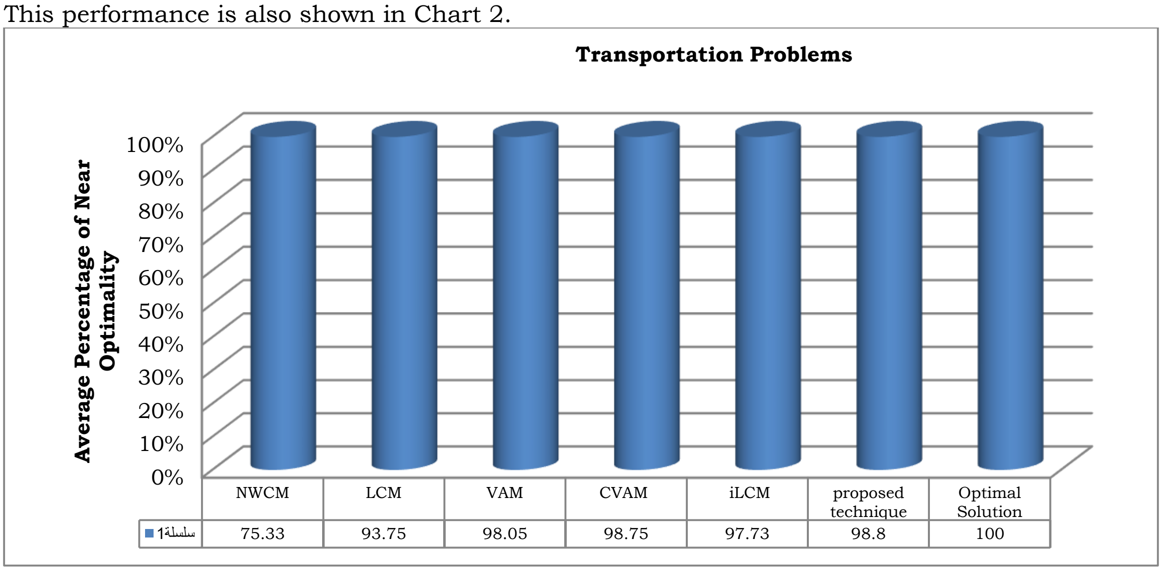

Chart 2: General performance of various methods

Conclusion:

Transportation models are the least cost means to transport a product manufactured at different factories (sources) to a number of different warehouses (destinations). Therefore, this finding is important in saving time and resources for minimization of transportation costs and optimizing transportation processes which could help significantly to improve the factory position in the market. In our study, we developed an efficient technique (LPAM) for cost minimization of transportation problem which is very easy to understand and provides better result in comparison to the existing techniques available in the literature technique. The solution obtained by the current method is near optimal or optimal, with lesser number of iterations and very easy computations. We also present MATLAB code for the developed technique and test the correctness of the code through different applications which proves that the code provides the identical result. This study will assist to achieve the aim of maximizing profits by minimizing the cost of transportation. From comparison table we can see that this new developed method is more effective than other existing which is totally new concept & result oriented for both the bigger & smaller size transportation problem. Future research might be carried out to develop this technique for the case of large number of factories (sources) and warehouses (destinations), and Further study may be carried out to evaluate the performance of the same operation on other techniques.

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