



Modelling and Simulation of X-Configuration QUAD-ROTOR

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ABSTRACT

This Paper presents modelling, simulation and flight testing of a quadrotor Unmanned Ariel Vehicle (UAV) Focus of this paper is to gain in-depth understanding of quadrotor dynamics to develop a simulator and a laboratory platform to facilitate future development of control algorithms. Quadrotor dynamics is modelled using Newton-Euler method. The model predicts the effects of the forces and torques generated by the four propellers on the quadrotor motion. Based on the Mathematical model, a quadrotor simulator was developed in MATLAB® Simulink, on which manually control Using joystick in Flight-Gear simulation environments to understand the Quadrotor behaviour in virtual reality. A Quadrotor platform will be developed composed of a Flight controller (KK-2) interfaced with several devices: power supply, receiving unit, BLDC-motor power boards, and the set of propellers.

النمذجة والمحاكاة للطائرة رباعية المراوح ذات شكل X

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الكلمات المفتاحية:

الطائرة بدون طيار
النمذجة
المحاكاة محاكي الطيران
كود كوبر

الملخص

تقدم هذه الورقة النمذجة والمحاكاة واختبار محاكي الطيران لمركبة جوية بدون طيار رباعية المراوح. يركز هذا العمل على اكتساب فهم متعمق لديناميكيات رباعية المراوح لتطوير جهاز محاكاة ومنصة مختبرية لتسهيل التطوير المستقبلي لخوارزميات التحكم. تم نمذجة ديناميكيات رباعية المراوح باستخدام طريقة نيوتن-أويلر. يتنبأ النموذج بتأثيرات القوى وعزم الدوران الناتجة عن المراوح الأربعة على حركة رباعية المراوح. بناءً على النموذج الرياضي، تم تطوير محاكي رباعي المراوح في MATLAB® Simulink، والذي يتم التحكم فيه يدويًا باستخدام عصا التحكم في بيئات محاكاة Flight-Gear لفهم سلوك رباعي الدوارات في الواقع الافتراضي. سيتم تطوير منصة رباعية المراوح تتكون من وحدة تحكم في الطيران KK-2 متصلة بعدة أجهزة: مصدر الطاقة، ووحدة الاستقبال، ولوحات طاقة المحرك BLDC، ومجموعة المراوح

1. Introduction

Unmanned aerial vehicle (UAV) is an aircraft that carries no human pilot or passengers, sometimes called drones, UAVs are aircrafts that fly autonomously or by remote control, UAV are gradually becoming more widely used across a wide range of real-world applications such as military operations, disaster relief, and exploration of hazardous remote areas, UAVs can be classified as fixed-wing and rotary-wing, fixed wing UAVs consist of a rigid wing which makes the flight capable to produce lift that caused by forward propulsion, The fixed-wing UAV features ailerons, elevator and rudder as its control surfaces to has three axis of rotation, In rotary-wing, the constant movement of the propellers produce airflow over the airofoil and generate lift. A Quadrotor, which is a type of UAV, is a small agile vehicle controlled by the rotational speed of the four rotors. The rotary-wing UAV has simple construction and three axis of rotation configuration is achieved by varying the thrust of the rotors [1].

2. X-QUADROTOR MATHEMATICAL MODELING.

2.1 Introduction.

The quadrotor's mathematical model is developed based upon the coordinate system and the Newton-Euler equations of motion. The system has a six degree of freedom and considered as a rigid body. Because of the system is highly non-linear system the following assumptions are considered in order to simplify the model [3]:

- The structure is rigid and symmetrical.
- The center of gravity of the quadrotor coincides with the body fixed frame origin
- The propellers are rigid
- Thrust and drag are proportional to the square of the propeller's speed.

The mathematical model was derived using two primary reference frames. The orthonormal basis vectors $E=(X, Y, \text{ and } Z)$ define the first frame, which is the inertial (Earth) frame reference for the model "E." The origin of this frame reference is fixed with respect to Earth, and positive is downward following the acceleration of gravity. The origin

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of this frame is fixed at the nominal center of mass, and the unit vectors spin in tandem with the body rotation. This second frame is the body frame reference Figure (1), which is entirely fixed within the rigid body[2].

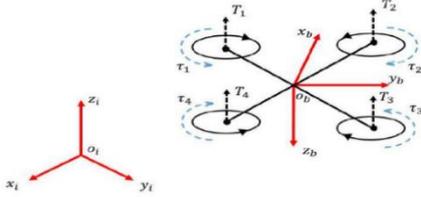


Fig. 1: body and Inertial Reference Frames

2.2 Direction Cosine Matrix [Rotation Matrix].

A rotation matrix, or DCM, is a mathematical representation of how the angular motion of a quadrotor is transformed from the body frame to the inertial frame. The Dual Channel Motion (DCM) is the result of a series of revolutions in which the Quadrotor is spun in three different directions: yaw (z-axis), pitch (y-axis), and roll (x-axis). Equation (1) is obtained by multiplying the rotation matrices for the yaw, pitch, and roll.

The Quadrotor's linear and angular positions in the body and inertial frames were referenced from [2] [8] [9] [10] and [15]. These equations can be expressed in the vector form as:

$$\begin{bmatrix} X^b \\ Y^b \\ Z^b \end{bmatrix} = \begin{bmatrix} c\theta \cdot s\psi & s\psi \cdot c\theta & -s\theta \\ c\psi \cdot s\phi \cdot s\theta - c\phi \cdot s\psi & s\phi \cdot s\psi \cdot s\theta + c\phi \cdot c\psi & c\theta \cdot s\phi \\ c\phi \cdot c\psi \cdot s\theta + s\phi \cdot s\psi & s\phi \cdot s\psi \cdot s\theta - c\psi \cdot s\phi & c\phi \cdot c\theta \end{bmatrix} \begin{bmatrix} X^G \\ Y^G \\ Z^G \end{bmatrix} \quad (1)$$

The transformation of the Quadrotor's angular velocities from the body frame coordinate to the inertial frame coordinate can be represented by the following equation [4]

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & s\phi \frac{s\theta}{c\theta} & c\phi \frac{s\theta}{c\theta} \\ 0 & c\phi & -s\phi \\ 0 & s\phi \frac{1}{c\theta} & c\phi \frac{1}{c\theta} \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \quad (2)$$

The gravitational force vector acting on the Quadrotor's CG in the inertia coordinate frame can be expressed as [5].

$$\overline{F_G^b} = \begin{bmatrix} -m \cdot g \cdot s\theta \\ m \cdot g \cdot c\theta \cdot s\phi \\ m \cdot g \cdot c\theta \cdot c\phi \end{bmatrix} \quad (3)$$

The rotational motion of the propeller-rotor combination generates a gyroscopic effect that acts on the Quadrotor in the body coordinate frame. The gyroscopic effect is contributed by the rotor's moment of inertia, the rotor's angular velocity, and the body attitude rate, which can be expressed by.

$$G^b = I_{rotor} \begin{bmatrix} q \\ p \\ 0 \end{bmatrix} \Omega = \begin{bmatrix} I_{rotor} \cdot q (\omega_1 - \omega_2 + \omega_3 - \omega_4) \\ I_{rotor} \cdot p (\omega_1 - \omega_2 + \omega_3 - \omega_4) \\ 0 \end{bmatrix} \quad (4)$$

The thrust from the propellers acting on the Quadrotor along the z-axis on the body coordinate frame (i.e., ZB) can be expressed as:

$$T^b = K_T (\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2) \quad (5)$$

$$T^b = U_1 \quad (6)$$

The roll moment for the Quadrotor along the x-axis on the body coordinate frame in cross configurations can be expressed as.

$$\tau_\phi = K_T \cdot l_y (-\omega_1^2 + \omega_2^2 + \omega_3^2 - \omega_4^2) \quad (7)$$

$$\tau_\phi = U_2 \quad (8)$$

The pitch moment for the Quadrotor along the y-axis on the body coordinate frame in cross configurations can be expressed as:

$$\tau_\theta = K_T \cdot l_x (-\omega_1^2 - \omega_2^2 + \omega_3^2 + \omega_4^2) \quad (9)$$

$$\tau_\theta = U_3 \quad (10)$$

The Yaw Moment for the Quadrotor along the y-axis on the body coordinate frame in cross configurations can be expressed as:

$$\tau_\psi = K_D (\omega_1^2 - \omega_2^2 + \omega_3^2 - \omega_4^2) \quad (11)$$

$$\tau_\psi = U_4 \quad (12)$$

3. Quadrotor model identification.

The system modelling component that was previously explained is expanded upon in this chapter. The model parameters will be estimated in this part using the physical system. This will help determine precise control system parameters in addition to increasing the simulated model's correctness [16].

1. Motor Moment of Inertia

The Motor moment of inertia about X, Y and Z axis, figure (2) defined with the following equations.

$$I_{XX_{motors}} = 4 \left(\frac{m_{motor} \cdot r^2}{4} + \frac{m_{motor} \cdot h^2}{12} \right) + 2 (m_{motor} \cdot L_{ym}^2 + m_{motor} \cdot L_{ym}^2) \quad (13)$$

$$I_{YY_{motor}} = 4 \left(\frac{m_{motor} \cdot r^2}{4} + \frac{m_{motor} \cdot h^2}{12} \right) + 2 (m_{motor} \cdot L_{xm}^2 + m_{motor} \cdot L_{xm}^2) \quad (14)$$

$$I_{ZZ_{motors}} = 4 \left(\frac{m_{motor} \cdot r^2}{4} + m_{motor} \cdot L_{zm}^2 \right) \quad (15)$$

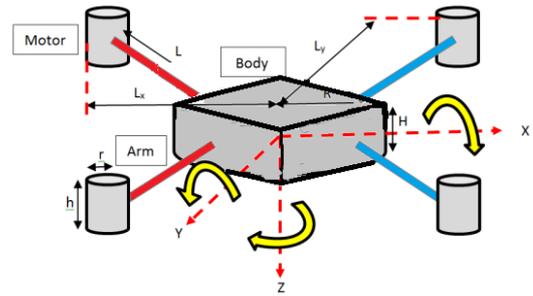


Fig. 2: Approximated Shapes and Dimensions for X-configuration Quad-rotor

2. Arms Moment of inertia

The Arms moment of inertia about X, Y, and Z axis, defined with the following equations.

$$I_{XX_{Arms}} = 4 \left(\frac{m_{arm} \cdot L^2}{12} + m_{arm} \cdot \left(\frac{L}{2} \right)^2 \right) \quad (16)$$

$$I_{YY_{arms}} = 4 \left(\frac{m_{arm} \cdot L^2}{12} + m_{arm} \cdot \left(\frac{L}{2} \right)^2 \right) \quad (17)$$

$$I_{ZZ_{arms}} = 4 \left(\frac{m_{arm} \cdot L^2}{12} + m_{arm} \cdot L_{arm}^2 \right) \quad (18)$$

3. Body Moment of Inertia

The body moment of inertia about X, Y, and Z axis defined with the following equations.

$$I_{XX_{body}} = \left(\frac{m_{body} \cdot w^2}{12} + \frac{m_{body} \cdot h^2}{12} \right) \quad (19)$$

$$I_{YY_{body}} = \left(\frac{m_{body} \cdot l^2}{12} + \frac{m_{body} \cdot h^2}{12} \right) \quad (20)$$

$$I_{ZZ_{motors}} = \left(\frac{m_{body} \cdot w^2}{12} + \frac{m_{body} \cdot l^2}{12} \right) \quad (21)$$

4. Measuring Quadrotor Properties.

For accurate representation of the quadrotor's mathematical model [7] and [12], its physical properties need to be measured and determined. A measuring tape and weighing scale, were used as the measuring equipment to determine the quadrotor's weight and length of its moment arms along the x and y-axes.

The results from the measurement of the quadrotor physical specifications are summarized in Table (1).

Table 1: Measurement of Quadrotor Physical Specifications.

Quadrotor Part	Mass (Grams)	X_Position cm	Y_Position cm	Z_Position cm
Motor + Propeller	4*66	16	16	22.5
Esc	4*25	7	7	10
Arms	4*48	11	11	11.25
Battery +Wire	165	-	-	-2
Electronic (KK2)	25	-	-	-
Receiver	10	-	-	-
Center Body wiring	63	-	-	-
Total component Mass	1.1	-	-	-

The Total Quadrotor moment of inertia Summarized in Table (2).

Table 2: Mass Moment of Inertia of the Quadrotor Using Analytical Method

Ixx kg.m ²	Iyy kg.m ² ,	Izz kg.m ²
0.0104	0.0104	0.008106

5. Quadrotor mathematical model Simulation.

The specifics of creating a reliable simulation environment are covered in this part. The equations of motion obtained in the System Modelling part (II) and the precise parameter values predicted in the Model Identification Part (III) are combined to create an accurate simulation. It takes all of these components to build a realistic simulation environment [15]. The steps required to construct a quadrotor simulator in MATLAB / Simulink are described in the following sections

1. Quadrotor Mathermatical model simulation

The "Body Euler Angle" toolbox in MATLAB and Simulink is used to simulate the resulting non-linear mathematical model in order to understand the behavior of the system. As illustrated in Figure (3), the system model is split into three sections. The trio consists of:

- 6-DOF rigid body dynamics Simulink Block
- Gravity subsystem Simulink Block
- BLDC Motors speeds subsystem Block as shown in figure (3)

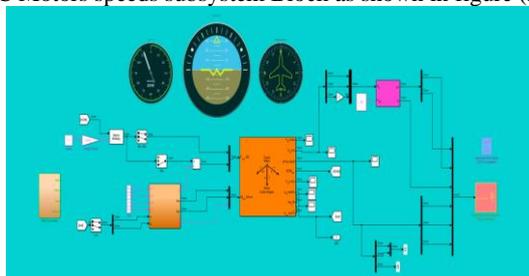


Fig. 3: Simulink Model of Quadrotor

2. 6-DOF Rigid Body Dynamics Block.

The Simulink block that is being used comes from the Simulink library's Aerospace toolbox. All of the fundamental equations of motion for rigid bodies in free space are included. The mass, the body's tensor of inertia, and the starting circumstances are the key variables in this block. Thrust forces and moments operating on the body along the three axes of the body-fixed frame are the inputs. Euler angles, angular rates, angular acceleration, and the direction cosine matrix for the transformation from Earth-fixed frame to body-fixed frame are among the outputs, together with location and velocity in the Earth-fixed frame and acceleration and velocity in the body-fixed frame.

3. Quadrotor Gravity Subsystem Block

This is just the application of the direction cosine matrix to convert the weight force acting along the z-direction from the Earth-fixed frame to the body-fixed frame figure (4). The total thrust force of the motors, which acts continuously along the z-direction in the body-fixed frame [12], is then subtracted from the z-component of the weight force in the body-fixed frame.

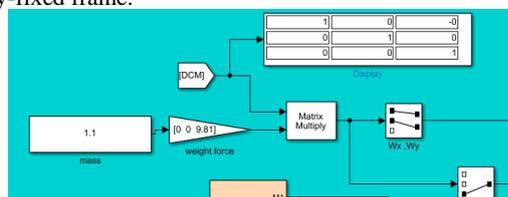


Fig. 4: Simulink Model of the Quadrotor Gravity Subsystem

4. BLDC Motors Speed Subsystem Block.

Equations (5) through (12) describe the thrust forces and moments produced by the motors subsystem. Rad per second rotational speeds are the inputs to the motors seen in Figures (5). The input U1 uses the total of the thrust forces to control the quadrotor's altitude. U2 uses the thrust forces from Motors 2 and 4 to control the roll angle of the quadrotor. U3 uses the thrust forces from Motors 1 and 3 to regulate the pitch angle of the quadrotor. U1, U2, and U3 control inputs are displayed in figure (5).

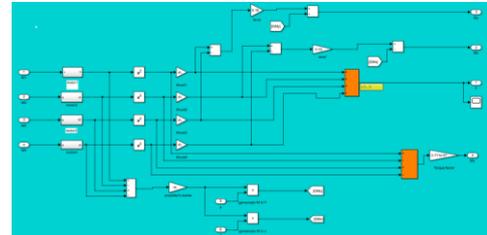


Fig. 5: Motor Subsystem Simulink block for Thrust, roll and pitch As seen in Figure (6), U4 is adjusting the yaw angle by measuring the torque differential between the two clockwise and counterclockwise turning motors.

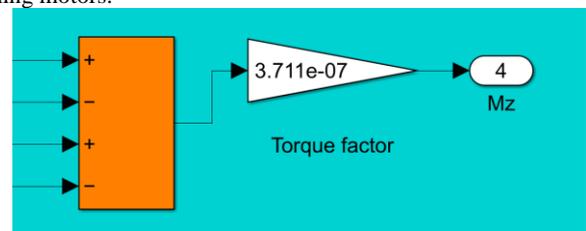


Fig. 6: Motor Subsystem Simulink block for yaw.

5. FlightGear subsystem.

The FlightGear subsystem is used for sending the position and attitude data to the FlightGear simulator which is used for visualization figure (7). The position needs to be converted from the Cartesian NED coordinates system to the spherical Geographic system. Some other minor conversions need to be done as well. The Aerospace blocks send data and Flightgear blocks receive these data from the Aerospace blockset in the Simulink library.

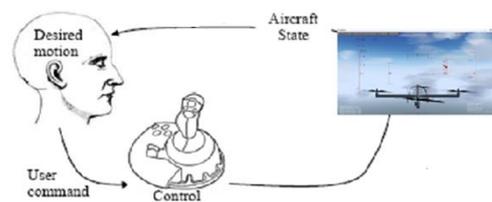
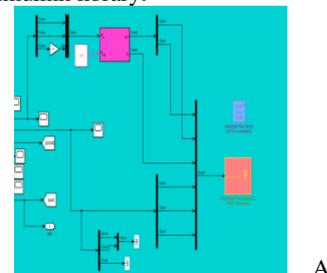


Fig. 7: FlightGear subsystem from the nonlinear model(A) and FlightGear visualization demonstration (B)

5 Simulation Results.

If all four motors are operating at the same speed (joystick only—no controller). the quadrotor will hover over the ground because the thrust coefficient multiplied by the total of the speeds causes the z-axis to vary in height without altering the x or y axes. The x, y, and z axes are displayed as changing in Figures (8) and (9).

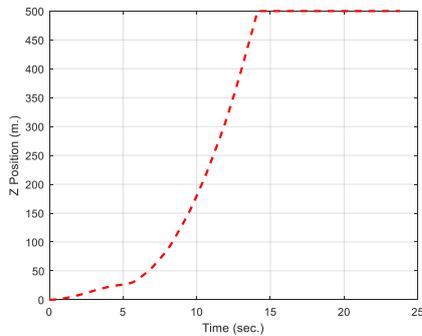


Fig. 8: Variation of Quadrotor Altitude

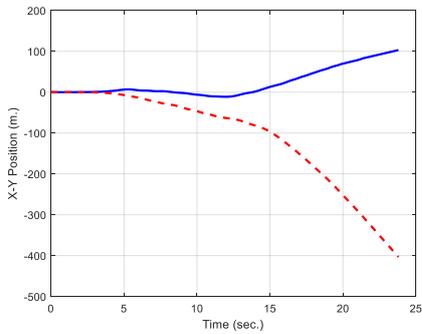


Fig. 9: Variation in x and y Axes

The roll, pitch, and yaw rotational angle changes as shown in Figure (10).

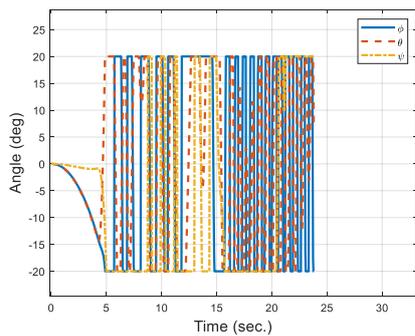


Fig. 10: Variation of the Angles Along the 3-axes

Previous Figures show that the quadrotor is unstable and that it cannot be flown with a joystick and a flight controller without one. The three movements that make up the attitude are roll, pitch, and yaw. Free flight The roll motion is generated by adjusting the roll angle about the x-axis using a joystick (with a PID controller in this case) [11] because of the difference in the speed motors' 2 and 3 with respect to 1 and 4. The angle (ϕ) with respect to the x-axis will go to the desired value as shown in Figure (11). At the same time, there is a shift in position in the y-direction, as shown in Figure (12).

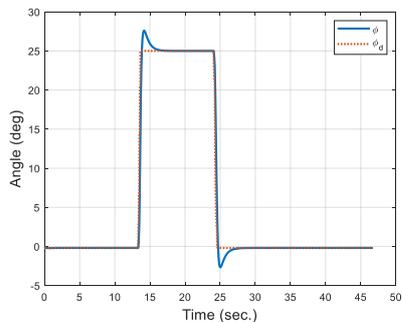


Fig. 11: Roll Angle (ϕ) Simulation Result Using joystick

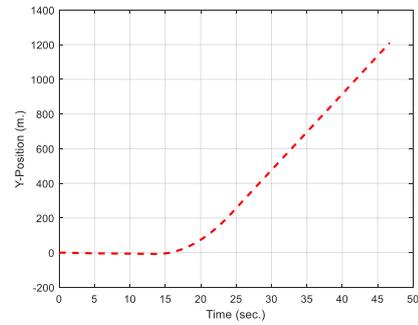


Fig. 12: Y-axis Simulation Result Using joystick

Using a joystick to adjust the pitch angle about the y-axis, which results in the pitch motion, causes the difference in speed between motors 3 and 4. The angle (θ) with respect to the y-axis will go to the desired value, as shown in Figure (13). At the same time, there is a shift in position in the x-direction, as shown in Figure (14).

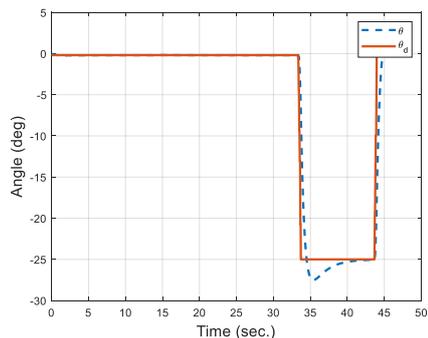


Fig. 13: Pitch Angle (θ) Simulation Result Using joystick

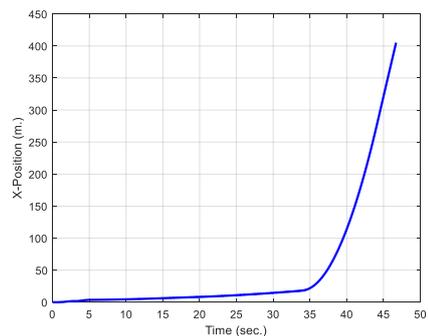


Fig. 14: X-axis Simulation Result Using joystick

The variation in motors 1 and 3's and motors 2 and 4's speeds yaw motion is produced by manipulating the yaw angle about the z-axis with a joystick. The angle (ψ) with respect to the z-axis will go to the desired value as shown in Figure (15). As can be observed in Figure (16), the XY position (16). As depicted in figure (17), the Attitude Angle (ϕ θ ψ).and the quadrotor simulation's 3D route, as seen in figure (18). From earlier Figures The quadrotor may be controlled to fly at any trajectory and at any altitude and attitude by utilizing a joystick and flightcontroller.

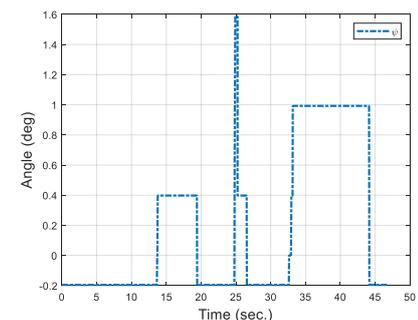


Fig. 15: Yaw Angle (ψ) Simulation Result Using joystick

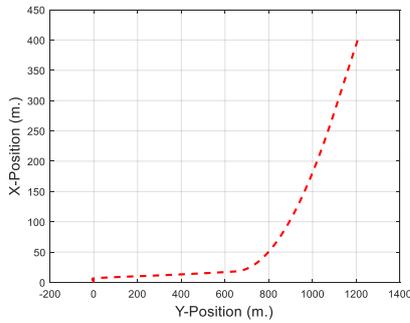


Fig. 16: X-Y Position Simulation Result Using joystick

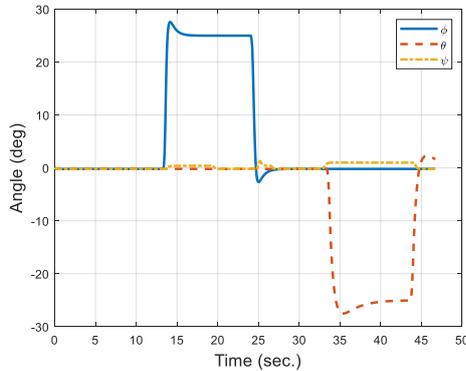


Fig. 17: All Attitude Angle Simulation Result Using joystick

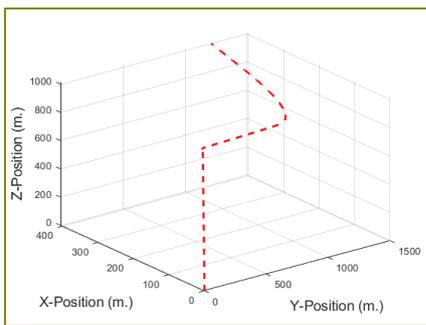


Fig. 18: 3D path Simulation Result Using joystick

6. Conclusion

This paper aims to construct and derive a mathematical model of a quadrotor. We have created a thorough model and simulation of a quadcopter drone with this paper. The quadcopter's propulsion system, flying dynamics, and physical structure are all depicted in depth in the model. Without the need for costly hardware prototypes, can be use this simulation environment to examine the behavior and performance of the quadcopter under a variety of operating situations using a joystick.

The overall conclusion of this Paper can be summarized as follow:

1. Rigid Body Dynamics Model: Using the Newton-Euler formal approach, the quadcopter's equations of motion were constructed, encapsulating the intricate relationship between translational and rotational dynamics.

2. Propulsion System Model, which included the four brushless DC motors of the quadcopter's thrust and torque generating.

3. Simulation Environment: Using MATLAB/Simulink, we created a high-fidelity simulation framework that let us see how the quadcopter moved, keep an eye on its internal states, and assess its performance indicators.

4. The moment of inertia of every part of the quadcopter was calculated, along with the motor and propeller's specifications, in order to calculate the thrust and torque of the motor.

5. The quadrotor becomes unstable after simulating with a joystick alone, and it is impossible to fly with a joystick without flight controller.

6. The quadrotor may be controlled and flown at any trajectory and at any altitude using a joystick and flight controller.

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