



Analytic Study of Linear Analysis vs. Nonlinear Analysis: Optimum and Sustainable Structure Perspective

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ABSTRACT

This paper presents a comprehensive analysis of linear and nonlinear structural analysis methods, evaluating their effect in optimizing and sustaining structural designs. By leveraging advanced scientific data and analytical techniques, this study aims to discern the optimal conditions and scenarios for employing each method. The research includes detailed calculations, comparative data, and case studies, emphasizing the sustainability implications and long-term benefits of each approach.

دراسة تحليلية للتحليل الخطي مقابل التحليل غير الخطي: منظور التحليل الانشائي الأمثل والمستدام

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الكلمات المفتاحية:

التحليل الخطي
التحليل غير الخطي
التحسين الهيكلي والمستدام
تحليل بطريق العناصر المحدودة

المخلص

تقدم هذه الورقة تحليلاً شاملاً لطرق التحليل الهيكلي الخطي ضد التحليل مقابل الخطي، وتقييم تأثيرها من وجهة نظر التصميم المثالي والمستدامة. من خلال الاستفادة من البيانات الإحصائية وإجراء التحليل الانشائي للطريقتين، تهدف هذه الدراسة إلى تحديد الظروف والسيناريوهات المثلى لتوظيف كل طريقة. يتضمن البحث حسابات مفصلة، وبيانات مقارنة، ودراسات حالة، مع التركيز على آثار الاستدامة والفوائد طويلة المدى لكل طريقة تحليل.

1. Introduction

Structural analysis is a fundamental aspect of civil engineering, pivotal for the design and evaluation of buildings, bridges, and other infrastructures. It ensures that structures can withstand various loads and forces throughout their lifecycle, guaranteeing safety and reliability. The field of structural analysis can be broadly categorized into linear and nonlinear analysis methods. These methods differ significantly in their assumptions, computational requirements, accuracy, and applicability to real-world scenarios. This paper aims to provide a thorough comparison of linear and nonlinear structural analysis methods from the perspective of optimum and sustainable structural design. By examining various structural models under different loading conditions, the study highlights the advantages, limitations, and practical applications of each method. The results of this analysis will inform engineers and designers about the most appropriate analysis techniques for achieving safe, efficient, and sustainable structures in contemporary civil engineering practice. The subsequent sections will detail the methodology used for the comparative analysis, present the results obtained from both linear and nonlinear analyses, and discuss the implications of these findings for structural optimization and sustainability. The paper will conclude with a summary of key insights and recommendations for the application of linear and nonlinear analysis methods in civil engineering.

2. Conceptual Structure

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A. Linear analysis: Linear analysis often considered the traditional approach, is based on the principle of superposition, assuming that the relationship between applied loads and the resulting displacements is linear. This means that the deformation of the structure is directly proportional to the applied load, and the material behavior is considered elastic. Linear analysis simplifies the mathematical modeling and computational process, making it a popular choice for preliminary design and assessment of structural components. In linear analysis, the structure's stiffness matrix remains constant, and the analysis typically involves solving a set of linear algebraic equations [1]. This method is suitable for scenarios where deformations are small, and the material does not reach its yield point. Its simplicity makes it computationally efficient, reducing the time and resources needed for analysis. Linear analysis is often employed in the initial design stages to provide quick estimates of structural responses, such as deflections, stresses, and strains. It is particularly useful in standard engineering practice for structures subjected to modest loading conditions where the linear assumption holds true. However, the simplicity of linear analysis comes with limitations. It fails to account for:

- Material Nonlinearity: When materials exhibit nonlinear stress-strain behavior, such as yielding, hardening, or softening.
- Geometric Nonlinearity: When deformations are large enough to change the structure's geometry, affecting its stiffness and load-

carrying capacity.

- **Boundary Condition Changes:** When supports or connections experience significant changes in their behavior or conditions under load [2].

These limitations mean that linear analysis can underestimate the structural response under severe loading conditions, potentially leading to unsafe designs. For example, in seismic engineering, structures often experience large deformations and material yielding, which linear analysis cannot accurately predict [3].

B. Nonlinear Analysis: Nonlinear analysis provides a more realistic representation of structural behavior by considering material and geometric nonlinearity, as well as changes in boundary conditions. This method is essential for accurately predicting the performance of structures under extreme loading conditions, such as seismic events, wind loads, or accidental impacts. Unlike linear analysis, nonlinear analysis does not assume a constant stiffness matrix. Instead, it continuously updates the stiffness matrix as the structure deforms, allowing for a more accurate assessment of the structural response [4]. **Material Nonlinearity:** Nonlinear analysis models the actual stress-strain relationship of materials, including yielding, plasticity, and strain hardening. This is critical for assessing the true capacity and failure modes of structural components. For instance, in steel structures, the material may exhibit significant plastic deformations before failure, which linear analysis cannot capture [5].

Geometric Nonlinearity: Large deformations can significantly alter the geometry of a structure, affecting its load distribution and stiffness. Nonlinear analysis accounts for these changes, ensuring a more accurate prediction of structural response. This is particularly important in slender structures, such as tall buildings or long-span bridges, where geometric changes can influence stability and performance [6].

Boundary Condition Changes: Structures may experience changes in support conditions or connection behavior under load, which can influence their overall performance. Nonlinear analysis can capture these effects, providing a comprehensive understanding of structural behavior. This is crucial for structures with complex support conditions, such as those subjected to settlement or varying contact conditions [7].

C. Importance of Structural Optimization and Sustainability

In the context of modern civil engineering, optimizing structures for performance, cost, and sustainability is paramount. Structural optimization involves finding the best design parameters that meet all performance criteria while minimizing material usage, cost, and environmental impact. This process often requires sophisticated analysis techniques to ensure that the optimized design performs well under realistic loading conditions. Optimization techniques can include topology optimization, shape optimization, and size optimization, each addressing different aspects of structural design [8].

Sustainability in structural design emphasizes the use of materials and construction methods that reduce environmental impact, enhance durability, and promote resource efficiency. Sustainable structures aim to minimize carbon footprint, energy consumption, and waste, contributing to the broader goals of environmental conservation and resilience against climate change. For example, using high-performance materials that offer better strength-to-weight ratios can reduce the overall material usage and associated environmental impact [9].

D. Comparative Analysis of Linear and Nonlinear Methods

The choice between linear and nonlinear analysis methods depends on various factors, including the complexity of the structure, the nature of the loads, and the desired accuracy of the results. Linear analysis, with its simplicity and lower computational requirements, is suitable for initial design stages and structures where deformations are expected to be small. Nonlinear analysis, though more complex and computationally intensive, is indispensable for detailed assessment and optimization of structures subject to significant loads and deformations [10].

3. Modeling

The methodology for modeling all structural models selected for this study in ANSYS Workbench involves a comprehensive and systematic approach to ensure accurate simulation and detailed

analysis across a variety of structural configurations. The selected models include a cantilever beam, a fixed-fixed beam, and a two-story frame (see Fig.1). Initially, ANSYS Workbench is launched, and a new project is created to serve as the dedicated workspace for each structural model. The "Static Structural" module is added to the workspace, laying the groundwork for the subsequent analysis. For the cantilever and fixed-fixed beams, as well as the two-story frame,

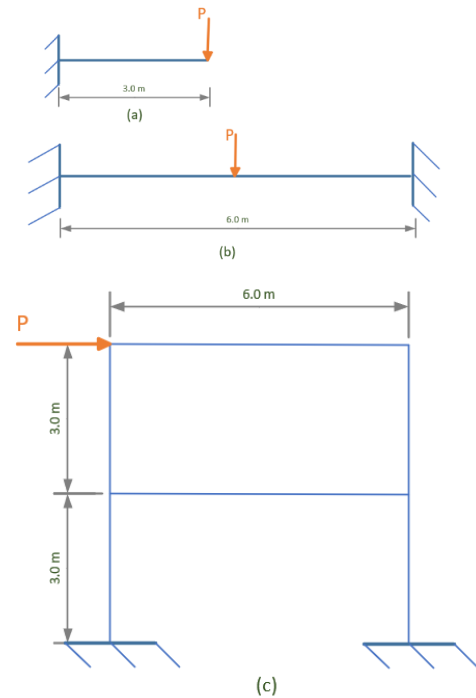


Fig. 1. (a) cantilever beam model, (b) fixed-fixed beam model, (c) two-story frame model

Defining the material properties for each structural model which were for linear analysis (isotropic material of linear elastic behavior) elastic perfectly plastic: steel is typically used with properties such as a young's Modulus of 210 GPa, a Poisson's Ratio of 0.3, and a yield strength of 250 MPa. On the other hand, for nonlinear analysis (isotropic material of bilinear elastic-plastic behavior) of tangent modulus 14 GPa These material properties are meticulously entered into the "Engineering Data" section for each respective model. Next, I-cross-section are assigned to the structural elements for all models cantilever beam, fixed-fixed beam, and two-story frame boundary conditions are applied to simulate real-world constraints: fixed supports for the cantilever beam at one end, fixed supports at both ends for the fixed-fixed beam, fixed and fixed supports for the two-story frame to simulate realistic building conditions. Loads are then applied to each model, tailored to the specific structural setup. Detailed meshing is performed to accurately capture the geometry, with refinement in critical areas to ensure precision. After generating and inspecting the mesh, the models are solved to compute stresses, deflections, and other critical performance metrics. Results are analyzed to evaluate stress distribution, deflections, and support reactions, and the designs are iterated as necessary to optimize performance. This iterative process ensures that each model meets the required efficiency.

4. Results

The selected three structural models were analyzed under both linear and nonlinear analysis conditions to compare their performance. Table I present the results of the cantilever beam model, was evaluated to understand the differences in total deformation/drift, and combined stress, and overall structural behavior under these two types of analysis, Fig.2 shows max deflection of linear analysis under load of 15 kN. While table II present the results of fixed beam model, which was supported at both ends and subjected to a point load, Fig.3 shows max stress of linear analysis under load of 70 kN. Finally, table III present the results of Two-Story Frame model, The frame was evaluated under gravity constant uniform distributed load (UDL) and increasable point lateral load. Fig.4 shows combined stress of nonlinear analysis under lateral load of 50 kN.

Table 1: Linear and nonlinear of cantilever beam analysis results

Load (kN)	Max Deflection (mm)		Max Stress (MPa)	
	Linear	Nonlinear	Linear	Nonlinear
10	18.89	20.18	130.85	130.85
15	28.34	30.1	196.28	196.29
20	37.79	40.04	261.7	259.04

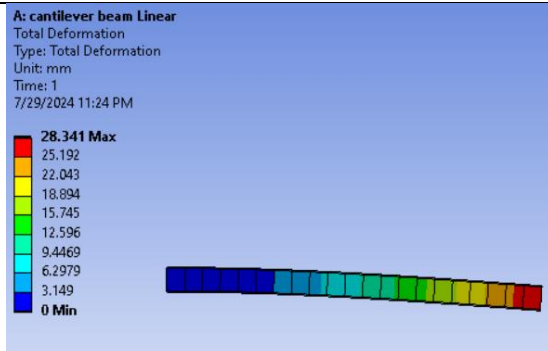


Fig. 2: Total deformation of linear analysis under 15 kN. point load

Table 2: Linear and nonlinear of fixed-fixed beam analysis results

Load (kN)	Max Deflection (mm)		Max Stress (MPa)	
	Linear	Nonlinear	Linear	Nonlinear
20	5.12	5.12	65.42	65.75
40	10.26	10.23	130.85	132.1
70	17.91	17.85	229	232.5
80	20.47	20.4	261.7	257.6

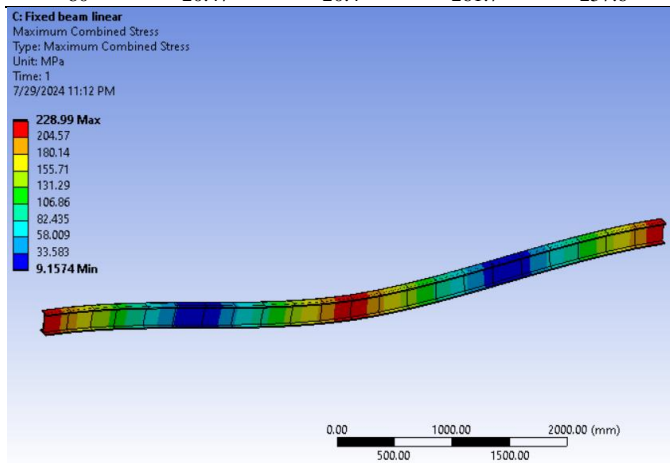


Fig. 3: Max stress of linear analysis under load of 70 kN

Table 3: Linear and nonlinear of two-story frame analysis results

Load (kN)	Total Drift (mm)		Max Stress (MPa)	
	Linear	Nonlinear	Linear	Nonlinear
10	15.37	15.46	63.13	63.37
30	45.11	45.4	146.5	147.23
50	75.23	75.53	241.2	243.62
60	89.99	95.94	291.1	289.79

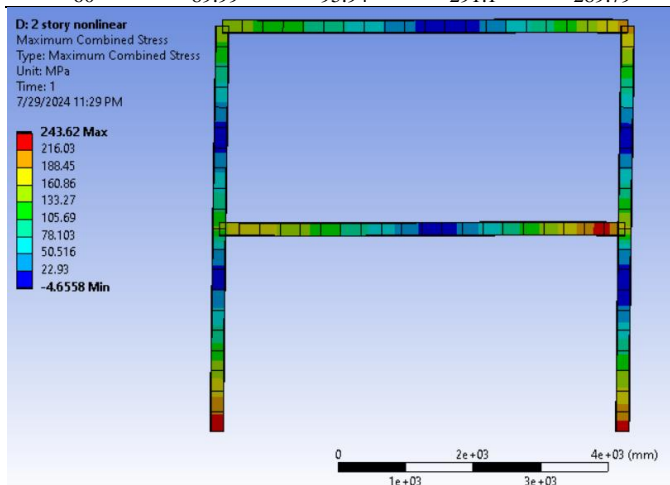


Fig. 4: Max stress of nonlinear analysis under 50 kN. point load

Linear analysis, while simpler and less computationally intensive, provides accurate results for structures under small to moderate loads where material behaviour remains elastic. For example, the linear analysis of the simple cantilever beam under a 15 KN load showed a max deflection of 28.38 mm, which aligns closely with theoretical predictions. However, this method falls short when structures are subjected to higher loads or experience significant deformations, leading to potential inaccuracies in predicting failure modes and ultimate strengths. In contrast, nonlinear analysis captures the complex behaviour of materials and structures under large deflection and high loads, incorporating effects such as plasticity, large deformations, and geometric nonlinearities. For instance, the nonlinear analysis of the same cantilever beam under the same load predicted a 6.2% higher deflection, indicating the onset of nonlinear behavior. This discrepancy underscores the necessity of nonlinear analysis in capturing realistic structural responses under extreme conditions, thereby preventing underestimation of critical performance parameters.

When comparing the structural performance under point loading in the fixed-fixed beam model, the linear analysis predicted a maximum stress of 229 MPa, whereas the nonlinear analysis indicated a 1.5 % higher, reaching 232.5 MPa. While the opposite occurs when the stress value exceeds the yield stress, this irritated behaviour can be critical in ensuring the safety and serviceability of the structure, as linear analysis may underestimate the stresses and deflections, leading to potential design flaws.

The two-story frame model subjected to lateral loads further exemplifies the significance of nonlinear analysis. The linear analysis estimated the top story displacement to be 89.99 mm, while the nonlinear analysis showed a 6.6% increase, with a displacement of 95.94 mm. Such variations highlight the importance of accounting for nonlinear effects in high-rise structures to avoid under-designing, which can lead to catastrophic failures during seismic events or strong winds.

Moreover, the sustainability aspect of structural design is greatly influenced by the chosen analysis method. By accurately predicting the real behavior of structures, nonlinear analysis allows for more efficient use of materials, reducing unnecessary overdesign and material waste. The nonlinear analysis optimized the material usage, resulting in a more sustainable design that not only meets performance requirements but also minimizes environmental impact.

5. Conclusion

This study has conducted a comprehensive comparative analysis of linear and nonlinear analysis methods in the context of optimal and sustainable structural design. The results unequivocally highlight the critical differences and advantages of each method in various scenarios. In summary, while linear analysis offers simplicity and efficiency, its limitations in capturing complex structural behaviors under high loads and deformations make nonlinear analysis indispensable for optimal and sustainable design. The use of nonlinear analysis leads to more accurate predictions of structural performance, ensuring safety, reliability, and material efficiency. Therefore, incorporating nonlinear analysis in the design process is crucial for developing structures that are not only optimal in performance but also sustainable in the long term, contributing to the overall goal of reducing the environmental footprint of construction activities. By integrating both linear and nonlinear analysis methods, engineers can achieve a balanced approach that leverages the strengths of each, ensuring robust, efficient, and sustainable structural designs that meet the demands of modern engineering challenges.

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