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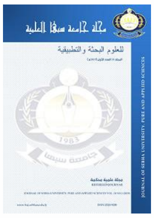

The study of some nuclear properties of even-even114-120Cd isotopes using interacting boson model-1

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ABSTRACT

Nuclear properties of even-even 114-120Cd are studied in the framework of the interacting boson model (IBM-1). In this study, we have determined the most appropriate Hamiltonian for the study of even-even Cd (114-120) to calculate, its energy levels. The values of parameters have been determined using the (IBM1) Hamiltonian which yield the best fit to the available experimental energy levels. In addition, the backbending (moment of inertia as a function of (ħω)) of the energy levels for each isotope was indicated through the experimental and calculated values of the energy levels. A simulation program with MATLAP-18 has been built to obtain the theoretical energy levels for Cadmium isotopes with neutron number N=16, 18, 20 and 22 up to spin-parity 〖14〗_1^+, 〖14〗_1^+, 〖14〗_1^+, 〖and16〗_1^+, respectively. The ratio of excitation energies of the first 4+ and the first 2+ excited states, R4/2 is also studied for the classification of symmetry of the nuclei. The results show that the 114-120Cd isotopes are γ – unstable rotor nuclei and they are dynamical symmetry O(6).

دراسة بعض الخصائص النووية لنظائر الكادميوم Cd120-114 الشفعية-شفعية بإستخدام نموذج البوزونات المتفاعلة -1

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الكلمات المفتاحية:
الإنحناء الخلفي
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النوى الشفعية-شفعية
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ظائر الكادميوم
نموذج البوزونات المتفاعلة-1.

الملخص

تمت دراسة الخصائص النووية لنظائر الكادميوم Cd120-114 الشفعية-شفعية بإستخدام نموذج البوزونات المتفاعلة (IBM-1). وقد قمنا في هذا البحث بتحديد الهاملتوني المناسب لدراسة نظائر الكادميوم (Cd114-120) الشفعية-شفعية لحساب مستويات الطاقة لها، ومن تم تحديد قيم البارامترات بإستخدام هاملتوني (IBM-1) والتي تعطي أفضل ملائمة لمستويات الطاقة المعملية المتاحة، كما تمت الإشارة إلى الإنحناء الخلفي (عزم القصور الذاتي كدالة في (ħω)) لمستويات الطاقة لكل نظير باستخدام قيم مستويات الطاقة المعملية والمحسوبة. تم تصميم برنامج محاكاة باستخدام (MATLAP-18) لحساب مستويات الطاقة لنظائر الكادميوم ذات العدد النيوتروني N=16, 18, 20, 22 حتى وصلنا للتماثل البرمي 〖14〗_1^+، 〖14〗_1^+، 〖14〗_1^+، 〖16〗_1^+ على التوالي.، كما وقدتمت دراسة نسب طاقات الإثارة للمستويات المثارة الأولى 2+ و 4+ لتصنيف تماثل النوى. وقد أظهرت النتائج أن نوى الكادميوم المدروسة ذات تناظر ديناميكي O(6) جاما الغير مستقرة.

Abbreviations and Acronyms
IBM: Interacting boson model, FDSM: The fermion dynamical symmetry model, PTSB: The pair-truncated shell model, U (5): Spherical limit, SU (3): Axially deformed shape limit O (6): , γ-unstable limit,

1. Introduction

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The low-lying states showing a rich collective structure in this region, were investigated extensively in terms of various models, such as the interacting boson model (IBM), the fermion dynamical symmetry model (FDSM)the pair-truncated shell model (PTSB) and the nucleon-pair shell model [1]. Many models have been developed to describe the collective properties of nuclei. Most of them, however, are usable only to a limited part of a major shell. One of the strengths of the Interacting Boson Model IBM [2, 5] represents significant step forward in understanding of nuclear structure. It offers a simple Hamiltonian capable of describing collective nuclear properties across a wide range of medium and heavy nuclei, and is found on rather general algebra group theoretical techniques which have also recently found application in problems in properties of nuclei [6, 7].

Cadmium isotopes are medium nuclei that the members of chain of nuclei around mass 114 and they represent an ideal case for studying the influence of the shape transition from spherical to deformed nuclei. The (IBM) presented by Arima and Iachello [2,5] and Casten [3] has become widely accepted as a manageable theoretical scheme, to predict and describe low-energy collective properties of several nuclei. In this model, the low-energy states of even-even nuclei are described in terms of interactions between s (J=0) and d (J=2) bosons [4].

(IBM-1) is the simplest version of this model, the nuclear properties are described by a system of a fixed number of bosons, where there is no distinction between proton bosons and neutron bosons; therefore, all states in IBM-1 are F-spin symmetric [8]. The most general two-body Hamiltonian within the boson space is expressed in terms of the generators of the group U (6), that is of the U (5), SU (3) and O (6) groups, associated with a spherical, axially deformed shape, and γ-unstable, respectively [9]. The phenomenon in which a plot of twice the moments of inertia versus the rigid value. where the nucleus then undergoes a phase transition from a superfluid state to a state of independent particle motion.

In this paper we determined the most appropriate Hamiltonian that is needed for present calculations of nuclei in the A = 114-120 region by the view of projection of IBM-1 parameters. To know some more information about the nucleus structures square of rotational frequency, for the various spin states has an S-shaped form, is called the backbending, the effect occurs due to the rapid increase of the moment of inertia with rotational frequency towards the, we have calculated moments of inertia of yrast states for even-even 114-120Cd versus the rotational frequency for the various spin states, in addition we calculate the energy ratios $R_{4/2}=E(4_1^+)/E(2_1^+)$ and $R_{6/2}=E(6_1^+)/E(2_1^+)$ are characteristics of different collective motions and would be investigated for O(6) (γ-unstable) symmetry of this nuclei.

IBM DESCRIPSION

The IBM is based on the well-known shell model and geometrical collective models of the atomic nucleus [10]. This model has to be able to describe nuclear properties such as spin energies of the lowest levels, decay probabilities for emission of gamma quants and multipole moments.

The IBM-1 was first introduced by Arima and Iachello [11]. In this model the collective properties are described in terms of pairs of nucleons paired to angular momentum L=0 and L=2, which are treated as bosons. The number of bosons depends on the number of active nucleons (particles or holes) outside close shell. The general formula of Hamiltonian [2] given:

$$H = \varepsilon \acute{n} d + a_0(\widehat{p}.\hat{p}) + a_1(\widehat{L}.\widehat{L}) + a_2(\widehat{Q}.\hat{Q}) + a_3(\hat{U}.\widehat{U}) + a_4(\widehat{V}.\hat{V}) \qquad (1)$$

Where ($\varepsilon=\varepsilon_d$) the boson energy in d-state and (a_0, a_1, a_2, a_3, a_4) are the strengths of pairing, angular momentum, quadrupole, octupole and hexadecpole interactions of each term in the equation.

This Hamiltonian can be expressed in terms of the generators of subgroup of U(6) that is U(5), SU(3) and O(6) as appearing in chains:

$$U(6) \supset U(5) \supset O(5) \supset O(3) \qquad (2)$$

$$U(6) \supset SU(3) \supset O(3) \qquad (3)$$

$$U(6) \supset O(6) \supset O(5) \supset O(3) \qquad (4)$$

In order to determine the behaviour of nuclei we compared it with three limits U(5) ,O(6) and SU(3).We must first examine a few appropriate Hamiltonian for the study of even-even Cd (114-120) through experimental data (E_J/E_2) that was agreed with O(6) limit, the eigenvalue for this limit was shown as [12,13]:

$$O(6): \quad E^{III} = K_3\big(N(N+4) - \sigma(\sigma+4)\big) + K_4\tau(\tau+3) + K_5L(L+1) \qquad (5)$$

Where K_3, K_4 and K_5 are other forms of the strength parameters and N is the total boson number, for the minimum momentum cases $= N$, it will be:

$$E^{III} = K_4\tau(\tau+3) + K_5L(L+1) \qquad (6)$$

Where K_4 and K_5 can be found by simulation program with MATLAP-18.

MOMENT OF INERTIA AND GAMMA ENEGY

The relation between the moment of inertia ($2J/\hbar^2$) and gamma energy E_γ is given by [14]:

$$2J/\hbar^2 = \frac{4I-2}{E_\gamma} \qquad (7)$$

While the relation between the ħω and E_γ is given by [12,14]:

$$\hbar\omega = \frac{E_\gamma}{\sqrt{(I+1} - \sqrt{(I-2)(I-1)}} \qquad (8)$$

RESULTS AND DISCUSSION

The evlution of the Nuclear structure, from a spherical to an axially deformed shape, of Cadmium isotopes in going from ^{114}Cd to ^{120}Cd was studied in the framework of the IBM-1 model. Cadmium isotopes (114-120) have atomic number Z=48 so they have one-hole proton boson and contain of (8,9,10,11) particle neutron bosons respectively.

By used simulation on (mathlab-18) program for IBM-1 Hamiltonian equation and applied the eq. (6) for γ - unstable group we have obtained the yrast band energy levels. The γ- unstable O(6) has ratios $R_{4/2}=E(4_1^+)/E(2_1^+)=2.5$ and $R_{6/2}=E(6_1^+)/E(2_1^+)=4.5$ .The variation of the values of these ratio as a function of even neutron numbers of cadmium isotopes for experimental values around (2.4,4) respectively, they are presented in Figure(1). The agreement between the experimental values and the calculated ones was very good, and these values were typical for the O(6) limit.

The calculated excitation energies (yrast band) for Cd$^{114-120}$ are presented in tables (1-4) as well as the experimental ones [15-18], and it has shown in fig (2) that the general agreement between experiment and calculation is rather good.

Tables (5-8) contains the experimental and calculated E_γ, (ħω) and (*2J/ħ²*) values. In order to know further about the nature of the backbending, we have plotted the E_γ as a function of (J) in fig (3), this figure appears at J=10 decreases sharply for E_γ value which indicate to backbending phenomena with increasing (J). The moment of inertia ($2J/\hbar^2$) and rotational frequency (ħω) are calculated from Eq. (7) and (8), therefor the moments of inertia are plotted versus the (ħω) as can be seen in Fig. (4). We notice that calculated moment of inertia is agreeable to experimental data and the backbending phenomena appear clearly. In general, the results were compared with previous experimental data and it was observed that they were on a good agreement.

Table 1: Energy levels for ^{114}Cd isotope

J^π	E_{exp}(KeV)	E_{cal}(KeV)
2_1^+	558.5	558.5
4_1^+	1283.7	1189.5
6_1^+	1990.3	1893.1
8_1^+	2669.3	2669.3
10_1^+	3143.3	3143.3
12_1^+	3711.3	3859.2
14_1^+	4604.2	4604.2

Table 2: Energy levels for ^{116}Cd isotope

J^π	E_{exp}(KeV)	E_{cal}(KeV)
2_1^+	513.5	513.5
4_1^+	1219.4	1155.5
6_1^+	2026.7	1925.9
8_1^+	2824.9	2824.9
10_1^+	3040.0	3040.0
12_1^+	3578.5	3701.4
14_1^+	4380.5	4380.5

Table 3: Energy levels for ^{118}Cd isotope

J^π	E_{exp}(KeV)	E_{cal}(KeV)
2_1^+	487.8	487.8
4_1^+	1164.9	1082.2
6_1^+	1935.9	1783.2
8_1^+	2590.9	2590.9
10_1^+	3017.9	3017.9
12_1^+	3579.0	3682.3
14_1^+	4367.0	4367.0

Table 4: Energy levels for ^{120}Cd isotope

J^π	E_{exp}(KeV)	E_{cal}(KeV)
2_1^+	505.9	505.9
4_1^+	1203.7	1155.6
6_1^+	2033.7	1949.0
8_1^+	2886.2	2886.2
10_1^+	3129.8	3129.8
12_1^+	3746.1	3884.5
14_1^+	4579.9	4682.1
16_1^+	5522.7	5522.7

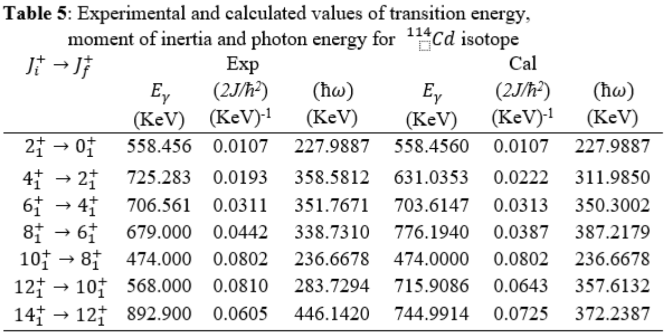

Table 5: Experimental and calculated values of transition energy, moment of inertia and photon energy for ^{114}Cd isotope

$J_i^+ \to J_f^+$	Exp			Cal		
	E_γ (KeV)	$(2J/\hbar^2)$ (KeV)$^{-1}$	$(\hbar\omega)$ (KeV)	E_γ (KeV)	$(2J/\hbar^2)$ (KeV)$^{-1}$	$(\hbar\omega)$ (KeV)
$2_1^+ \to 0_1^+$	558.456	0.0107	227.9887	558.4560	0.0107	227.9887
$4_1^+ \to 2_1^+$	725.283	0.0193	358.5812	631.0353	0.0222	311.9850
$6_1^+ \to 4_1^+$	706.561	0.0311	351.7671	703.6147	0.0313	350.3002
$8_1^+ \to 6_1^+$	679.000	0.0442	338.7310	776.1940	0.0387	387.2179
$10_1^+ \to 8_1^+$	474.000	0.0802	236.6678	474.0000	0.0802	236.6678
$12_1^+ \to 10_1^+$	568.000	0.0810	283.7294	715.9086	0.0643	357.6132
$14_1^+ \to 12_1^+$	892.900	0.0605	446.1420	744.9914	0.0725	372.2387

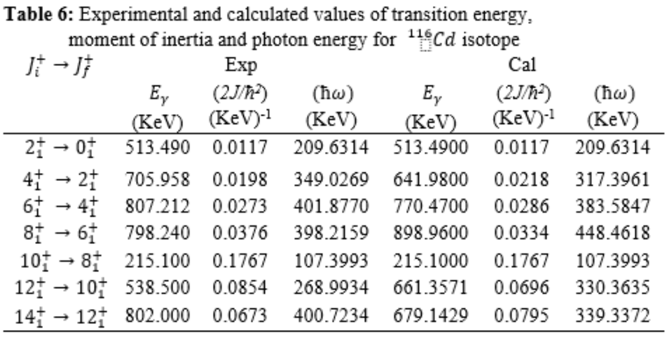

Table 6: Experimental and calculated values of transition energy, moment of inertia and photon energy for ^{116}Cd isotope

$J_i^+ \to J_f^+$	Exp			Cal		
	E_γ (KeV)	$(2J/\hbar^2)$ (KeV)$^{-1}$	$(\hbar\omega)$ (KeV)	E_γ (KeV)	$(2J/\hbar^2)$ (KeV)$^{-1}$	$(\hbar\omega)$ (KeV)
$2_1^+ \to 0_1^+$	513.490	0.0117	209.6314	513.4900	0.0117	209.6314
$4_1^+ \to 2_1^+$	705.958	0.0198	349.0269	641.9800	0.0218	317.3961
$6_1^+ \to 4_1^+$	807.212	0.0273	401.8770	770.4700	0.0286	383.5847
$8_1^+ \to 6_1^+$	798.240	0.0376	398.2159	898.9600	0.0334	448.4618
$10_1^+ \to 8_1^+$	215.100	0.1767	107.3993	215.1000	0.1767	107.3993
$12_1^+ \to 10_1^+$	538.500	0.0854	268.9934	661.3571	0.0696	330.3635
$14_1^+ \to 12_1^+$	802.000	0.0673	400.7234	679.1429	0.0795	339.3372

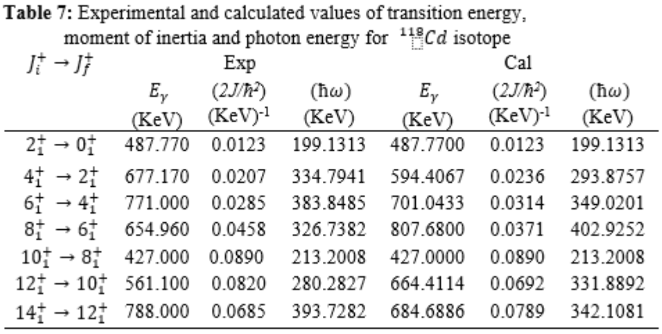

Table 7: Experimental and calculated values of transition energy, moment of inertia and photon energy for ^{118}Cd isotope

$J_i^+ \to J_f^+$	Exp			Cal		
	E_γ (KeV)	$(2J/\hbar^2)$ (KeV)$^{-1}$	$(\hbar\omega)$ (KeV)	E_γ (KeV)	$(2J/\hbar^2)$ (KeV)$^{-1}$	$(\hbar\omega)$ (KeV)
$2_1^+ \to 0_1^+$	487.770	0.0123	199.1313	487.7700	0.0123	199.1313
$4_1^+ \to 2_1^+$	677.170	0.0207	334.7941	594.4067	0.0236	293.8757
$6_1^+ \to 4_1^+$	771.000	0.0285	383.8485	701.0433	0.0314	349.0201
$8_1^+ \to 6_1^+$	654.960	0.0458	326.7382	807.6800	0.0371	402.9252
$10_1^+ \to 8_1^+$	427.000	0.0890	213.2008	427.0000	0.0890	213.2008
$12_1^+ \to 10_1^+$	561.100	0.0820	280.2827	664.4114	0.0692	331.8892
$14_1^+ \to 12_1^+$	788.000	0.0685	393.7282	684.6886	0.0789	342.1081

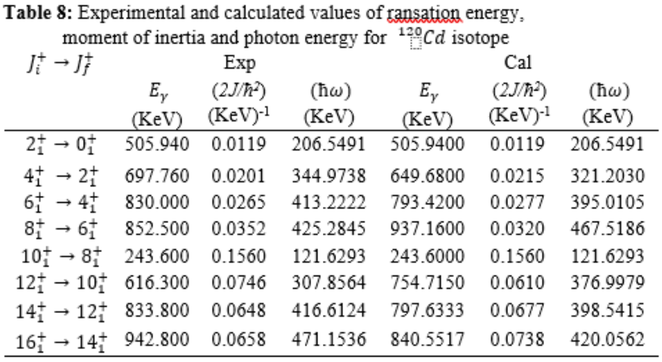

Table 8: Experimental and calculated values of ransation energy, moment of inertia and photon energy for ^{120}Cd isotope

$J_i^+ \to J_f^+$	Exp			Cal		
	E_γ (KeV)	$(2J/\hbar^2)$ (KeV)$^{-1}$	$(\hbar\omega)$ (KeV)	E_γ (KeV)	$(2J/\hbar^2)$ (KeV)$^{-1}$	$(\hbar\omega)$ (KeV)
$2_1^+ \to 0_1^+$	505.940	0.0119	206.5491	505.9400	0.0119	206.5491
$4_1^+ \to 2_1^+$	697.760	0.0201	344.9738	649.6800	0.0215	321.2030
$6_1^+ \to 4_1^+$	830.000	0.0265	413.2222	793.4200	0.0277	395.0105
$8_1^+ \to 6_1^+$	852.500	0.0352	425.2845	937.1600	0.0320	467.5186
$10_1^+ \to 8_1^+$	243.600	0.1560	121.6293	243.6000	0.1560	121.6293
$12_1^+ \to 10_1^+$	616.300	0.0746	307.8564	754.7150	0.0610	376.9979
$14_1^+ \to 12_1^+$	833.800	0.0648	416.6124	797.6333	0.0677	398.5415
$16_1^+ \to 14_1^+$	942.800	0.0658	471.1536	840.5517	0.0738	420.0562

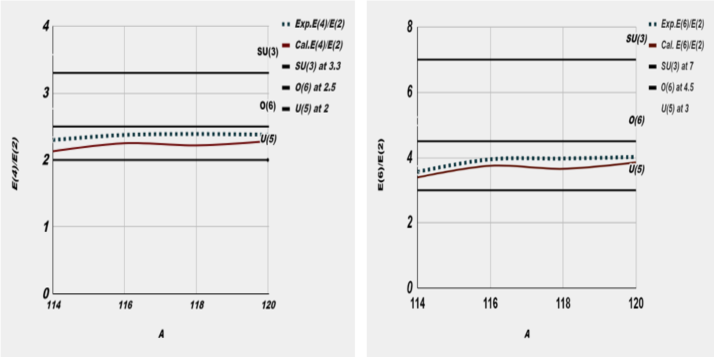

Fig. 1: The ratio E(4)/E(2) versus to the atomic number (A) and the ratio E(6)/E(2) versus to (A), respectively

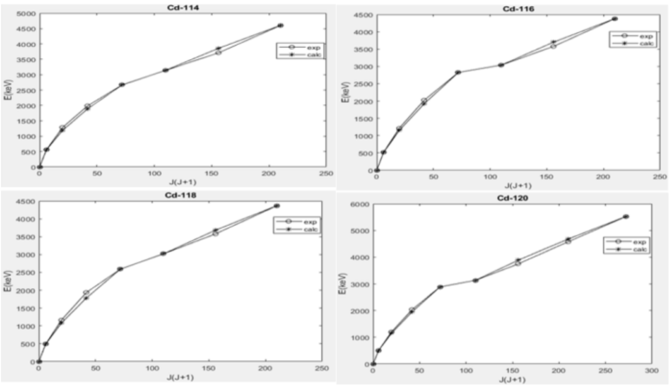

Fig. 2: Experimental and Calculated energy levels as a function of J(J+1) for $^{114-120}Cd$ isotopes

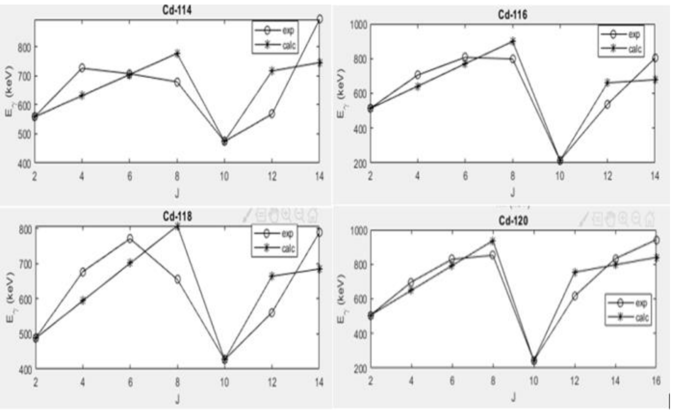

Fig. 3: Experimental and Calculated transition energy as a function of spin for $^{114-120}Cd$ isotopes

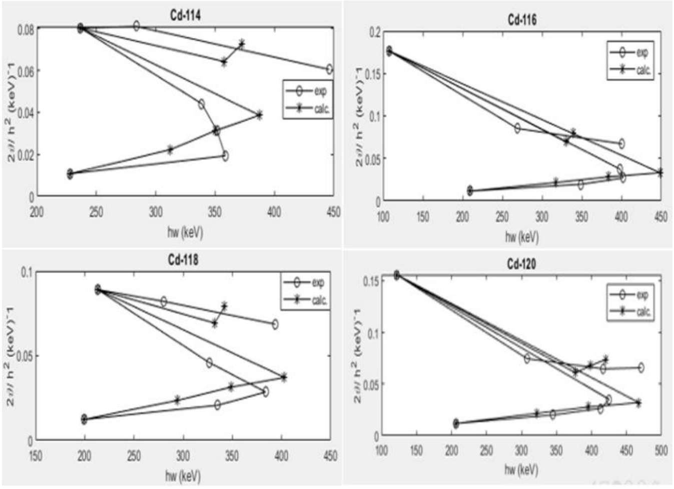

Fig. 4: Experimental and Calculated moment of inertia and photon energy for $^{114-120}Cd$ isotopes

Conclusion

In this work we carried out a systematic investigation of even-even $^{114-120}$cd isotopes by using IBM-1 framework, we have calculated the yrast band energy levels, addition to calculated moment of inertia and ħω, the calculated results are in very good agreement with the experimental data. The ratio (E_4^+/E_2^+) and (E_6^+/E_2^+) values indicate that the classify is γ- unstable. In these isotopes the backbending phenomena appears clearly at J =10.

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