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Arabic Font Desigin Using Quadratic Bezier-Like Curve

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ABSTRACT

This paper has constructed a linear quadratic Bézier-like curve with two shape parameters. The proposed work is to design the Arabic fonts by using Bezier-like curves. The focus of this study is to study the new functions of the Bezier-like curve and look at their application in designing Arabic fonts. Our new procedures involve a combination of quadratic and linear polynomial basis functions. The new functions of rational and non-rational Bezier-like curve are used to design Arabic fonts. The shape of the curve is modified as desired by simply altering the values of the shape parameters without changing the control polygon. The present study also looks at the extent of the approximation of the Arabic script design to its digitized image using the Bezier-like curves. The results of this study confirm that all generated functions can give the very visually pleasing shapes of the Arabic fonts similar to the original.

تصميم الخط العربي باستخدام منحنى شبيه-بيزير التربيعي

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الكلمات المفتاحية:
منحنى بيزير
كثيرات الحدود من الدرجة الثانية
الدوال الأساسية للمنحنى
المنحنيات الخطية-التربيعية لمنحنيات شبيه-بيزير
معلمات الشكل
الخطوط والحروف العربية

الملخص

في هذا البحث تم بناء منحنى خطي تربيعي يشبه منحنى بيزير بمعلمتين. العمل المقترح هو تصميم الخطوط العربية باستخدام منحنيات تشبه منحنيات بيزير. تركز هذه الدراسة على انشاء دوال جديدة لمنحنى شبه بيزير وإلقاء نظرة على تطبيقها في تصميم الخطوط العربية. تتضمن الدوال الجديدة مجموعة من دوال اساسية متعددة الحدود التربيعية والخطية. نستخدم الدوال الجديدة لمنحنى شبه بيزير النسبية وغير النسبية لتصميم الخطوط العربية. كرة العمل تمت بتعديل شكل المنحنى وذلك بتغيير قيم معلمات الشكل دون تغيير نقاط التحكم. تبحث الدراسة الحالية أيضًا في مدى تقريب تصميم الخط العربي إلى صورته الرقمية باستخدام منحنيات تشبه بيزير. تحيث ؤكد نتائج هذه الدراسة أن جميع الدوال التي تم إنشاؤها يمكن أن تعطي أشكال للخطوط العربية مشابهة للأصل.

Introduction

Bezier is one of the influent polynomial and an essential tool for interpolation. Its interpolating curve always lies within the convex hull and never oscillates wildly away from the control points. Bezier polynomial has several applications in designing, engineering, science, and technology, such as highway or railway route networks. The Arabic font design is one of these applications. Bezier curves can design the nice shapes and curves of Arabic fonts. Fonts are an essential part of any computer system, and two fundamental approaches for storing fonts in the computer are bitmap and outline [1], [2]. The shape represents the fonts with many advantages over the bitmap. The outlines may be arbitrarily translated, rotated, scaled, and clipped. Therefore, most modern desktop publishing systems are based on the outlined font [3]. The Arabic script is used for many languages and cultures, such as Urdu, Persian, Kurdish, etc., and uses various acceptable calligraphic styles. This condition makes the Arabic script complex and challenging [4]–[6]. There are many studies regarding designing Arabic fonts. Even some works are still in progress [7] whereby they try to achieve more elegant results by using different parametric techniques, segment subdivision, and cubic models. To lengthen the chain of studying and designing the

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Arabic font, we refer to [8]. Curves manipulation and construction is one of the main subjects in computer graphics. They have been used to make the model of objects in the real world that shaped non-uniform but smooth. Computer-Aided Geometric Design (CAGD) is a branch of applied mathematics concerned with algorithms to design soft curves and their efficient mathematical representations. Today, the Bezier curve is an essential part of almost every computer graphics illustration program and computer-aided geometric design (CAGD) system. It is used in many ways, from designing the curves and surfaces of automobiles to defining the shape of letters in type fonts. Jeok & Ong [9] have investigated the cubic Bezier-like curves in monotonicity, preserving, and constrained interpolation. The cubic Bezier-like polynomial functions are extensions of the Bezier polynomial functions with two additional parameters that can be used to modify the curve. In the 1990s, Zhang presented the Bezier-like curves, called C-Bezier, for the space $\Gamma = span\{1, t, sin\ t, cost\}$ [10]. In 2004, Chen e al. extended them to the general algebraic, trigonometric polynomial space [11]. This paper presents the linear-quadratic Bezier-like curve with two shape parameters, The effect on the shape of the curve by altering the value of the shape parameter is given as well. The quadratic linear rational Bezier-like curve is also given.

Preliminary Concepts

In this section, we define some basic concepts **Bernstein Polynomials**

The general formula of Bernstein polynomials of degree n is defined as

$$f_{i,n}(t) = \binom{n}{i} t^i (1-t)^{n-i}\ , \quad 0 \le t \le 1 \qquad (1)$$

for $i = 1,2, \dots, n$. Or similarly

$$f_{i,n}(t) = \frac{n!}{i!\,(n-i)!} t^i (1-t)^{n-i} \qquad (2)$$

where n is the degree of the curve. We can obtain the coefficients $\binom{n}{i}$ and observe the relationship between $t\ and\ i$ from Pascal's triangle; the exponents on the t term increase by one as i increases, and the exponents on the $(1-t)$ term decrease by one as i increases [12]. If we substitute $n = 1$ in the equation (2), we will get a Bernstein polynomials basis function at degree 1 and when $n = 2$, the function will be a quadratic polynomial basis function.

Bezier Curve

By using the Bernstein basis functions, the Bezier polynomial of degree n can be defined as:

$$r(t) = \sum_{i=0}^{n} f_{i,n}(t)\, p_i\,, \quad 0 \le t \le 1 \qquad (3)$$

with $p_i \in R^r (r = 1,2,3)$.

Control Points

In a font design project, determining the correct control points, which are points in two or more dimensions, plays an essential role in drawing the curve. We can determine the start and end points of the curve, but the middle control point will not be specific because the same curve with the same start and end point can be drawn with a different middle point, so there is no local control point this far. Still, we have to change the shape parameters; we observe that by drawing one character two times with different middle points. Also, we have to select corner points and decide the number of segments, and we can modify the curve by adding and moving the control points.

In general, our Bezier-like curve interpolation is the fixed interpolation which means that the shape of the interpolating curve is set for the given interpolating data and control polygon since the interpolating function is unique for the given control points. Therefore, to modify the shape of the interpolating curve, the control points need to be changed

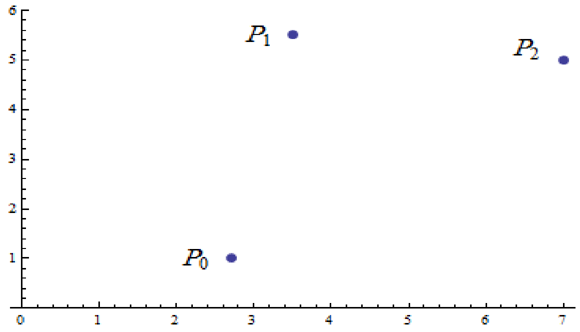

Fig. 1: Control points.

Fig. 1 shows the control points. We will have the control polygon if we connect the control points with line segments. Fig. 2 shows a control polygon comprising three control points associated with line segments.

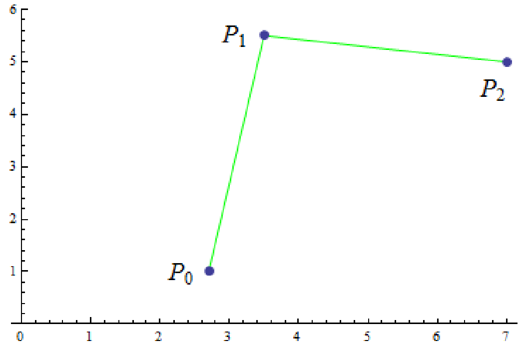

Fig. 2: Control polygon

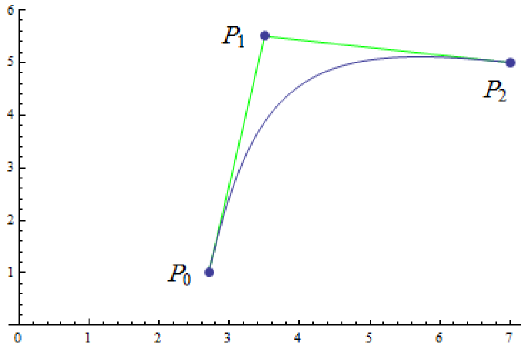

Fig. 3: Bezier curve of degree 2

Quadratic and linear Bezier-like curve

In this section, we will derive new functions from the general functions of the Bezier curve, which are a combination of quadratic and linear polynomial basis functions to use in our applications. To do that, we must have shape parameters such as γ, μ, which help us to know how the influence of the control points on the curve is.

Definition 1

For two arbitrarily selected actual values of γ and μ where $\gamma, \mu\ \epsilon[-2,1]$, the following three functions of $t(t\epsilon[0,1])$ are defined as a combination of quadratic and linear polynomial basis function with the control points $p_i (i = 0,1,2)$, and their Bezier-like curve is,

$$r(t) = \sum_{i=0}^{2} p_i f_{i,2}(t)\,, \qquad t\epsilon[0,1], \qquad \gamma, \mu\epsilon[-2,1] \qquad (4)$$

where the new functions are given by

$$f_{0,2}(t) = (1-\gamma t)(1-t)^2 \qquad (5a)$$
$$f_{1,2}(t) = t(t-1)(-2+\gamma(-1+t)-\mu t) \qquad (5b)$$
$$f_{2,2}(t) = \big(1-\mu(1-t)\big)t^2 \qquad (5c)$$

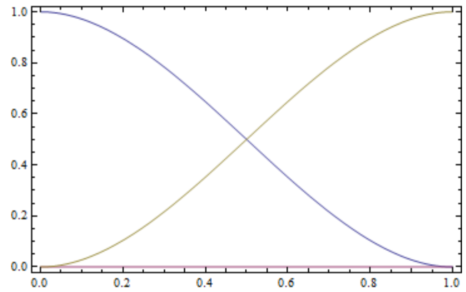

Fig. 4: Combination of quadratic and linear polynomial basis functions with $\gamma = -2\,, \mu = -2$

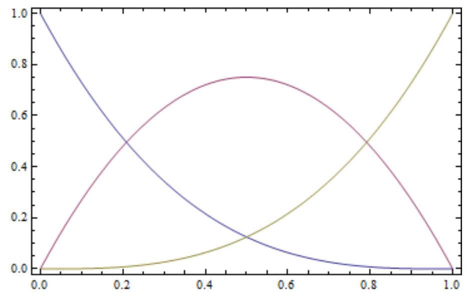

Fig. 5: Combination of quadratic and linear polynomial basis functions with $\gamma = 1, \mu = 1$

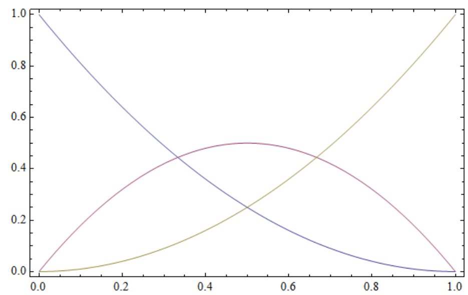

Fig. 6: Ordinary combination quadratic and linear polynomial basis functions with $\gamma = 0\,, \mu = 0$

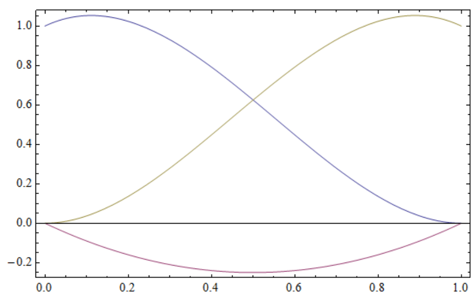

Fig. 7: Combination of quadratic and linear polynomial basis functions with $\gamma = -3\ \mu = -3$

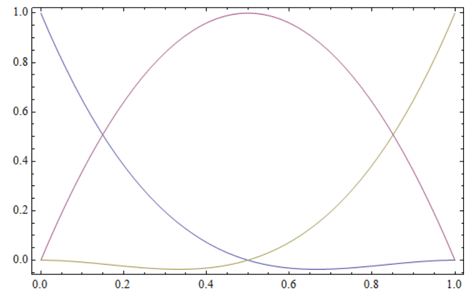

Fig. 8: Combination of quadratic and linear polynomial basis functions with $\gamma = 2, \mu = 2$

Figures 4, 5, 6, 7, and 8 show combination quadratic and linear polynomial basis functions, plotted as a function of t, which goes from 0 to 1. A critical characteristic of these polynomials is that for every value of t in the interval from 0 to 1, the sum of the functions evaluated at t is 1. The graph, taken together with the equation (4), shows the relative importance of the three control points as t varies. At $t = 0$, only $p_0(t)$ is nonzero. So only the point p_0 has to influence the curve segment, $r(t)$. At $t = 1$, the point p_2 takes over while the influence of the other control points disappears. In essence, p_0 and p_2 set the starting and ending points, respectively, of the curve segment. Once p_0 is established, the control point p_1 sets the direction of the curve as it leaves p_0. Similarly, the control point p_1, along with p_2, sets the direction of the curve as it nears the ending point, p_2. Thus, with three control points, it is possible to control the starting and end directions of the curve independently. This ability makes a quadratic Bezier-like curve a very flexible and efficient design tool. These properties will generally be proved in the following theorem.

Theorem 1

The functions (5) have the following properties:

- Non-Negativity: $f_{i,2}(t) \geq 0, \quad i = 0,1,2$
- partition of unity: $\sum_{i=0}^{2} f_{i,2}(t) = 1$
- Symmetry: $f_{i,2}(t;\gamma,\mu) = f_{2-i,2}(1-t;\mu,\gamma),\ for \quad i = 0,1,2$

Proof

- Non-Negativity:

For $t \in [0,1]$ and $\gamma, \mu \in [-2,1]$, it is observed that

$$f_i(t) \geq 0, i = 0,1,2$$

$$f_{0,2}(t) = (1-\gamma t)\,(1-t)^2 \geq 0,$$

because when we substitute in it $\gamma = -2$, we get

$$(1+2t)\,(1-t)^2 \geq 0$$

and when $\gamma = 1$, we get

$$(1-t)(1-t)^2 \geq 0,$$

then in both last functions when $t = 1$ they will be 0, and when $t = 0$ they will be 1.

$$f_{2,2}(t) = \big(1-\mu(1-t)\big)t^2 \geq 0,$$

because when we substitute it $\mu = -2$ we get

$$\big(1+2(1-t)\big)t^2 \geq 0$$

and when $\mu = 1$, we get

$$t^3 \geq 0$$

then in both last functions when $t = 1$ they will be *one*, and when $t = 0$ they will be 0.

$$f_{1,2}(t) = (-1+t)t(-2+\gamma(-1+t)-\mu t) \ \geq 0$$

because when we substitute it $\mu = -2$ and $\gamma = -2$, we get

$$1-((1+2t)(1-t)^2-\big(1+2(1-t)\big)t^2) \geq 0$$

and when we substitute in it $\mu = 1 and\ \gamma = 1$, we get

$$1-((1-t)(1-t)^2-t^3) \geq 0$$

then in both last functions, when $t = 1$, they will be 0, and when $t = 0$ they will be 0.

- partition of unity

It is easy to see that

$$\sum_{i=1}^{2} f_i(t) = (1-\gamma t)(1-t)^2 + 1 - t^3 - (1-t)^2(1+2t) + (1-\mu(1-t))t^2) = 1$$

- Symmetry

The control points, p_0, p_1, p_2 *and* $p_{2,}p_1, p_0$ define the same Bezier-like curve in different parameterizations, i.e.

$$r(t;\gamma;\mu;p_{0,}p_1,p_2) = r(1-t;\mu;\gamma;p_2,p_1,p_0)$$

for $0 \le t \le 1$ and the shape parameters $-2 \le \gamma, \mu \le 1$. So, it is clear that

$$f_{0,2}(t;\gamma,\mu) = f_{2,2}(1-t;\mu,\gamma),$$

because

$$f_{2,2}(1-t,\mu,\gamma) = \Big(1-\gamma\big(1-(1-t)\big)\Big)(1-t)^2$$
$$= (1-\gamma t)(1-t)^2$$
$$= f_{0,2}(t,\gamma,\mu)$$
$$f_{1,2}(t;\gamma,\mu) = f_{1,2}(1-t;\mu,\gamma),$$

because

$$f_{1,2}(1-t,\mu,\gamma) = (1-t)\big((1-t)-1\big)\Big(-2+\mu\big(-1+(1-t)\big) - \gamma(1-t)\Big)$$
$$= \ t(t-1)(-2-\mu t) - \gamma(1-t))$$
$$= \ t(t-1)(-2+\gamma(-1+t)-\mu t)$$
$$= f_{1,2}(t,\gamma,\mu)$$

The curve (4) is linear when $\gamma, \mu = -2$ because

$$(1+2t)(1-t)^2 + \big(1+2(1-t)\big)t^2 = 1$$

and

$$r'(0) = 0 \ \text{ at } \ \gamma = -2,$$

and

$$r'(1) = 0 \text{ at } \ \mu = -2.$$

Theorem 2

The quadratic linear Bezier-like curve has the following properties:

- Endpoint properties:

$$r(0) = p_0$$
$$r(1) = p_2$$
$$r'(0) = (2+\gamma)(\mathrm{p}_1 - \mathrm{p}_0)$$
$$r'(1) = (2+\mu)(\mathrm{p}_2 - \mathrm{p}_1)$$

- Symmetry p_0, p_1, p_2 *and* $p_{2,}p_1, p_0$ define the same quadratic Bezier-like curve in different parameterizations, i.e.,

$$r(t;\gamma;\mu;p_{0,}p_1,p_2) = r(1-t;\mu;\gamma;p_2,p_1,p_0) \ 0 \le t \le 1, -2 \le \gamma, \mu \le 1$$

- Convex hull Property: From the non-negativity and partition of unity of basis functions, the whole curve is located in the convex hull generated by its control points**.**

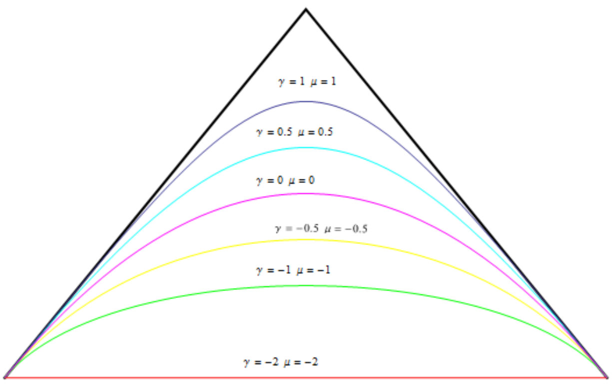

Fig. 9: The parameters γ, μ in different values

The parameters $\boldsymbol{\gamma}, \boldsymbol{\mu}$ control the shape of the curve (4). The quadratic linear Bezier-like curve $\boldsymbol{r(t)}$ gets closer to the control polygon as the value of the parameter increases gradually in $[-\mathbf{2}, \mathbf{1}]$.

When $\boldsymbol{\gamma}$ ***and*** $\boldsymbol{\mu}$ increase by the same value and $\boldsymbol{\gamma}, \boldsymbol{\mu}\boldsymbol{\epsilon}[-\mathbf{2}, \mathbf{1}]$, we interpret it as fullness; in figure 10, we only increase $\boldsymbol{\mu}$ and fix $\boldsymbol{\gamma}$, so the shape of the curve can be changed by the parameters $\boldsymbol{\gamma}, \boldsymbol{\mu}$. When $\boldsymbol{\gamma}, \boldsymbol{\mu} = -\mathbf{2}$ the curve will be linear as in figure 9, and when $\boldsymbol{\gamma}, \boldsymbol{\mu} < -\mathbf{2}$, the curve will come down to the negative trend as in figure11, and when $\boldsymbol{\gamma}, \boldsymbol{\mu} = \mathbf{0}$ the curve will be a standard quadratic Bezier-like curve, and when $\boldsymbol{\gamma}, \boldsymbol{\mu} > \mathbf{1}$ the curve will come outside the range of the function as in figure 11. Also, from figures 9 and 10, we can observe that the range of $\boldsymbol{\gamma}, \boldsymbol{\mu}$ between $[-\mathbf{2}, \mathbf{1}]$, while figure 11 shows that when $-\mathbf{2} > \boldsymbol{\gamma}, \boldsymbol{\mu} > \mathbf{1}$ how the curve goes looks.

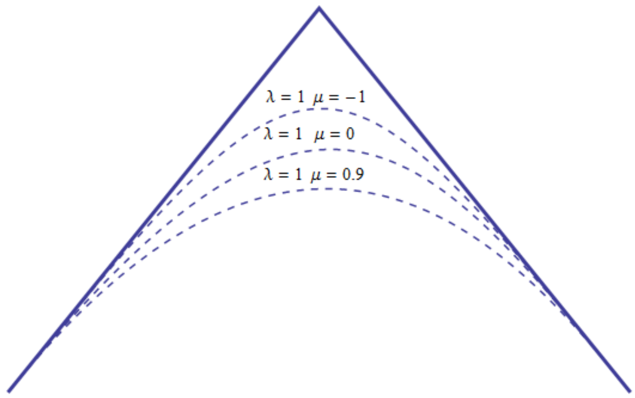

Fig. 10: The shape parameters by increasing μ and fix γ

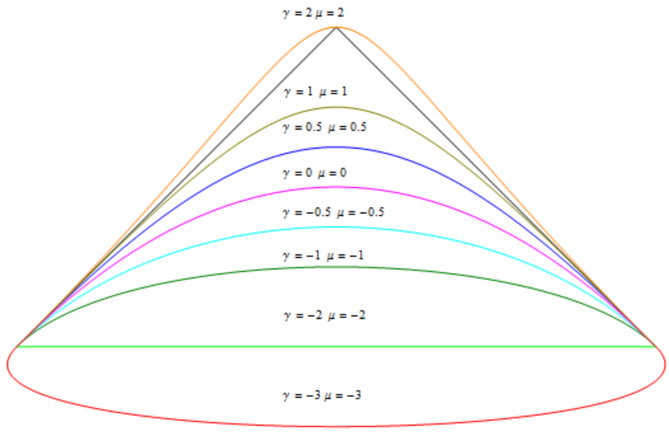

Fig. 11: The parameters γ, μ with values outside the range [-2,1]

Rational Bezier-like curve with a combination of quadratic and linear basis functions

After considering a non-rational Bezier-like curve with a combination of quadratic and basis functions, we will introduce a Rational Bezier-like curve that can represent a wide range of curves and more surfaces. We will use the same equations (5) to present a rational Bezier-like curve.

Definition 2

The general formula of a rational Bezier curve is,

$$r(t) = \frac{\sum_{i=0}^{n} f_{i,n}(t) H_i p_i}{\sum_{i=0}^{n} f_{i,n}(t) H_i} \ , 0 \le t \le 1 \ \ i = 0,1,2,\ldots,n \ , H_i > 0 \quad (6)$$

The idea is to use equations (5), a combination of quadratic and linear polynomial basis functions to introduce a new rational Bezier-like curve. To do that, we must have arbitrary values such as $\gamma, \mu, v1$ shape parameters that help us know how the influence of the control points on the curve is.

Definition 3

For two arbitrarily selected real values of γ and μ where $\gamma, \mu\epsilon[-2,1]$, the following three functions of t, ($t\epsilon[0,1]$) are defined as combination quadratic and linear basis functions, and their rational Bezier- like curve is,

$$r(t) = \frac{\sum_{i=0}^{2} f_{i,2}(t) H_i p_i}{\sum_{i=0}^{2} f_{i,2}(t) H_i}, \qquad 0 \le t \le 1, \qquad H_i > 0 \qquad (7)$$

where

$$f_{0,2}(t) = (1-\gamma t)(1-t)^2 \quad (8a)$$
$$f_{1,2}(t) = (-1+t)t(-2+\gamma(-1+t)-\mu t) \quad (8b)$$
$$f_{2,2}(t) = \left(1-\mu(1-t)\right)t^2 \quad (8c)$$

where

$$i = 0,1,2 \quad ,H_0 = 1, H_1 = v_1, H_2 = 1$$

when $\gamma, \mu = -2$, the curve of the quadratic and linear basis functions will be linear as in figure 4, and when $\gamma, \mu < -2$, the curve will come down to the negative trend as in figure 7, and when $\gamma, \mu = 0$, the curve will be a standard quadratic Bezier-like curve as in figure 6, and when $\gamma, \mu > 1$ the curve will come outside the rang of the function as figure 8.

The functions (8) have the following properties:

- Non-Negativity: $f_{i,2}(t) \geq 0, i = 0,1,2$
- partition of unity: $\sum_{i=0}^{2} f_{i,2}(t) = 1$
- Symmetry: $f_{i,2}(t;\gamma,\mu) = f_{2-i,2}(1-t;\mu,\gamma),\ for\ \ i = 0,1,2.$

Their rational Bezier-like curve also satisfies the properties:

$$r(0) = p_0 \quad (9a)$$
$$r(1) = p_2 \quad (9b)$$
$$r'(0) = (2+\gamma)(\mathrm{p}_1 - \mathrm{p}_0)v_1 \quad (9c)$$
$$r'(1) = (2+\mu)(\mathrm{p}_2 - \mathrm{p}_1)v_1 \quad (9d)$$

Proof

It is similar to the explanation of theorem 1.

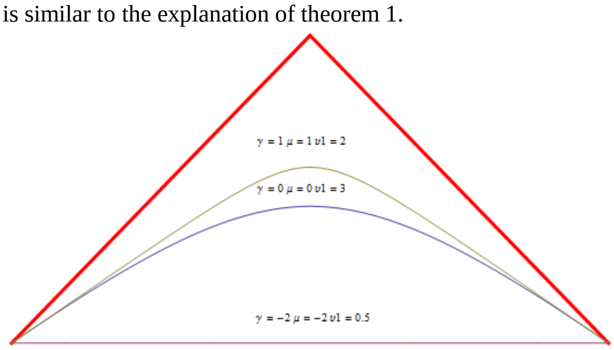

Fig. 12: The shape parameters $\boldsymbol{\gamma}, \boldsymbol{\mu}$ with values between the range $[-\mathbf{2}, \mathbf{1}]$ and $\boldsymbol{v1} > \mathbf{0}$.

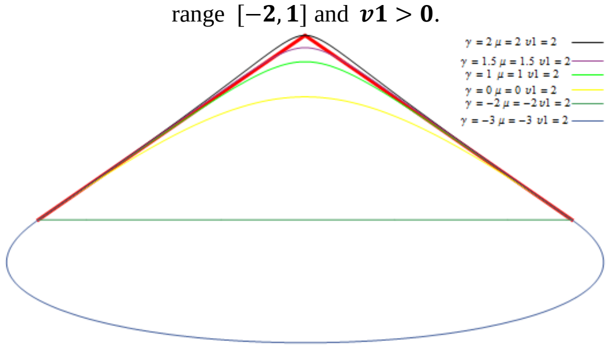

Fig. 13: The shape parameters $\boldsymbol{\gamma}, \boldsymbol{\mu}$, with values outside the range $[-\mathbf{2}, \mathbf{1}]$ and fix $\boldsymbol{v1}$

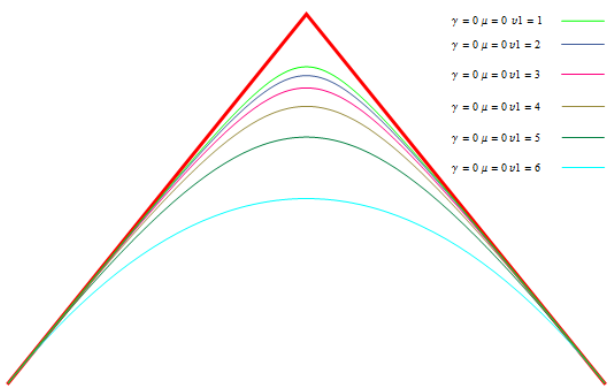

Fig. 14: The fix shape parameters $\boldsymbol{\gamma} = \mathbf{0}, \boldsymbol{\mu} = \mathbf{0}$, and increase the value $\boldsymbol{v1}$

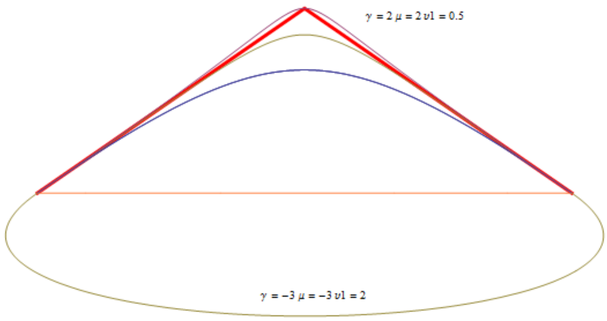

Fig. 15: The shape parameters $\boldsymbol{\gamma}, \boldsymbol{\mu}, \boldsymbol{v1}$ with different values outside the range $[-\mathbf{2}, \mathbf{1}]$.

The range of the values γ and μ and the value $v1$ can be interpreted geometrically; it is clear from figure 12 that when γ and μ increase by the same value and $\gamma, \mu \epsilon[-2,1]$ with different $v1$, we interpret it as fullness and the range is $[-2,1]$, while in figure 13 shows the shape of the curve when the values γ and μ are outside the curve. In contrast, in figure 14, we only increase $v1$ and fix μ and γ. Hence, it is clear that the shape of the parameter $v1$ can change the shape of the curve. For the set of $v1$ and changed values of μ and γ, we conclude that μ and γ affect the shape of the curve (see figure 15).

Designing Arabic Font Characters

Arabic characters are different from other characters such as English, Latin, etc. The Arabic alphabet, written from right to left, comprises 28 basic letters. There is no difference between the upper and lower cases or written and printed letters. Most letters connect directly to the letter, which immediately follows and gives the written text an overall cursive appearance. The Arabic script is rich in different font formats, and its cursive nature requires much more attention. For processing on drawing fonts which want to be at the outset crafts fee is required paper charts and then we appoint coordinate level on points relevant to the fonts. After that, it is needed fee Recta between these points, and then we craft in the form of the curves. If the format of the letter was inappropriate, we climate the site points until we have the appropriate form and post. These forces apply the Bezier-like curve with the appropriate programming language

Example 1.

This example of designing the Arabic font shows the Dal (د) character through a non-rational Bezier-like curve of equation (3), in which its functions are a combination of quadratic and linear polynomial basis functions (4) by using Computer Programming (see figures 16, 17 and 18).

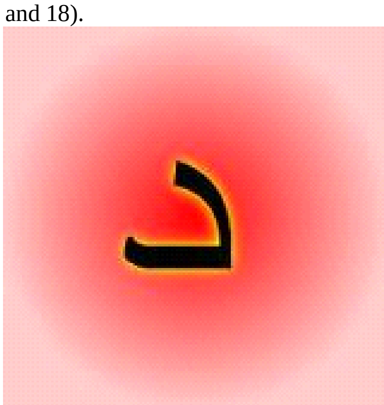

Fig. 16: The original Arabic font (Dal)

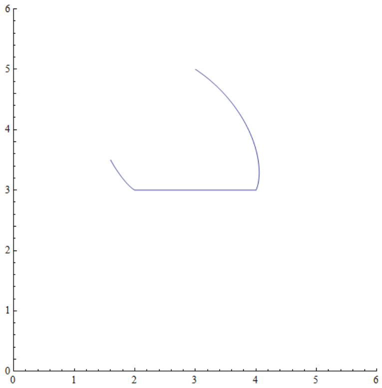

Fig. 17: The external curve of the Arabic font Dal.

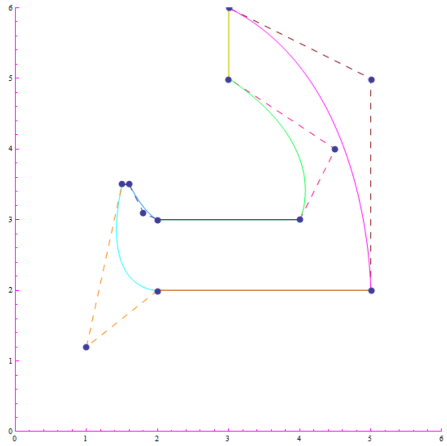

Fig. 18: Dal character through non-rational Bezier-like curve, whose functions combine quadratic and linear polynomial basis functions with its control polygon.

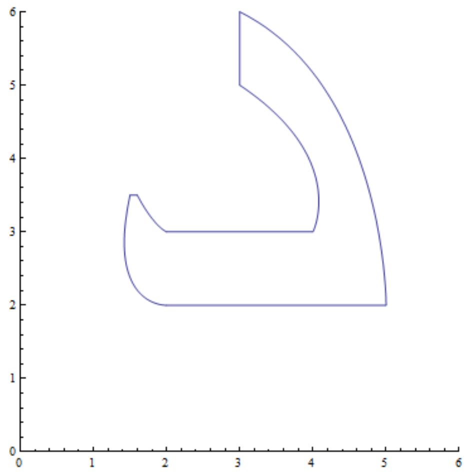

Fig. 19: Dal character through non-rational Bezier-like curve which functions are combination quadratic and linear polynomial basis functions.

Example 2.

This example of designing the Arabic font shows the Dal (د) character through a rational Bezier-like curve of equation (6), in which its functions are a combination of quadratic and linear polynomial basis functions (7) (see figures 20 and 21).

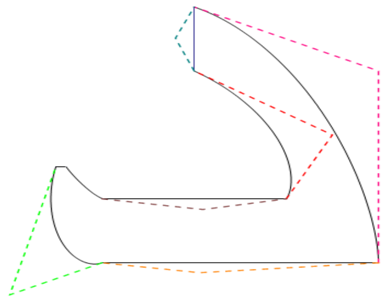

Fig. 20: Dal character through rational Bezier-like curve which its functions are combination quadratic and linear polynomial basis functions with its control polygons

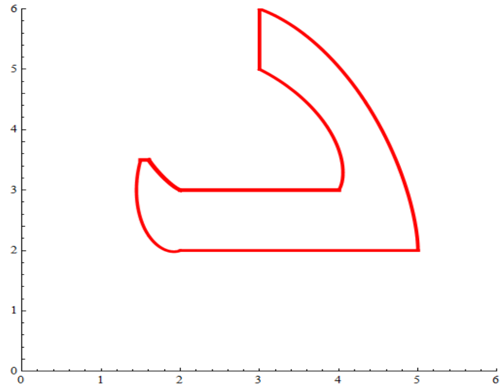

Fig. 21: Dal character through rational Bezier-like curve which functions are combination quadratic and linear polynomial basis functions.

Conclusion

Bezier curves have been applied in many applications, including describing object shapes. The critical aspect of using the Bezier curve is the proper selection of the control points. For that, the Bezier method and its properties have been reviewed. Our primary goal in this work is to design a font using new functions of Bezier, and their curve is a Bezier-like curve. To achieve that goal, we have derived new functions of Bezier functions: a combination of quadratic and linear polynomial basis functions. All the properties of the new functions have been manually proven and geometrically satisfied by using Computer Programming. We used rational and non-rational Bezier-like curves to design the Arabic fonts as an application. For example, the Dal is carried out. This study shows that we can get fonts similar to the original fonts by using rational and non-rational Bezier-like curves. Also, we can get the same font by using different Bezier-like curves and different functions of Bezier-like functions with varying points of control and shape parameters. We also conclude that the middle control point will not be specific because the same curve with the same start and end point can be drawn with a different middle point by changing the shape parameters. The results show that all the derived functions can give visually pleasing shapes.

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