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The Quadratic Trigonometric Bezier like Curve With two Shape Parameters

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ABSTRACT

The quadratic trigonometric Bezier model serves as a potent and valuable tool in computer-aided geometric design, boasting exceptional geometric properties. In this study, Novel functions are introduced by leveraging a trigonometric Bezier-like curve with adjustable shape parameters. Each curve segment is formed by three consecutive control points, providing precise control over shape by adjusting these parameters while keeping the control polygon unchanged. These curves exhibit superior proximity to the control polygon compared to quadratic Bezier curves across all shape parameter values, gradually approaching the control polygon as the shape parameter increases. Moreover, for practical application, the quadratic trigonometric Bezier-like curve was utilized to artfully design the Arabic word "الله" (Allah) and define flower contours, employing MATHEMATICA software. The results underscore the model's effectiveness in accurately representing intricate shapes, offering a versatile and efficient solution for diverse design challenges. Furthermore, the fusion of the quadratic trigonometric Bezier-like curve with shape parameters significantly augments design flexibility and adaptability, making it a valuable asset in computer graphics, geometric modeling, and artistic design across multiple domains.

منحنى شبه بيزير المثلثي التربيعي بمعلمتي

*مبروكة أبولقاسم محمد الكيلاني و المبروك حسين السنوسي عمر و إيمان عيسى الغناي أحمد و فاطمة الزهراء محمد علي أبوزيّان

قسم الرياضيات، كلية العلوم، جامعة سبها، ليبيا

الكلمات المفتاحية:
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منحنى شبه بيزير المثلثي التربيعي
معلمة الشكل
الخط العربي

الملخص

يعتبر النموذج التربيعي المثلثي لبيزير أداة قوية وقيمة في تصميم الهندسة المعززة بالحاسوب، حيث يتميز بخصائص هندسية ممتازة. في هذه الدراسة، نقدم وظائف جديدة باستخدام منحنى يشبه بيزير المثلثي مع معلمات الشكل قابلة للتعديل في مجال محدد. بحيث يتم تكوين كل قطعة من المنحنى من ثلاث نقاط تحكم متتالية، مما يتيح لنا التحكم بشكل دقيق في شكل المنحنى عن طريق ضبط معلمات الشكل مع الحفاظ على مضلع التحكم ثابتًا. تظهر هذه المنحنيات تقاربًا متفوقًا لمضلع التحكم مقارنة بمنحنيات بيزير التربيعية عبر جميع قيم معلمة الشكل، مع التقارب التدريجي للمنحنى نحو مضلع التحكم مع زيادة معلمة الشكل في المجال المحدد. وعلاوة على ذلك، كتطبيق عملي، استخدمنا منحنى بيزير التربيعي المثلثي لتصميم كلمة "الله" بشكل ماهر وتحديد الشكل الخارجي للزهور، باستخدام برنامج MATHEMATICA . حيث اكدت النتائج فعالية هذا النموذج في تمثيل الأشكال المعقدة بدقة، مما يوفر حلاً متعدد الاستخدامات وفعالاً لتحديات التصاميم المتنوعة. علاوة على ذلك، فإن دمج منحنى بيزير التربيعي المثلثي مع معلمات الشكل يعزز بشكل كبير مرونة وقابلية التكيف في عملية التصميم، مما يجعله أداة قيمة في مجالات متعددة، بما في ذلك الرسومات الحاسوبية والنمذجة الهندسية والتصميم الفني.

1. Introduction

The paper In curve design, users prefer easily understandable and manipulable curves. The influence of artistic design on the development of free-form curves is notable. A well-known type is the curved Bezier, initially intended for automotive body design and now a captivating area of applied mathematics. Bezier curves, rooted in Bernstein's borders, find wide-ranging applications in computer

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graphics and solid modeling. They are extensively used for crafting diagrams, line designs, and animations. Moreover, their versatility allows combination into slides and extension to higher dimensions for crafting Bezier surfaces. The significance of curve and surface design lies in their diverse applications across various domains, especially in Computer-Aided Geometric Design (CAGD) systems. Bezier curves, due to their mathematical properties facilitating easy manipulation and analysis, are widely used in CAGD, a highly valuable and popular discipline in applied mathematics.

In 2009, Han et al. introduced a novel cubic trigonometric Bezier curve with two shape parameters, showcasing a significant advantage in closeness to the control polygon compared to traditional Bezier curves by adjusting these parameters [1]. Trigonometric Bezier curves stand out for their integration of trigonometric functions, providing smoother and more flexible shape representations. This characteristic empowers designers with finer control over the curve's curvature, facilitating the creation of organic and visually appealing shapes. In 2012, an innovative quadratic Bezier curve was introduced, incorporating a shape parameter, offering designers increased versatility in shaping the curve as needed. The curve progressively aligns with the control polygon as the shape parameter value increases, allowing seamless transition between various curve configure rations and highly customizable representations [2]. A ground-breaking development emerged in 2018, introducing a cubic trigonometric Bezier-like curve enriched with a shape parameter, akin to the familiar cubic Bezier curve. This advancement provided enhanced control over the curvature, leading to smoother and more customizable shapes. Additionally, researchers explored the seamless connection between non-adjacent cubic trigonometric Bezier-like curves, aiming for continuous and smooth transitions between individual curves to construct complex and coherent shapes through their cohesive assembly [3]. In 2019, a remarkable advancement came with the introduction of a generalized Hybrid Trigonometric Bezier curve, demonstrating its potential in generating revolutionary engineering curves and symmetric rotation surfaces. By adjusting the shape parameters, designers could easily modify shapes, showcasing outstanding capabilities and promising to significantly enhance the design process in various fields [4]. In 2020, a significant breakthrough unveiled new trigonometric cubic Bezier-like curves, complemented by novel trigonometric cubic Bernstein-like basis functions. These curves, with a single shape parameter, adhered to essential continuity conditions, revolutionizing curve construction and enabling the creation of intricate shapes with enhanced ease and accuracy. The newfound versatility and efficiency of these curves open up new avenues for design and modelling in various domains, promising to influence and enrich the fields of computer graphics, font creation, and artistic design [5].

This paper introduces a quadratic trigonometric Bezier-like curve with adjustable shape parameters and explores its applications, including designing Arabic words and intricate shapes. The main goal is to create new functions for this curve type while studying Bezier curves and investigating quadratic trigonometric Bezier-like curves.

1. Preliminary concepts

In this section, the fundamental concepts essential for understanding the topic will be introduced. These concepts serve as a foundation for the ensuing discussion, elucidating key terms and theories.

2.1. Bernstein basis

The general formula of the ith Bernstein polynomials is defined as

$$B_{i,n}(t) = \binom{n}{i} [(1-t)]^{(n-i)} t^i, t \in [0,1].$$

For $i = 1,2, \dots n$, where n is the degree of the curve [5].

2.2. Control Points

The precise positioning of control points significantly influences the curve's shape. While the start and end points can be determined, the middle control point remains more flexible, as different middle points can produce the same curve with identical start and end points. Hence, there is no local control point in this regard. To achieve desired variations in the curve, the shape parameters need to be altered, the selection of corner points and the determination of segment numbers further impact the curve's appearance. Moreover, the curve can be easily modified by adding or repositioning the control points. These factors collectively contribute to the versatility and adaptability of the curve, empowering designers with creative control over the final outcome. [6].

A control polygon serves as a vital tool in shaping objects, consisting of a sequence of control points arranged in space. When these control points are connected by straight lines, the resulting polygon is referred to as the control polygon. This geometric construct holds significant importance in computer graphics, design, and various modeling applications. By manipulating the positions of the control points, designers can easily alter the shape of the object or curve, allowing for creative and intuitive control over its final form. The control polygon provides a visual representation of how the control points influence the object's appearance, making it easier to understand and adjust the design. It serves as a fundamental element in many geometric modeling techniques, facilitating the creation of smooth curves, surfaces, and shapes in diverse fields such as computer-aided design (CAD), computer graphics, and animation. [7].

2.3. Bezier Curve

The Bezier curve is a fundamental concept in the field of computer graphics and geometric modeling. It was introduced by Pierre Bezier in the 1960s and has since become a widely used technique for designing smooth curves and surfaces. Bezier curves are defined by a set of control points that dictate the curve's shape and behavior. With their intuitive control and versatility, Bezier curves find applications in various domains, including computer-aided design, animation, and font creation. In this introduction, the key properties and applications of Bezier curves are explored, shedding light on their significance in modern design and visualization. The Bezier curve defined by control points $P_i \in \mathbb{R}^N$, is the nth-degree polynomial given by

$$r(t) = \sum_{i=0}^{n} B_{i,n}(t) P_i \text{ for } t \epsilon [0,1].$$

The multipliers at the points in the above equation are called Bernstein basis and are defined as

$$B_{i,n}(t) = B_i^n(t) = \binom{n}{i} t^i (1-t)^{n-i} \text{ [7].}$$

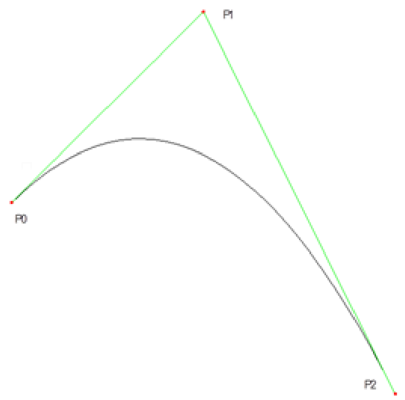

Figure. 1: Bezier curve of degree 2.

2.4. The shape parameters

Let three P_0, P_1, P_2 control points, a family of generalized quadratic trigonometric Bezier-like curves can be generated by the shape parameters α, β take on different values in their respective value ranges. Due to the fact that each generalized quadratic trigonometric Bezier-like curve provides three shape parameters, the shape of a quadratic trigonometric Bezier-like curve can be adjusted flexibly by modifying its shape parameters [7].

2. The main results

In our endeavour to enhance the versatility of Bezier curves for practical applications, the aim is to derive new functions that combine quadratic trigonometric and linear polynomial basis functions. These innovative functions will play a pivotal role in our various applications. To achieve this, shape parameters, denoted as α and β, are introduced, enabling us to gauge the influence of the control points on the curve's shape. By leveraging these shape parameters, the curves can be precisely controlled and tailored to meet specific design requirements, providing us with a powerful and flexible tool for curve manipulation and customization.

Definition1

For two arbitrarily selected real values of α and β, where $\beta \in [0,1]$, the following four functions of t ($t \in [0,1]$) are defined as quadratic trigonometric Bezier basis functions with two shape parameters α and β:

$$r(t) = \sum_{i=0}^{2} p_i B_{i,2}(t)\,, \qquad t\epsilon[0,1],\ \ \alpha,\beta\epsilon[0,1] \qquad (1)$$

where

$$B_{0,2}(t) = (1-\alpha t)\cos\left(\frac{\pi t}{2}\right)^2,$$
$$B_{1,2}(t) = \frac{1}{2}[\beta + (\alpha-\beta)t + (\beta-(\alpha+\beta)t)\sin(\pi t)] \qquad (2)$$
$$B_{2,2}(t) = \big(1+\beta(t-1)\big)\sin\left(\frac{\pi t}{2}\right)^2$$

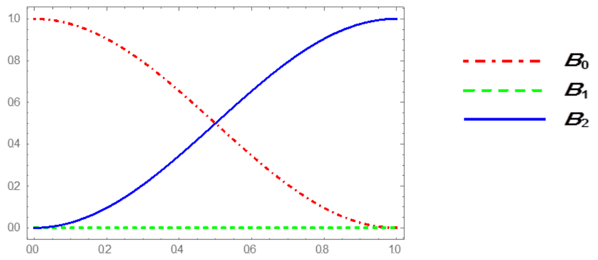

Figure.2: Combination quadratic trigonometric and linear polynomial basis functions with $\alpha = 0, \beta = 0$.

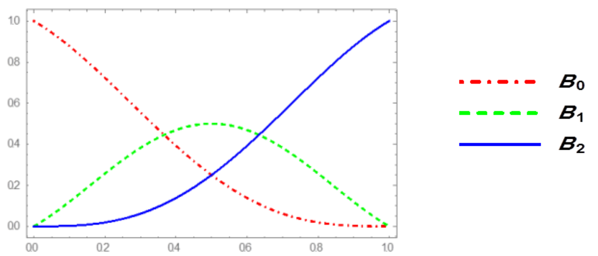

Figure. 2: Combination quadratic trigonometric and linear polynomial basis functions with $\alpha = 1, \beta = 1$.

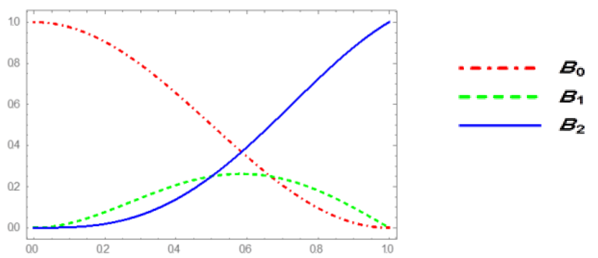

Figure. 3: Combination quadratic trigometric and linear polynomial basis functions with $\boldsymbol{\alpha = 0, \beta = 1}$.

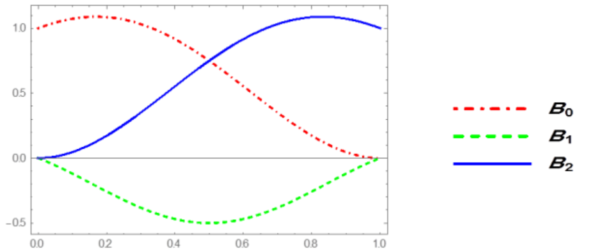

Figure. 4: Combination quadratic trigometric and linear polynomial basis functions with $\alpha = -1, \beta = -1$.

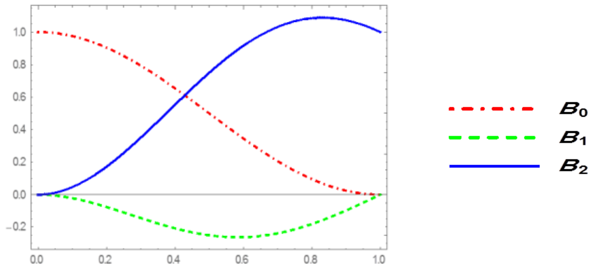

Figure. 5: Combination quadratic trigometric and linear polynomial basis functions with $\boldsymbol{\alpha = 0, \beta = -1}$.

Figurers **1**, **2**, **3** and **6** showcases the combination of quadratic trigonometric and linear polynomial basis functions plotted as a function of t, ranging from 0 to 1. A significant characteristic of these polynomials is that, for any value of t within the interval [0, 1], the sum of the functions evaluated at t is always 1. The graph, in conjunction with equation (1), reveals the varying importance of the three control points as t changes. At $t = 0$, only $p_0(t)$ is nonzero, so only the point p_0 has to influence the curve segment, $r(t)$. At $t = 1$, the point p_2 takes over while the influence of the other control points disappears. In essence, p_0 and p_2 set the starting and ending points, respectively, of the curve segment. Once p_0 is established, the control point p_1 sets the direction of the curve as it leaves p_0. Similarly, the control point p_1, along with p_2, sets the direction of the curve as approaches the ending point, p_2. Thus, with just three control points, it becomes possible to independently control the starting and ending directions of the curve, providing a powerful and concise means of curve manipulation.

The remarkable flexibility and efficiency of a quadratic trigonometric Bezier-like curve as a design tool stem from its unique ability. This distinct characteristic enables the curve to attain a wide range of shapes and behaviors, making it highly adaptable to various design requirements. These exceptional properties will be demonstrated and validated in the ensuing theorem.

Theorem1

The new basis functions of degree 2, associated with the shape parameters α, β, have the following properties:

i. Non-Negativity: $B_{i,2}(t) \geq 0, \quad i = 0,1,2$
ii. Normalization: $\sum_{i=0}^{2} B_i(t) = 1$
iii. Symmetry:
$B_{i,2}(t;\alpha,\beta) = B_{2-i,2}(1-t;\alpha,\beta), for\ i = 0,1,2$

Their Bezier-like has the following properties:

$$r(0) = p_0 \qquad (3)$$

$$r(1) = p_2 \qquad (4)$$

$$r'(0) = \alpha(p_1 - p_0) \qquad (5)$$

$$r'(1) = \beta(p_2 - p_1) \qquad (6)$$

Proof

1) For $t \in [0,1]$ and $\alpha, \beta \in [0,1]$, then

2) $\left((1-\alpha t)\cos^2\left(\frac{\pi t}{2}\right)\right) \geq 0$,

because when $\alpha = 0$ then

$$\cos^2\left(\frac{\pi t}{2}\right) \geq 0$$

and when $\alpha = 1$ then

$$(1-t)\cos^2\left(\frac{\pi t}{2}\right) \geq 0$$

therefore, in both last functions when $t = 1$ they will be 0 and when $t = 0$ they will be 1.

3) $\left(1+\beta(t-1)\right)\sin^2\left(\frac{\pi t}{2}\right) \geq 0$, because when $\beta = 0$ then

$$\sin^2\left(\frac{\pi t}{2}\right) \geq 0,$$

and when $\beta = 1$

$$\left(1+(t-1)\right)\sin^2\left(\frac{\pi t}{2}\right) \geq 0$$

then in both last functions when $t = 1$ they will be 0 and when $t = 0$ they will be 1.

4) $\frac{1}{2}[\beta + (\alpha-\beta)t + (\beta-(\alpha+\beta)t)\sin(\pi t)] \geq 0$,

because when $\alpha = 0$ and $\beta = 0$, then

$$1-\left((1-\alpha t)\cos^2\left(\frac{\pi t}{2}\right) - \left(1+\beta(t-1)\right)\sin^2\left(\frac{\pi t}{2}\right)\right) \geq 0,$$

and when $\alpha = 1$ and $\beta = 1$ then

$$1-\left((1-t)\cos^2\left(\frac{\pi t}{2}\right) - \left(1+(t-1)\right)\sin^2\left(\frac{\pi t}{2}\right)\right) \geq 0,$$ then

in both last functions, when $t = 1$ they will be 0 and when $t = 0$ they will be 0.

5) $\sum_{i=0}^{2} B_i(t) = 1$,

since $\left((1-\alpha t)\ \cos^2\left(\frac{\pi}{2}t\right) + \frac{1}{2}[\beta + (\alpha-\beta)t + (\beta-(\alpha+\beta)t)\sin(\pi t)] \ + \left(\left(1+\beta(t-1)\right)\sin^2\left(\frac{\pi}{2}t\right)\right)\right) = 1.$

6) Symmetry p_0, p_1, p_2 and p_2, p_1, p_0 define the same trigonometric curve in different parameterizations, i.e.,

7) $r(t;\alpha;\beta;p_0 p_1, p_2) = r(1-t;\beta;\alpha;p_2, p_1, p_0)$

Where $0 \leq t \leq 1, 0 \leq \alpha, \beta \leq 1$. Then

1. $B_{0,2}(t;\alpha,\beta) = B_{2,2}(1-t;\beta,\alpha)$, because

$$B_{2,2}(1-t,\beta,\alpha) = \left(1+\beta((1-t)-1)\right)\sin^2\left(\frac{\pi}{2}(1-t)\right)$$

$$= (1+\beta-\beta t-\beta)\left(sin^2\left(\frac{\pi}{2}\right) - \sin^2\left(\frac{\pi}{2}\right)t\right)$$

$$= (1-\beta t)\left(1-cos^2\left(\frac{\pi}{2}\right) - \left(1-cos^2\left(\frac{\pi}{2}\right)t\right)\right)$$

$$= (1-\beta t)\left(cos^2\left(\frac{\pi}{2}\right)t\right) = B_{0,2}(t,\alpha,\beta).$$

2. $B_{1,2}(t;\alpha,\beta) = B_{1,2}(1-t;\beta,\alpha)$, because

$$B_{1,2}(1-t,\beta,\alpha) == \frac{1}{2}[\beta + (\alpha-\beta)(1-t) + (\beta-(\alpha+\beta)(1-t))sin(\pi(1-t)]$$

$$= \frac{1}{2}[\beta + (\alpha-\beta) - (\alpha-\beta)t + (\beta-(\alpha+\beta)-(\alpha+\beta)t)sin(\pi-\pi t)]$$

$$= \frac{1}{2}[\beta + (\alpha-\beta) - (\alpha-\beta)t + (\beta-(\beta+\alpha)-(\beta+\alpha)t)sin(\pi-\pi t)]$$

$$= \frac{1}{2}[\alpha + (\beta-\alpha) - (\beta-\alpha)t + (\alpha-(\beta+\alpha)-(\beta+\alpha)t)sin(\pi)cos(\pi t) + \cos(\pi)sin(\pi t)] = B_{1,2}(t,\alpha,\beta)$$

The curve (3.1) is linear when $\alpha, \beta = 0$ because

$$\left(1+\cos^2\frac{\pi}{2}t\right)/2 + \left(1-\cos^2\frac{\pi}{2}t\right)/2 = 1.$$

And $r'(0) = 0$ at $\alpha = 0$.

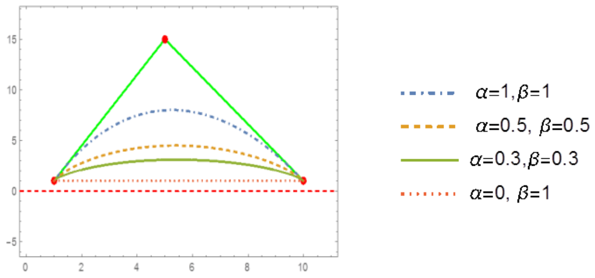

Figure. 6: The shape parameter α, β with values between the rang [0,1].

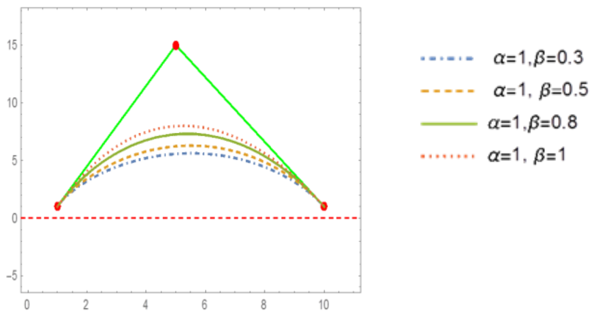

Figure.7: The parameters by increasing **β** and fixing **α**.

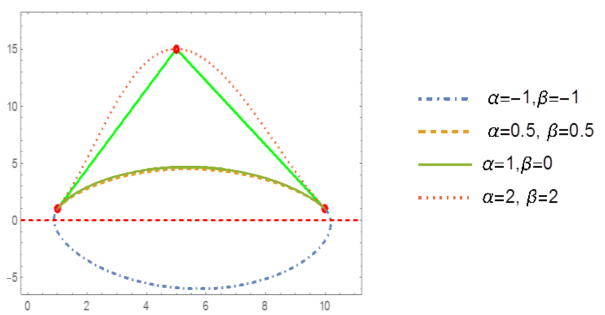

Figure. 8: The parametersα , β with values outside the range [0,1].

Figurer **7** Upon increasing α and β by the same value, where $\alpha, \beta \in [0,1]$, this increment can be interpreted as controlling the fullness of the curve. In Figurer 8, the observed experiment, β is increased while α remains fixed, illustrating how the curve's shape can be altered by manipulating the parameters α and β. Specifically, when α and β both equal -1, the curve takes a linear form, as depicted in Figurer 4. Conversely, when α and β are both set to 0, the curve transforms into an ordinary quadratic Bezier-like curve, as seen in Figurer 1. Notably, Figurer **7** provides further insights, showcasing that the range of α and β spans between [0,1], underscoring the versatility and control that these parameters offer in shaping the curve.

3. Arabic Script

Arabic script presents a unique and intricate challenge, setting it apart from other fonts like English. Consisting of 28 fundamental letters, written from right to left, the Arabic alphabet relies on multiple segments that closely relate to the letter's shape. Notably, most letters directly connect to the succeeding letter, imparting an overall cursive look to written text. In generating Arabic script, two key factors play a crucial role: the careful positioning of control points and the precise determination of segment numbers. These elements hold utmost significance in achieving the graceful and flowing appearance characteristic of Arabic writing [8]. Drawing the script requires the control points for all segments that determine the shape of the letter. To adjust the shape of the letter must change the inner points for all segments.

4. Methodology

The process of creating fonts involves several important steps:

1) Firstly, paper charts are used to outline the design of the fonts. On these charts, relevant points are designated with specific coordinate levels.
2) Next, straight lines are drawn between these points, forming the initial structure of the curves.
3) If the letter's shape is not yet satisfactory, further adjustments are made by modifying the points until the desired form is achieved. This iterative process ensures the creation of appropriately shaped letters in the font.Now apply these steps on the quadratic trigonometric Bezier-like curve with the appropriate programming language, and this study uses the MATHEMATICA software.

5. Design Arabic Font

In the forthcoming sections, the process of designing an Arabic font using MATHEMATICA software will be illustrated. To achieve the desired modifications in the curves, two key methods will be employed: first, by adding and adjusting control points, and second, by manipulating the shape parameters. These actions allow us to finely tune the curves, ensuring precise control over the font's appearance and achieving the desired artistic outcome.

Example 1

The MATHEMATICA software will be employed to design the out face of a flower using a quadratic trigonometric Bezier-like curve. This curve is characterized by a combination of quadratic and linear polynomial basis functions (2). With the aid of MATHEMATICA, we can seamlessly integrate these functions to create an aesthetically pleasing and intricate representation of the flower's outer shape.

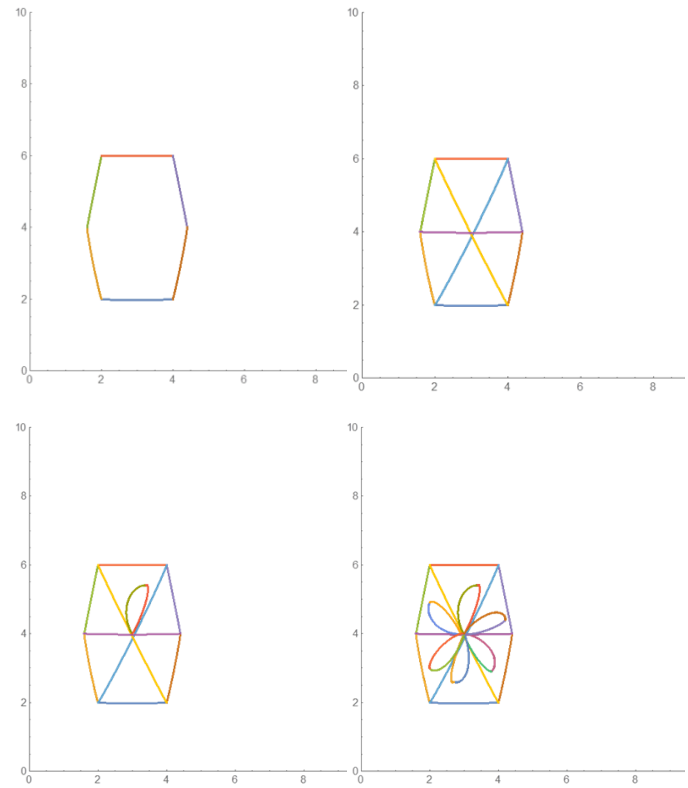

Figure. 9: The quadratic, trigonometric Bezier-like curve with different shape parameters (flower).

Example 2:

In this font design example, the representation of the Arabic word "الله" (Allah) will be illustrated using a quadratic trigonometric Bezier-like curve governed by equations (1). Though MATHEMATICA software will be relied on for this demonstration, our goal is to approximate the drawing of the Arabic word **"الله"** to resemble the original one shown in Figurer **11**. This creative endeavour allows us to craft a visually captivating and accurate rendition of the word, paying homage to its cultural and linguistic significance.

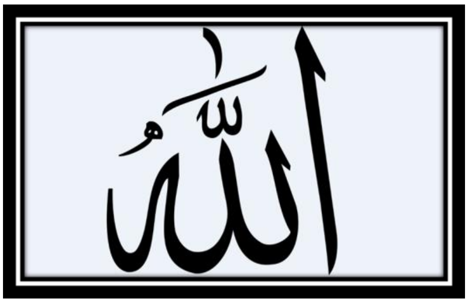

Figure. 10: The original Arabic Word (الله).

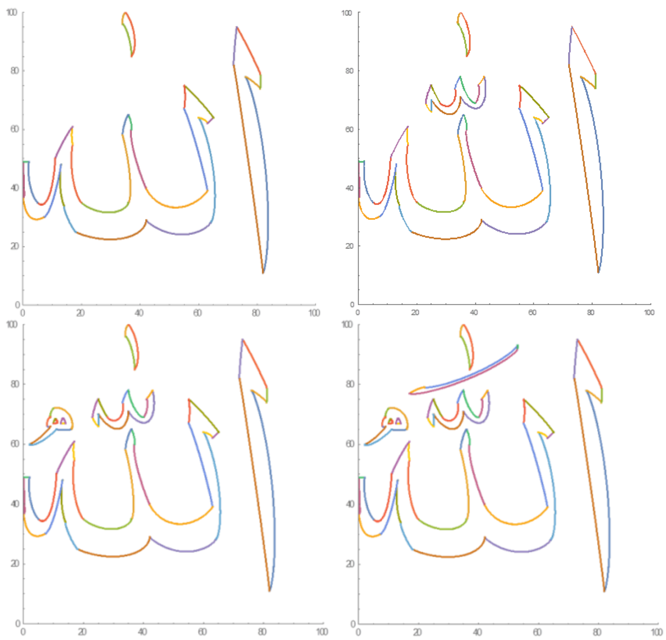

Figure. 11: The quadratic, trigonometric Bezier-like curve with different shape parameters .

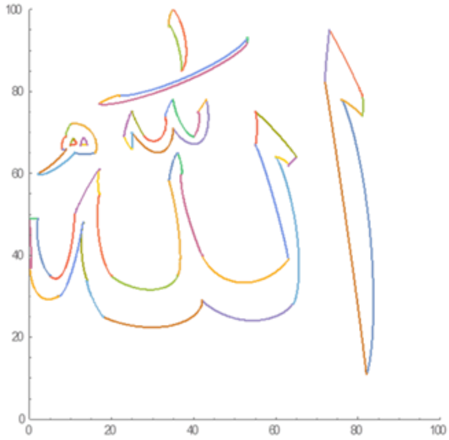

Figure. 12: The Allah (الله) word through quadratic trigonometric Bezier like curve.

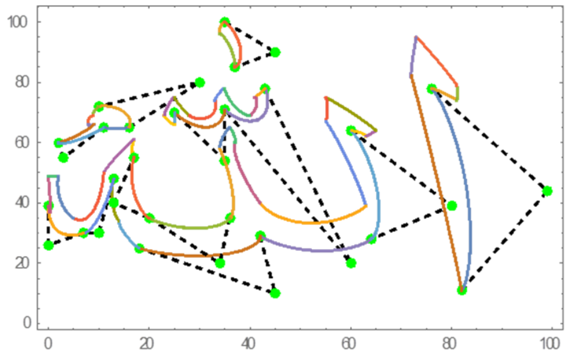

Figure. 13: The Allah (الله) word through quadratic trigonometric Bezier like curve with control polygon.

In the first example, the focus was on the quadratic trigonometric Bezier-like curve, a powerful design tool that combines quadratic and linear polynomial basis functions. The control points and shape parameters played a crucial role in shaping the curve, allowing for precise adjustments to achieve different curve configurations. Additionally, the flexibility of this curve type was showcased through its ability to change its fullness by adjusting α and β by the same value, providing designers with a versatile and efficient tool for curve manipulation.

In the second example, the design of an Arabic font using MATHEMATICA software demonstrated the application of quadratic trigonometric Bezier-like curves. By skillfully adding, moving, and adjusting control points, as well as manipulating shape parameters, designers were able to create intricate and visually captivating Arabic fonts. The use of this curve type allowed for precise control over the font's appearance, enabling the faithful representation of the word "الله" (Allah) while honoring the original design.

Both examples exemplify the power and versatility of quadratic trigonometric Bezier-like curves in design and visualization. These curves offer a robust foundation for generating smooth and aesthetically pleasing shapes, making them indispensable tools in various applications, including font designing, curve representation, and geometric modeling. The control points and shape parameters provide designers with fine-tuned control over the curves, enabling the creation of complex shapes with ease and accuracy. Moreover, the integration of MATHEMATICA software in the design process emphasizes the significance of computational tools in modern design practices, facilitating precise and efficient curve manipulation.

The examples highlight the importance of innovative curve representations and their applications in design. The quadratic trigonometric Bezier-like curves, with their combination of quadratic and linear polynomial basis functions, offer a powerful and flexible approach to curve design. Whether it is designing Arabic fonts or creating complex geometric shapes, these curves empower designers to achieve visually appealing and sophisticated results. Their widespread applications in various domains underscore their significance as indispensable tools in the field of computer graphics and design.

6. The discussion

The examples had illustrated the potency of quadratic trigonometric Bezier-like curves, effectively merging quadratic and linear polynomial basis functions to allow precise curve manipulation. In the initial instance, we had showcased the curve's adaptability by altering control points and shape parameters, revealing its capacity to achieve diverse configurations. The second example had demonstrated the curve's effectiveness in faithfully representing intricate Arabic fonts while maintaining flexibility and precision through control points and shape parameter adjustments. These results had affirmed that quadratic trigonometric Bezier-like curves are indispensable tools, providing fine-tuned control for creating aesthetically appealing designs across various applications like font design and geometric modelling.

Conclusion

Our primary focus in this study is to devise new functions using the quadratic trigonometric Bezier-like curve, with the aim of designing the Arabic word "الله" (Allah) and creating an outface of a flower. We have meticulously demonstrated and validated all properties of these new functions, manually verifying their geometric coherence through the use of MATHEMATICA software.

The results of our study reveal that fonts that closely resemble the original ones can be achieved by leveraging the quadratic trigonometric Bezier-like curve. Furthermore, by employing different Bezier-like curves and diverse functions of Bezier-like functions, along with varying control points and shape parameters, the same font with intriguing variations in shape and appearance can be obtained. The flexibility of this approach is evident, as the middle control point is not specifically defined. This allows for the generation of diverse curves with the same start and end points by merely adjusting the shape parameters. Our findings indicate that all derived functions yield visually pleasing and aesthetically satisfying shapes. The successful application of quadratic trigonometric Bezier-like curves in designing fonts and flower outfaces underscores their efficacy as versatile and efficient tools in the realm of computer graphics and geometric modeling. The comprehensive analysis, along with the manual and geometric validation, reinforces the significance of these newly derived functions, offering valuable insights into their creative potential and broad applications.

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